In this example, we work on a low-noise microwave amplifier based on a M-Pulse Microwave MP42141 silicon npn BJT low noise microwave transistor operating at 2 GHz with V_{CE} = 10 V and I_C = 5 mA.

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Transistor info:

$Z0 := 50\ \Omega$ $\quad FdB := 3.4$ dB (typical) $\quad Rn := 4\ \Omega$ $\quad \Gamma opt := 0.5\cdot e^{j\cdot 160\cdot\frac{\pi}{180}}$

$F := 10^{\frac{FdB}{10}}$ $\quad F = 2.188$ $\quad rn := \frac{Rn}{Z0}$ $\quad rn = 0.08$

$S11 := 0.561\cdot e^{j\cdot 166.2\cdot\frac{\pi}{180}}$ $\quad S12 := 0.078\cdot e^{j\cdot 32.6\cdot\frac{\pi}{180}}$

$S21 := 1.894\cdot e^{j\cdot 43.2\cdot\frac{\pi}{180}}$ $\quad S22 := 0.571\cdot e^{j\cdot -63.6\cdot\frac{\pi}{180}}$

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Is the transistor stable at 2 GHz?

$\Delta := S11\cdot S22 - S12\cdot S21$ $\quad \boxed{|\Delta| = 0.19989}$ $\quad$ < 1, good

$$K := \frac{1 - (|S11|)^2 - (|S22|)^2 + (|\Delta|)^2}{2\,|S12\cdot S21|}$$ $\quad \boxed{K = 1.3511}$ $\quad$ > 1, good

Yes, stability Conditions ARE met at 2 GHz.

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Since the transistor is unconditionally stable at 2 GHz, we do NOT need to bother with calculating the input or output stability circles.

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The unilateral constant-gain circles should NOT apply since $|S_{21}| = 0.078$. However, let's check on the unilateral figure of merit to see if the unilateral case might be useful.

$$U := \frac{|S12|\cdot|S21|\cdot|S11|\cdot|S22|}{\left[1-(|S11|)^2\right]\left[1-(|S22|)^2\right]}$$ $U = 0.102464$

$$\frac{1}{(1+U)^2} = 0.82276$$ $$\frac{1}{(1-U)^2} = 1.24136$$

$$10\cdot\log\left[\frac{1}{(1+U)^2}\right] = -0.84729 \quad \text{dB}$$ $$10\cdot\log\left[\frac{1}{(1-U)^2}\right] = 0.93897 \quad \text{dB}$$

Unilateral design will <u>NOT</u> be very useful.

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The next possibility to examine is whether a simultaneous conjugate match (bilateral case) is viable for both good gain ($G_{T,max}$), VSWR, and low noise. Since we know the transistor is unconditionall stable, we'll start by calculating $\Gamma_{MS} = \Gamma_{IN}^*$ and $\Gamma_{ML} = \Gamma_{OUT}^*$, choosing the "-" solutions.

$$B1 := 1 + (|S11|)^2 - (|S22|)^2 - (|\Delta|)^2$$ $B1 = 0.949$

$$B2 := 1 + (|S22|)^2 - (|S11|)^2 - (|\Delta|)^2$$ $B2 = 0.971$

$$C1 := S11 - \Delta\cdot\overline{S22}$$ $|C1| = 0.455$ $\quad \arg(C1)\cdot\frac{180}{\pi} = 161.4$ deg

$$C2 := S22 - \Delta\cdot\overline{S11}$$ $|C2| = 0.467$ $\quad \arg(C2)\cdot\frac{180}{\pi} = -68.2$ deg

$$\Gamma MS := \frac{B1 - \sqrt{B1^2 - 4\cdot(|C1|)^2}}{2\cdot C1}$$ $|\Gamma MS| = 0.74761$ $\quad \arg(\Gamma MS)\cdot\frac{180}{\pi} = -161.4$ deg

$$\Gamma ML := \frac{B2 - \sqrt{B2^2 - 4\cdot(|C2|)^2}}{2\cdot C2}$$ $|\Gamma ML| = 0.75297$ $\quad \arg(\Gamma ML)\cdot\frac{180}{\pi} = 68.2$ deg

How does Γ_{MS} do for noise performance? First, plot it on a Smith chart. Then, calculate and plot noise figure circles (in black) for F = 3.5, 3.7, & 4 dB.

$F35 := 10^{\frac{3.5}{10}}$ $F35 = 2.239$

$N35 := \frac{F35 - F}{4 \cdot rn} \cdot \left(\left|1 + \Gamma opt\right|\right)^2$ $N35 = 0.049$

$CF35 := \frac{\Gamma opt}{1 + N35}$ $\left|CF35\right| = 0.47646$ $\arg(CF35) \cdot \frac{180}{\pi} = 160$ deg

$rF35 := \frac{1}{1 + N35} \cdot \sqrt{N35^2 + N35 \cdot \left[1 + \left(\left|\Gamma opt\right|\right)^2\right]}$ $rF35 = 0.2415$

$F37 := 10^{\frac{3.7}{10}}$ $F37 = 2.344$

$N37 := \frac{F37 - F}{4 \cdot rn} \cdot \left(\left|1 + \Gamma opt\right|\right)^2$ $N37 = 0.152$

$CF37 := \frac{\Gamma opt}{1 + N37}$ $\left|CF37\right| = 0.4341$ $\arg(CF37) \cdot \frac{180}{\pi} = 160$ deg

$rF37 := \frac{1}{1 + N37} \cdot \sqrt{N37^2 + N37 \cdot \left[1 + \left(\left|\Gamma opt\right|\right)^2\right]}$ $rF37 = 0.4004$

$F40 := 10^{\frac{4}{10}}$ $F40 = 2.512$

$N40 := \frac{F40 - F}{4 \cdot rn} \cdot \left(\left|1 + \Gamma opt\right|\right)^2$ $N40 = 0.314$

$CF40 := \frac{\Gamma opt}{1 + N40}$ $\left|CF40\right| = 0.3804$ $\arg(CF40) \cdot \frac{180}{\pi} = 160$ deg

$rF40 := \frac{1}{1 + N40} \cdot \sqrt{N40^2 + N40 \cdot \left[1 + \left(\left|\Gamma opt\right|\right)^2\right]}$ $rF40 = 0.5335$

As can be seen on the Γ_S Smith chart, the Γ_{MS} point is inside the F = 4 dB noise figure circle. An exact calculation of F_{MS} is shown below.

$$FMS := F + \frac{4\cdot rn\cdot\left(\left|\Gamma MS - \Gamma opt\right|\right)^2}{\left[1 - \left(\left|\Gamma MS\right|\right)^2\right]\cdot\left[1 - \left(\left|\Gamma opt\right|\right)^2\right]}$$

$FMS = 2.405$

$10\cdot\log(FMS) = 3.811$ dB

So, F_{MS} is 0.411 dB above F_{min} = 3.4 dB. This might be perfectly acceptable.

How did we do for gain and VSWRs at the simultaneous complex conjugate match points?

$$GTmax := \frac{1}{1 - \left(\left|\Gamma MS\right|\right)^2}\cdot\left(\left|S21\right|\right)^2\cdot\frac{\left[1 - \left(\left|\Gamma ML\right|\right)^2\right]}{\left(\left|1 - S22\cdot\Gamma ML\right|\right)^2}$$

$GTmax = 10.746$ $\quad 10\cdot\log(GTmax) = 10.3125$ dB

For comparison, the maximum stable (transducer) gain for the transistor is:

$$G\_MSG := \frac{|S21|}{|S12|}$$

$G\_MSG = 24.282$ $\quad 10\cdot\log(G\_MSG) = 13.853$ dB

$$\Gamma IN := S11 + \frac{S12\cdot S21\cdot\Gamma ML}{1 - S22\cdot\Gamma ML}$$

$\left|\Gamma IN\right| = 0.7476$ $\quad \arg(\Gamma IN)\cdot\frac{180}{\pi} = 161.405$ deg

$$\Gamma OUT := S22 + \frac{S12\cdot S21\cdot\Gamma MS}{1 - S11\cdot\Gamma MS}$$

$\left|\Gamma OUT\right| = 0.753$ $\quad \arg(\Gamma OUT)\cdot\frac{180}{\pi} = -68.19$ deg

$$\Gamma a\_mag := \left|\frac{\Gamma IN - \overline{\Gamma MS}}{1 - \Gamma IN\cdot\Gamma MS}\right| \qquad VSWR\_IN := \frac{1 + \Gamma a\_mag}{1 - \Gamma a\_mag}$$

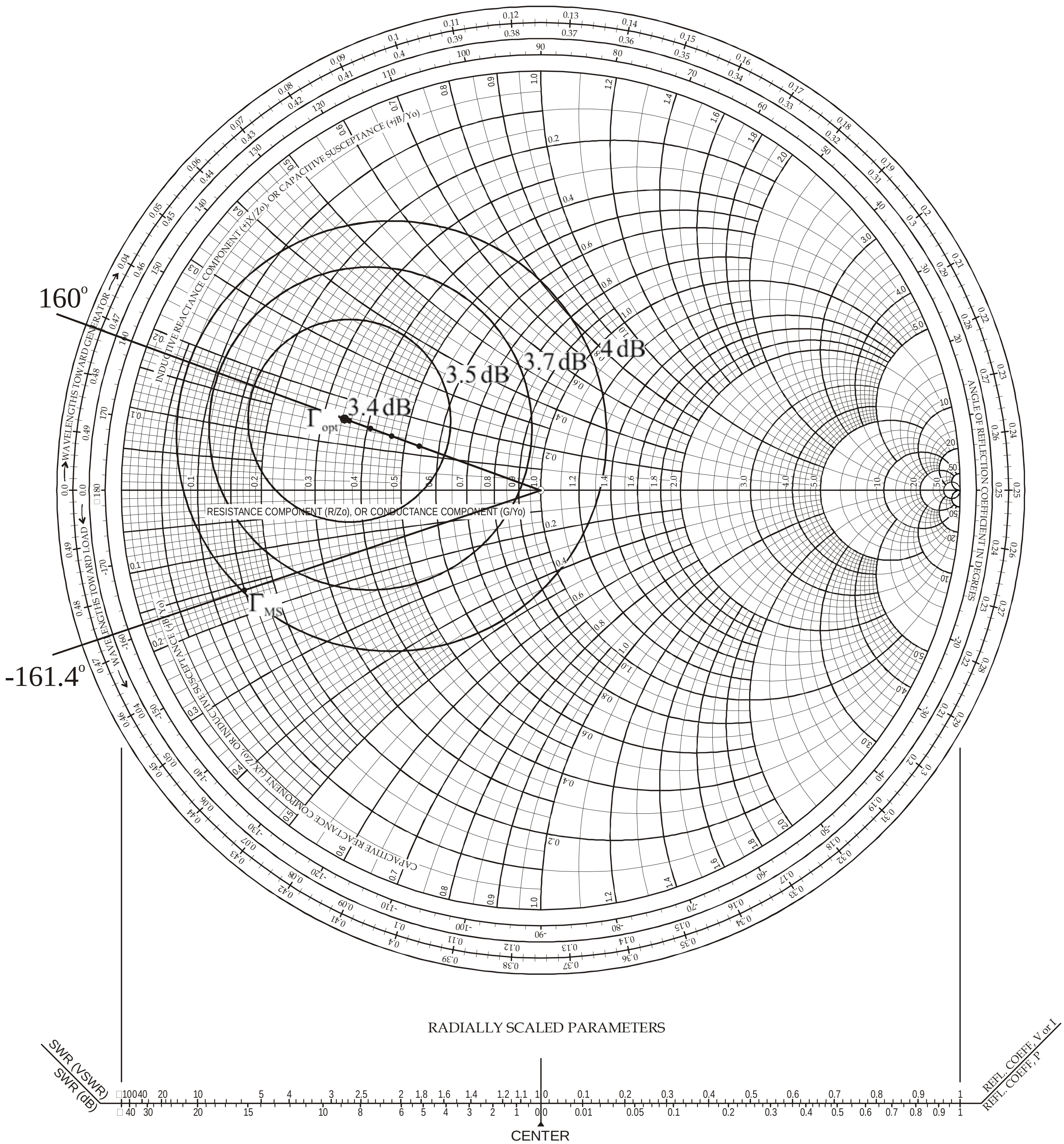
$VSWR\_IN = 1$

$$\Gamma b\_mag := \left|\frac{\Gamma OUT - \overline{\Gamma ML}}{1 - \Gamma OUT\cdot\Gamma ML}\right| \qquad VSWR\_OUT := \frac{1 + \Gamma b\_mag}{1 - \Gamma b\_mag}$$

$VSWR\_OUT = 1$

The gains ($G_{Tmax} = G_{Pmax} = G_{Amax}$) are a little above 10 dB and about 3.5 dB below the maximum stable gain. As expected, $VSWR_{OUT} = VSWR_{IN}$ = **1** at the simultaneous complex conjugate match points.

Γ_S Smith Chart



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How much gain and VSWR would we need to give up to achieve a lower noise figure?

Since we are working on the Γ_S Smith chart, let's plot circles of constant available power gain (in blue) for G_A = 9.4, 9.7 & 10 dB.

$$GA10 := 10^{\frac{10}{10}} \qquad ga10 := \frac{GA10}{(|S21|)^2} \qquad GA10 = 10 \qquad ga10 = 2.788$$

$$GA97 := 10^{\frac{9.7}{10}} \qquad ga97 := \frac{GA97}{(|S21|)^2} \qquad GA97 = 9.333 \qquad ga97 = 2.602$$

$$GA94 := 10^{\frac{9.4}{10}} \qquad ga94 := \frac{GA94}{(|S21|)^2} \qquad GA94 = 8.71 \qquad ga94 = 2.428$$

$$Ca10 := \frac{ga10\cdot\overline{C1}}{1 + ga10\cdot\left[(|S11|)^2 - (|\Delta|)^2\right]} \qquad Ca97 := \frac{ga97\cdot\overline{C1}}{1 + ga97\cdot\left[(|S11|)^2 - (|\Delta|)^2\right]}$$

$$Ca94 := \frac{ga94\cdot\overline{C1}}{1 + ga94\cdot\left[(|S11|)^2 - (|\Delta|)^2\right]}$$

$$ra10 := \frac{\left[1 - 2\cdot K\cdot|S12\cdot S21|\cdot ga10 + (|S12\cdot S21|)^2\cdot ga10^2\right]^{0.5}}{\left|1 + ga10\cdot\left[(|S11|)^2 - (|\Delta|)^2\right]\right|}$$

$$ra97 := \frac{\left[1 - 2\cdot K\cdot|S12\cdot S21|\cdot ga97 + (|S12\cdot S21|)^2\cdot ga97^2\right]^{0.5}}{\left|1 + ga97\cdot\left[(|S11|)^2 - (|\Delta|)^2\right]\right|}$$

$$ra94 := \frac{\left[1 - 2\cdot K\cdot|S12\cdot S21|\cdot ga94 + (|S12\cdot S21|)^2\cdot ga94^2\right]^{0.5}}{\left|1 + ga94\cdot\left[(|S11|)^2 - (|\Delta|)^2\right]\right|}$$

$|Ca10| = 0.71821$ $\quad \arg(Ca10)\cdot\frac{180}{\pi} = -161.4$ deg $\quad ra10 = 0.13494$

$|Ca97| = 0.69026$ $\quad \arg(Ca97)\cdot\frac{180}{\pi} = -161.4$ deg $\quad ra97 = 0.19268$

$|Ca94| = 0.66262$ $\quad \arg(Ca94)\cdot\frac{180}{\pi} = -161.4$ deg $\quad ra94 = 0.23951$

Next, since we are working on the Γ_S Smith chart, let's plot circles of constant input VSWR (in red) for $VSWR_{IN}$ = 1.5, 1.75 & 2.

$$\Gamma a15 := \frac{1.5-1}{1.5+1} \qquad \Gamma a15 = 0.2 \qquad \Gamma a175 := \frac{1.75-1}{1.75+1} \qquad \Gamma a175 = 0.2727$$

$$\Gamma a2 := \frac{2-1}{2+1} \qquad \Gamma a2 = 0.3333$$

$$Cvi15 := \frac{\overline{\Gamma IN}\cdot\left(1-\Gamma a15^2\right)}{1-\Gamma a15^2\cdot\left(|\Gamma IN|\right)^2} \qquad rvi15 := \frac{\Gamma a15\cdot\left[1-\left(|\Gamma IN|\right)^2\right]}{1-\Gamma a15^2\cdot\left(|\Gamma IN|\right)^2}$$

$$Cvi175 := \frac{\overline{\Gamma IN}\cdot\left(1-\Gamma a175^2\right)}{1-\Gamma a175^2\cdot\left(|\Gamma IN|\right)^2} \qquad rvi175 := \frac{\Gamma a175\cdot\left[1-\left(|\Gamma IN|\right)^2\right]}{1-\Gamma a175^2\cdot\left(|\Gamma IN|\right)^2}$$

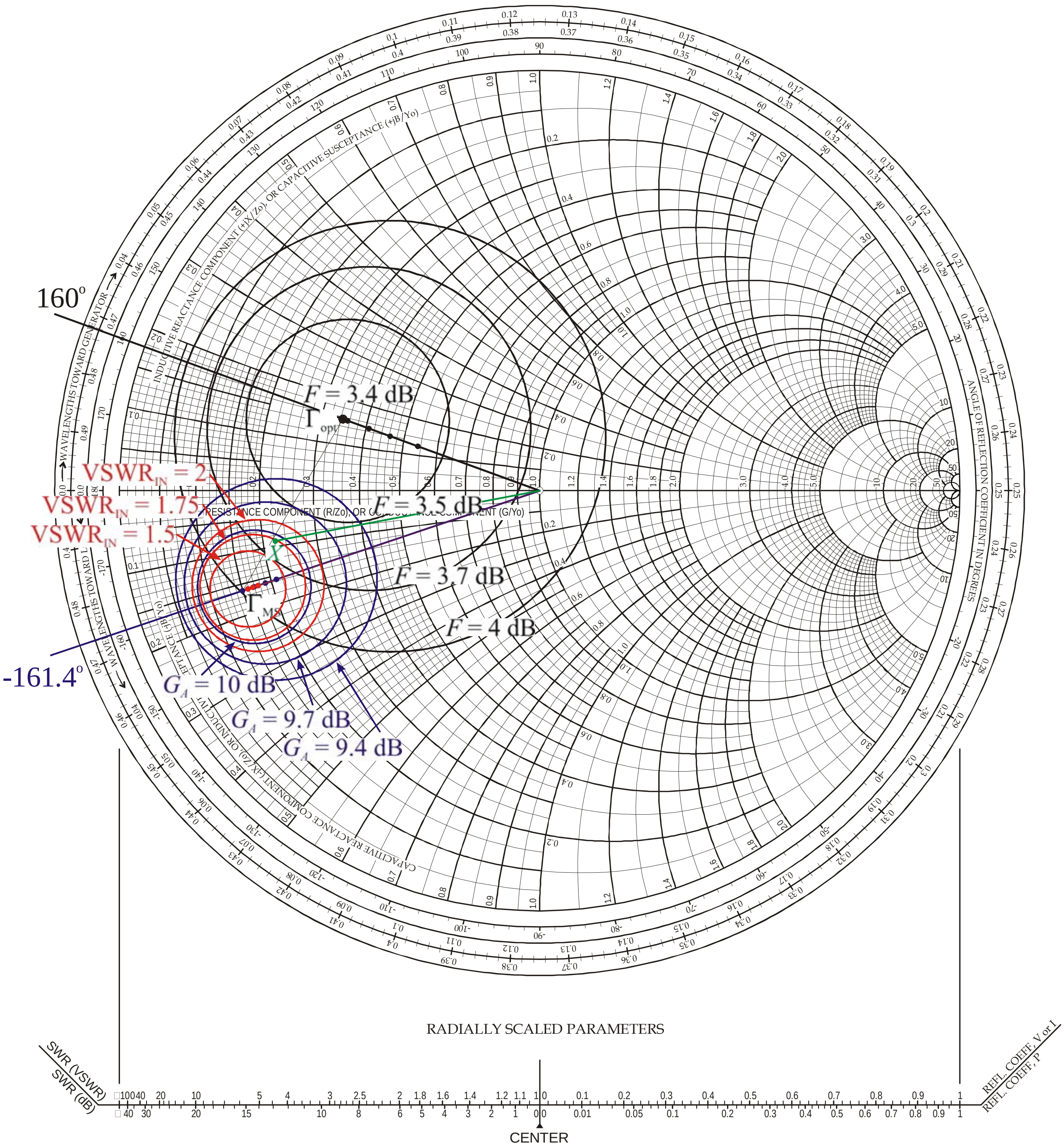
$$Cvi2 := \frac{\overline{\Gamma IN}\cdot\left(1-\Gamma a2^2\right)}{1-\Gamma a2^2\cdot\left(|\Gamma IN|\right)^2} \qquad rvi2 := \frac{\Gamma a2\cdot\left[1-\left(|\Gamma IN|\right)^2\right]}{1-\Gamma a2^2\cdot\left(|\Gamma IN|\right)^2}$$

$|Cvi15| = 0.73411$ $\quad \arg(Cvi15)\cdot\frac{180}{\pi} = -161.4$ deg $\quad rvi15 = 0.0902$

$|Cvi175| = 0.72201$ $\quad \arg(Cvi175)\cdot\frac{180}{\pi} = -161.4$ deg $\quad rvi175 = 0.1255$

$|Cvi2| = 0.70854$ $\quad \arg(Cvi2)\cdot\frac{180}{\pi} = -161.4$ deg $\quad rvi2 = 0.1568$

Γ_S Smith Chart



On examining the F (black), G_A (blue) and VSWR_{IN} (red) circles on the Γ_S Smith chart, let's select the point X (green) as a potential compromise and see how it works out for gains, VSWRs, and noise figure.

$$zSX := 0.22 - j\cdot 0.09 \qquad \Gamma SX := \frac{zSX - 1}{zSX + 1}$$

$$|\Gamma SX| = 0.6418 \qquad \arg(\Gamma SX)\cdot\frac{180}{\pi} = -169.2 \text{ deg}$$

$$\Gamma OUTX := S22 + \frac{S12\cdot S21\cdot \Gamma SX}{1 - S11\cdot \Gamma SX} \qquad |\Gamma OUTX| = 0.701 \qquad \arg(\Gamma OUTX)\cdot\frac{180}{\pi} = -69.9 \text{ deg}$$

Choose to complex conjugate match Γ_L to Γ_{OUTX} for best VSWR_{OUT}.

$$\Gamma LX := \overline{\Gamma OUTX} \qquad |\Gamma LX| = 0.7015 \qquad \arg(\Gamma LX)\cdot\frac{180}{\pi} = 69.93 \text{ deg}$$

$$\Gamma INX := S11 + \frac{S12\cdot S21\cdot \Gamma LX}{1 - S22\cdot \Gamma LX} \qquad |\Gamma INX| = 0.727 \qquad \arg(\Gamma INX)\cdot\frac{180}{\pi} = 162.4 \text{ deg}$$

$$\Gamma aX := \left|\frac{\Gamma INX - \overline{\Gamma SX}}{1 - \Gamma INX\cdot \Gamma SX}\right| \qquad \Gamma aX = 0.218 \qquad VSWR\_INX := \frac{1 + \Gamma aX}{1 - \Gamma aX}$$

$$\Gamma bX := \left|\frac{\Gamma OUTX - \overline{\Gamma LX}}{1 - \Gamma OUTX\cdot \Gamma LX}\right| \qquad \Gamma bX = 0 \qquad VSWR\_OUTX := \frac{1 + \Gamma bX}{1 - \Gamma bX}$$

$$VSWR\_INX = 1.559 \qquad VSWR\_OUTX = 1$$

The input VSWR is now 1.559 (corresponds to a reflection coefficient magnitude of 0.218) which is not too bad while the output VSWR is still 1.

$$GTX := \frac{1-(|\Gamma SX|)^2}{(|1-\Gamma INX\cdot\Gamma SX|)^2}\cdot(|S21|)^2\cdot\frac{1-(|\Gamma LX|)^2}{(|1-S22\cdot\Gamma LX|)^2}$$

$|GTX| = 10.1172$

$10\cdot\log(|GTX|) = 10.051$ dB

$$GPX := \frac{1}{1-(|\Gamma INX|)^2}\cdot(|S21|)^2\cdot\frac{1-(|\Gamma LX|)^2}{(|1-S22\cdot\Gamma LX|)^2}$$

$|GPX| = 10.6244$

$10\cdot\log(|GPX|) = 10.263$ dB

$$GAX := \frac{1-(|\Gamma SX|)^2}{(|1-S11\cdot\Gamma SX|)^2}\cdot(|S21|)^2\cdot\frac{1}{1-(|\Gamma OUTX|)^2}$$

$|GAX| = 10.1172$

$10\cdot\log(|GAX|) = 10.051$ dB

The gains are only slightly below the the maximum transducer gain of 10.3125 dB.

$$FX := F + \frac{4\cdot rn\cdot(|\Gamma SX - \Gamma opt|)^2}{\left[1-(|\Gamma SX|)^2\right]\cdot\left[1-(|\Gamma opt|)^2\right]}$$

$FX = 2.268$

$10\cdot\log(FX) = 3.557$ dB

The noise figure is only slightly above the minimum noise figure of 3.4 dB.

Not too bad of a compromise. Still have very good gains, VSWRs, and only a small degradation in the noise figure *F*.