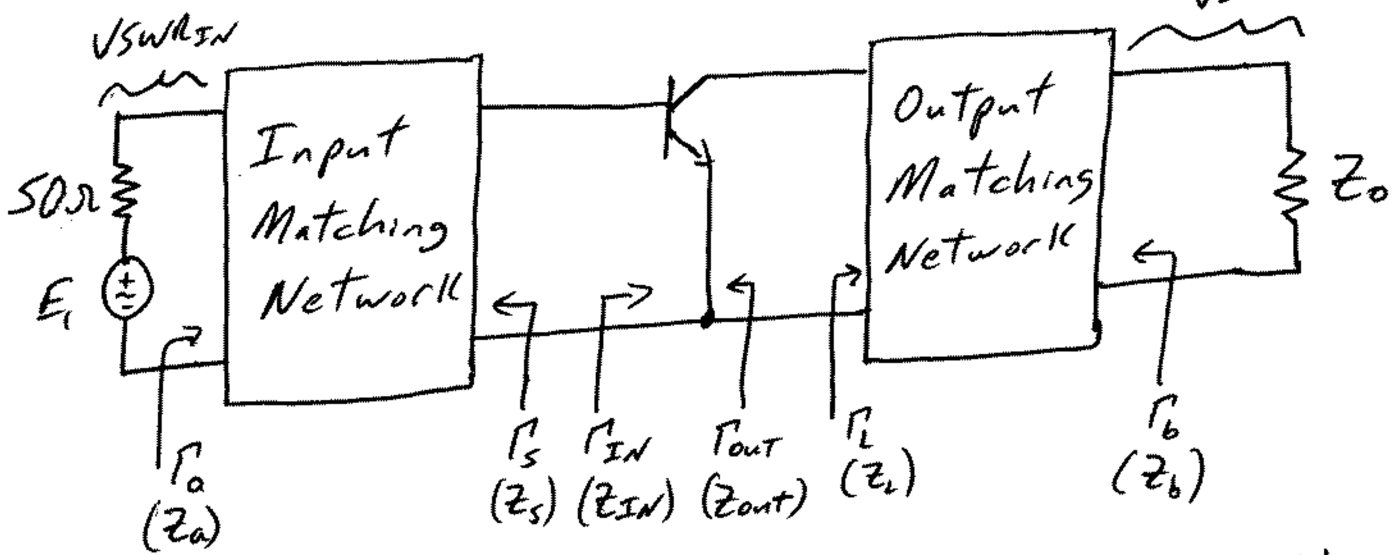
3.8 Constant VSWR Circles

* Maximum $VSWR_{IN}$ and $VSWR_{OUT}$ are usually part of amplifier design specifications.

* Can draw circles of constant $VSWR_{IN}$ on Γ_s Smith Chart (or Γ_s plane).

* Can draw circles of constant $VSWR_{OUT}$ on Γ_L Smith Chart (or Γ_L plane).



$$VSWR_{IN} = \frac{1+|\Gamma_a|}{1-|\Gamma_a|} \qquad VSWR_{OUT} = \frac{1+|\Gamma_b|}{1-|\Gamma_b|}$$

$$|\Gamma_a| = \left| \frac{\Gamma_{IN} - \Gamma_s^*}{1-\Gamma_{IN}\Gamma_s} \right| \qquad |\Gamma_b| = \left| \frac{\Gamma_{OUT} - \Gamma_L^*}{1-\Gamma_{OUT}\Gamma_L} \right|$$

Γ_a & Γ_s related by bilinear transformation ⇓ Circle on Γ_s plane

Γ_L & Γ_b related by bilinear transformation ⇓ Circle on Γ_L plane

3.8 cont.

$VSWR_{IN}$ (or $|\Gamma_a|$) Circle on Γ_s plane

Circle equation $|\Gamma_s - C_{V_i}| = r_{V_i}$
for $VSWR_{IN}$

Center of $VSWR_{IN}$ Circle $\equiv C_{V_i} = \dfrac{\Gamma_{IN}^* (1 - |\Gamma_a|^2)}{1 - |\Gamma_a \Gamma_{IN}|^2}$

$= \dfrac{\Gamma_{IN}^* (1 - |\Gamma_a|^2)}{1 - |\Gamma_a|^2 |\Gamma_{IN}|^2}$ ← complex #

Radius of $VSWR_{IN}$ Circle $\equiv r_{V_i} = \dfrac{|\Gamma_a| (1 - |\Gamma_{IN}|^2)}{1 - |\Gamma_a \Gamma_{IN}|^2}$

$= \dfrac{|\Gamma_a| (1 - |\Gamma_{IN}|^2)}{1 - |\Gamma_a|^2 |\Gamma_{IN}|^2}$ ← Real #

How do we get $|\Gamma_a|$?

For a given $VSWR_{IN}$, $|\Gamma_a| = \dfrac{VSWR_{IN} - 1}{VSWR_{IN} + 1}$

What about $VSWR_{IN} = 1$? (a point)

↳ $\Gamma_s = \Gamma_{IN}^*$ and $|\Gamma_a| = 0$

So $C_{V_i}\big|_{|\Gamma_a|=0} = \Gamma_{IN}^*$

$r_{V_i}\big|_{|\Gamma_a|=0} = 0$

} $VSWR_{IN}$ circle reduces to a point at $\Gamma_s = \Gamma_{IN}^*$

3.8 cont.

$VSWR_{OUT}$ (or $|\Gamma_b|$) Circle on Γ_L plane

Circle Equation for $VSWR_{OUT}$ $\quad |\Gamma_L - C_{V_o}| = r_{V_o}$

Center of $VSWR_{OUT}$ Circle $\equiv C_{V_o} = \dfrac{\Gamma_{OUT}^* (1-|\Gamma_b|^2)}{1-|\Gamma_b \Gamma_{OUT}|^2}$

$$= \frac{\Gamma_{OUT}^* (1-|\Gamma_b|)}{1-|\Gamma_b|^2|\Gamma_{OUT}|^2} \leftarrow \text{complex \#}$$

Radius of $VSWR_{OUT}$ Circle $\equiv r_{V_o} = \dfrac{|\Gamma_b|(1-|\Gamma_{OUT}|^2)}{1-|\Gamma_b \Gamma_{OUT}|^2}$

$$= \frac{|\Gamma_b|(1-|\Gamma_{OUT}|^2)}{1-|\Gamma_b|^2|\Gamma_{OUT}|^2}$$

How do we get $|\Gamma_b|$?

For a given (selected) $VSWR_{OUT}$, $|\Gamma_b| = \dfrac{VSWR_{OUT}-1}{VSWR_{OUT}+1}$

What about $VSWR_{OUT} = 1$?

↳ $\Gamma_L = \Gamma_{OUT}^*$ and $|\Gamma_b| = 0$

So $\quad C_{V_o}\big|_{|\Gamma_b|=0} = \Gamma_{OUT}^*$

$\quad r_{V_o}\big|_{|\Gamma_b|=0} = 0$

} $VSWR_{OUT}$ circle reduces to a point at $\Gamma_L = \Gamma_{OUT}^*$

VSWR Circles applied to the unconditionally stable modified MLN2037F RF power transistor at 1 GHz where G_P = 10 dB.

In the prior example, we achieved a $G_p = G_T$ = 10 dB gain with $VSWR_{IN}$ = 1 (let $\Gamma_S = \Gamma_{IN}^*$) but $VSWR_{OUT}$ = 2.672. Here, we'll try reducing $VSWR_{OUT}$ to < 2 by allowing for a higher $VSWR_{IN}$ (new Γ_S).

$Z0 := 50 \quad \Omega$

$S11 := 0.8128 \cdot e^{j \cdot 156 \cdot \frac{\pi}{180}}$ $\qquad S12m := 0.08 \cdot e^{j \cdot 63 \cdot \frac{\pi}{180}}$

$S21 := 1.81 \cdot e^{j \cdot 55 \cdot \frac{\pi}{180}}$ $\qquad S22 := 0.3801 \cdot e^{j \cdot -136 \cdot \frac{\pi}{180}}$

$\Delta := S11 \cdot S22 - S12m \cdot S21$ $\qquad \boxed{|\Delta| = 0.35898}$ < 1, good

$$K := \frac{1 - (|S11|)^2 - (|S22|)^2 + (|\Delta|)^2}{2\,|S12m \cdot S21|}$$ $\qquad \boxed{K = 1.1179}$ > 1, good

Stability Conditions ARE met at 1 GHz.

$C2 := S22 - \Delta \cdot \overline{S11}$ $\qquad |C2| = 0.162 \qquad \arg(C2) \cdot \frac{180}{\pi} = -90$ deg

$G\_MSG := \frac{|S21|}{|S12m|}$ $\qquad \boxed{G\_MSG = 22.625} \qquad \boxed{10 \cdot \log(G\_MSG) = 13.546}$ dB

Choose to plot GP for 10 dB. $\qquad GP := 10^{\frac{10}{10}} \qquad GP = 10$

$$gP := \frac{GP}{(|S21|)^2}$$ $\qquad gP = 3.052$

$$CP := \frac{gP \cdot \overline{C2}}{1 + gP \cdot \left[(|S22|)^2 - (|\Delta|)^2 \right]}$$ $\qquad \boxed{|CP| = 0.472} \qquad \boxed{\arg(CP) \cdot \frac{180}{\pi} = 90}$ deg

$$rP := \frac{\left[1 - 2 \cdot K \cdot |S12m \cdot S21| \cdot gP + (|S12m \cdot S21|)^2 \cdot gP^2 \right]^{0.5}}{\left| 1 + gP \cdot \left[(|S22|)^2 - (|\Delta|)^2 \right] \right|}$$ $\qquad \boxed{rP = 0.434}$

Plot G_P = 10 dB circle on Γ_L Smith Chart.

Again select Point A on the Γ_L Smith Chart. Here the normalized impedance $z_L = 1.6 + j0.4$ corresponds to $\Gamma_L = 0.274 / 24.94^o$.

$zL := 1.6 + j\cdot 0.4$ $\quad \Gamma L := \frac{zL-1}{zL+1}$ $\quad |\Gamma L| = 0.274$ $\quad \arg(\Gamma L)\cdot\frac{180}{\pi} = 24.944$ deg

For maximum output power, select $\Gamma_S = \Gamma_{IN}^*$. Check input & output VSWR.

$\Gamma IN := S11 + \frac{S12m\cdot S21\cdot\Gamma L}{1 - S22\cdot\Gamma L}$ $\quad |\Gamma IN| = 0.849$ $\quad \arg(\Gamma IN)\cdot\frac{180}{\pi} = 155.188$ deg

$\Gamma S := \overline{\Gamma IN}$ $\quad |\Gamma S| = 0.849$ $\quad \arg(\Gamma S)\cdot\frac{180}{\pi} = -155.188$ deg

$\Gamma a\_mag := \left|\frac{\Gamma IN - \overline{\Gamma S}}{1 - \Gamma IN\cdot\Gamma S}\right|$ $\quad VSWR\_IN := \frac{1+\Gamma a\_mag}{1-\Gamma a\_mag}$ $\quad VSWR\_IN = 1$

$\Gamma OUT := S22 + \frac{S12m\cdot S21\cdot\Gamma S}{1 - S11\cdot\Gamma S}$ $\quad |\Gamma OUT| = 0.496$ $\quad \arg(\Gamma OUT)\cdot\frac{180}{\pi} = -84.24$ deg

$\Gamma b\_mag := \left|\frac{\Gamma OUT - \overline{\Gamma L}}{1 - \Gamma OUT\cdot\Gamma L}\right|$ $\quad VSWR\_OUT := \frac{1+\Gamma b\_mag}{1-\Gamma b\_mag}$ $\quad VSWR\_OUT = 2.672$

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Let's shoot for $VSWR_{IN}$ = **1.666** and see what we can get for $VSWR_{OUT}$.

$\Gamma a\_1666 := \frac{1.666-1}{1.666+1}$ $\quad \Gamma a\_1666 = 0.2498$

$$Cvi1666 := \frac{\overline{\Gamma IN}\cdot\left(1 - \Gamma a\_1666^2\right)}{1 - \Gamma a\_1666^2\cdot\left(|\Gamma IN|\right)^2} \qquad rvi1666 := \frac{\Gamma a\_1666\cdot\left[1 - \left(|\Gamma IN|\right)^2\right]}{1 - \Gamma a\_1666^2\cdot\left(|\Gamma IN|\right)^2}$$

$|Cvi1666| = 0.83354$ $\quad \arg(Cvi1666)\cdot\frac{180}{\pi} = -155.188$ deg $\quad rvi1666 = 0.073$

Plot $VSWR_{IN}$ = 1.666 circle on Γ_S Smith Chart along with $\Gamma_S = \Gamma_{IN}^*$ point.

Select Γ_S values on the $VSWR_{IN}$ = 1.666 circle at the points A-D (placed at 90° intervals on circle) and evaluate how the VSWRs change.

$$\Gamma SA := Cvi1666 + rvi1666 \cdot e^{j \cdot 0} \qquad \left|\Gamma SA\right| = 0.768 \qquad \arg(\Gamma SA) \cdot \frac{180}{\pi} = -152.901 \text{ deg}$$

$$\Gamma OUTA := S22 + \frac{S12m \cdot S21 \cdot \Gamma SA}{1 - S11 \cdot \Gamma SA} \qquad \left|\Gamma OUTA\right| = 0.41 \qquad \arg(\Gamma OUTA) \cdot \frac{180}{\pi} = -92.56 \text{ deg}$$

$$\Gamma aA := \left|\frac{\Gamma IN - \overline{\Gamma SA}}{1 - \Gamma IN \cdot \Gamma SA}\right| \qquad VSWR\_INA := \frac{1 + \Gamma aA}{1 - \Gamma aA} \qquad VSWR\_INA = 1.666$$

$$\Gamma bA := \left|\frac{\Gamma OUTA - \overline{\Gamma L}}{1 - \Gamma OUTA \cdot \Gamma L}\right| \qquad VSWR\_OUTA := \frac{1 + \Gamma bA}{1 - \Gamma bA} \qquad VSWR\_OUTA = 2.406$$

$$\Gamma SB := Cvi1666 + rvi1666 \cdot e^{j \cdot \frac{\pi}{2}} \qquad \left|\Gamma SB\right| = 0.806 \qquad \arg(\Gamma SB) \cdot \frac{180}{\pi} = -159.907 \text{ deg}$$

$$\Gamma OUTB := S22 + \frac{S12m \cdot S21 \cdot \Gamma SB}{1 - S11 \cdot \Gamma SB} \qquad \left|\Gamma OUTB\right| = 0.52 \qquad \arg(\Gamma OUTB) \cdot \frac{180}{\pi} = -96.13 \text{ deg}$$

$$\Gamma aB := \left|\frac{\Gamma IN - \overline{\Gamma SB}}{1 - \Gamma IN \cdot \Gamma SB}\right| \qquad VSWR\_INB := \frac{1 + \Gamma aB}{1 - \Gamma aB} \qquad VSWR\_INB = 1.666$$

$$\Gamma bB := \left|\frac{\Gamma OUTB - \overline{\Gamma L}}{1 - \Gamma OUTB \cdot \Gamma L}\right| \qquad VSWR\_OUTB := \frac{1 + \Gamma bB}{1 - \Gamma bB} \qquad VSWR\_OUTB = 3.188$$

$$\Gamma SC := Cvi1666 + rvi1666 \cdot e^{j \cdot \pi} \qquad \left|\Gamma SC\right| = 0.9 \qquad \arg(\Gamma SC) \cdot \frac{180}{\pi} = -157.138 \text{ deg}$$

$$\Gamma OUTC := S22 + \frac{S12m \cdot S21 \cdot \Gamma SC}{1 - S11 \cdot \Gamma SC} \qquad \left|\Gamma OUTC\right| = 0.6 \qquad \arg(\Gamma OUTC) \cdot \frac{180}{\pi} = -81.73 \text{ deg}$$

$$\Gamma aC := \left|\frac{\Gamma IN - \overline{\Gamma SC}}{1 - \Gamma IN \cdot \Gamma SC}\right| \qquad VSWR\_INC := \frac{1 + \Gamma aC}{1 - \Gamma aC} \qquad VSWR\_INC = 1.666$$

$$\Gamma bC := \left|\frac{\Gamma OUTC - \overline{\Gamma L}}{1 - \Gamma OUTC \cdot \Gamma L}\right| \qquad VSWR\_OUTC := \frac{1 + \Gamma bC}{1 - \Gamma bC} \qquad VSWR\_OUTC = 3.395$$

$\Gamma SD := Cvi1666 + rvi1666 \cdot e^{j \cdot 1.5\pi}$ $\quad \left|\Gamma SD\right| = 0.867 \quad \arg(\Gamma SD)\cdot\frac{180}{\pi} = -150.802$ deg

$\Gamma OUTD := S22 + \frac{S12m \cdot S21 \cdot \Gamma SD}{1 - S11 \cdot \Gamma SD}$ $\quad \left|\Gamma OUTD\right| = 0.42 \quad \arg(\Gamma OUTD)\cdot\frac{180}{\pi} = -74.81$ deg

$\Gamma aD := \left|\frac{\Gamma IN - \overline{\Gamma SD}}{1 - \Gamma IN \cdot \Gamma SD}\right|$ $\quad VSWR\_IND := \frac{1 + \Gamma aD}{1 - \Gamma aD} \quad VSWR\_IND = 1.666$

$\Gamma bD := \left|\frac{\Gamma OUTD - \overline{\Gamma L}}{1 - \Gamma OUTD \cdot \Gamma L}\right|$ $\quad VSWR\_OUTD := \frac{1 + \Gamma bD}{1 - \Gamma bD} \quad VSWR\_OUTD = 2.073$

The Γ_S values on the $VSWR_{IN} = 1.666$ circle near point D yielded the lowest $VSWR_{OUT}$. Now, iteratively search on either side of point D for the best Γ_S.

$\Gamma SE := Cvi1666 + rvi1666 \cdot e^{j \cdot 1.574\pi}$ $\quad \left|\Gamma SE\right| = 0.851 \quad \arg(\Gamma SE)\cdot\frac{180}{\pi} = -150.365$ deg

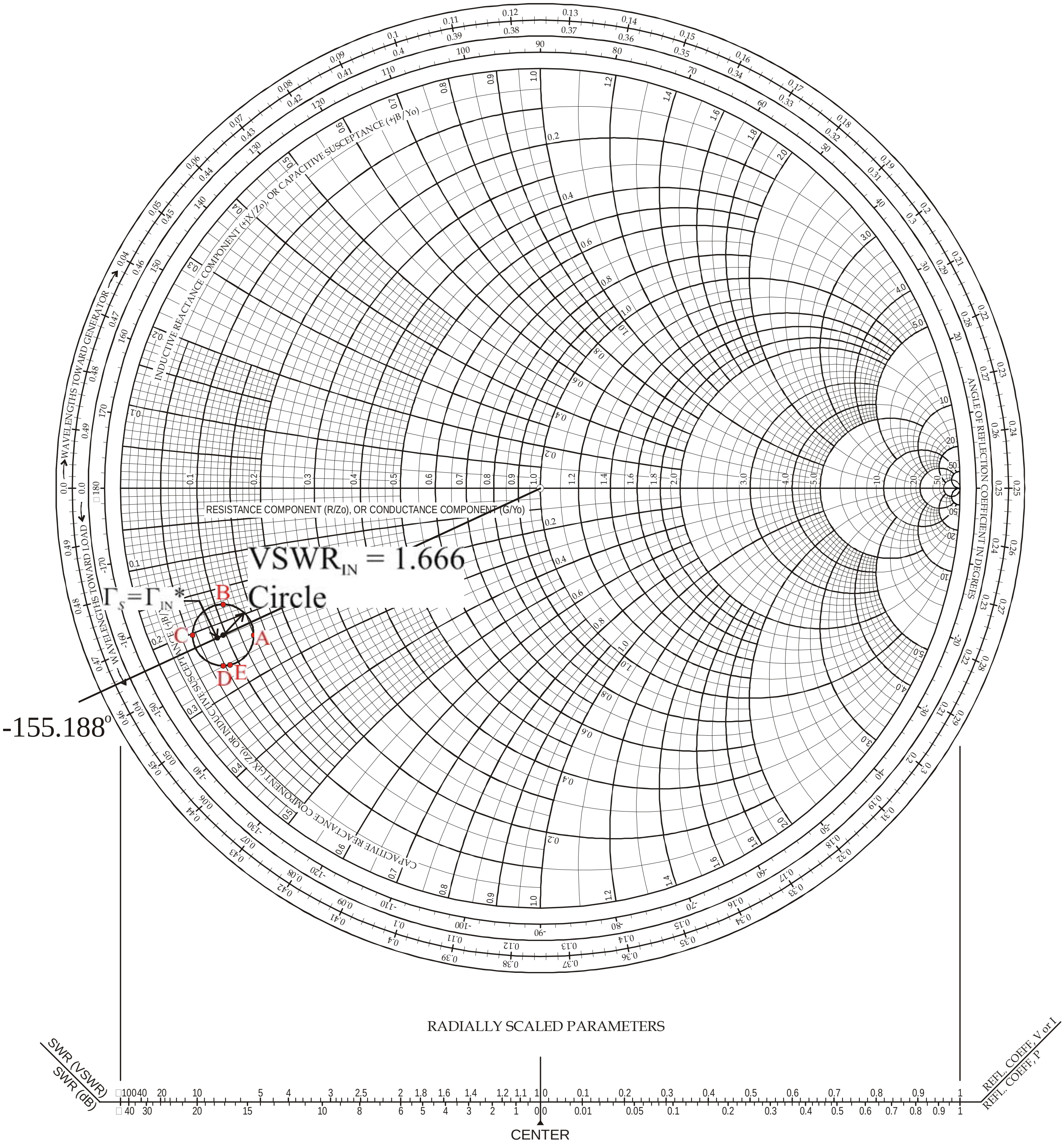
$\Gamma OUTE := S22 + \frac{S12m \cdot S21 \cdot \Gamma SE}{1 - S11 \cdot \Gamma SE}$ $\quad \left|\Gamma OUTE\right| = 0.41 \quad \arg(\Gamma OUTE)\cdot\frac{180}{\pi} = -77.27$ deg

$\Gamma aE := \left|\frac{\Gamma IN - \overline{\Gamma SE}}{1 - \Gamma IN \cdot \Gamma SE}\right|$ $\quad VSWR\_INE := \frac{1 + \Gamma aE}{1 - \Gamma aE} \quad VSWR\_INE = 1.666$

$\Gamma bE := \left|\frac{\Gamma OUTE - \overline{\Gamma L}}{1 - \Gamma OUTE \cdot \Gamma L}\right|$ $\quad VSWR\_OUTE := \frac{1 + \Gamma bE}{1 - \Gamma bE} \quad VSWR\_OUTE = 2.0532$

We allowed $VSWR_{IN} = 1.666$ and achieved a $VSWR_{OUT} = 2.0532$ (not quite < 2).

Γ_S Smith Chart



How are the gains with the new Γ_S value at point E?

With $\Gamma_S = \Gamma_{IN}{}^*$, we had $G_T = G_P = 9.999$ dB and $G_A = 11.008$ dB.

$$GTE := \frac{1-(|\Gamma SE|)^2}{(|1-\Gamma IN\cdot\Gamma SE|)^2}\cdot(|S21|)^2\cdot\frac{1-(|\Gamma L|)^2}{(|1-S22\cdot\Gamma L|)^2}$$

$|GTE| = 9.373$

$10\cdot\log(|GTE|) = 9.719$ dB

$$GP := \frac{1}{1-(|\Gamma IN|)^2}\cdot(|S21|)^2\cdot\frac{1-(|\Gamma L|)^2}{(|1-S22\cdot\Gamma L|)^2}$$

$|GP| = 9.997$

$10\cdot\log(|GP|) = 9.999$ dB

$$GA := \frac{1-(|\Gamma SE|)^2}{(|1-S11\cdot\Gamma SE|)^2}\cdot(|S21|)^2\cdot\frac{1}{1-(|\Gamma OUTE|)^2}$$

$|GA| = 10.639$

$10\cdot\log(|GA|) = 10.269$ dB

As expected, G_P is unchanged.

However, G_T decreased by 0.28 dB and G_A decreased by 0.739 dB.

Pretty minimal changes for a substantial improvement in $VSWR_{OUT}$ albeit at the cost of $VSWR_{IN}$ increasing to 1.666.

3.8 cont.

Next, note that $\Gamma_{OUT} = \dfrac{S_{22} - \Delta\Gamma_S}{1 - S_{11}\Gamma_S}$

and $\Gamma_S = \dfrac{S_{22} - \Gamma_{OUT}}{\Delta - S_{11}\Gamma_{OUT}}$

are related by a bilinear transformation. Therefore, it is possible to map circles of constant G_A on the Γ_S plane into circles on the Γ_{OUT} plane and in turn onto circles on the Γ_L plane where $\Gamma_L = \Gamma_{OUT}^*$

For a G_A circle, $|\Gamma_S - C_a| = r_a$, where

$$C_a = \frac{g_a C_1^*}{1 + g_a(|S_{11}|^2 - |\Delta|^2)}$$

$$r_a = \frac{[1 - 2K|S_{12}S_{21}|g_a + |S_{12}S_{21}|^2 g_a^2]^{1/2}}{|1 + g_a(|S_{11}|^2 - |\Delta|^2)|}$$

w/ $g_a = \dfrac{G_A}{|S_{21}|^2}$ and $C_1 = S_{11} - \Delta S_{22}^*$.

This maps into a circle on the Γ_{OUT} plane $|\Gamma_{OUT} - C_{OUT}| = r_{OUT}$ where

$$C_{OUT} = \frac{(1 - S_{11}C_a)^*(S_{22} - \Delta C_a) - r_a^2 \Delta S_{11}^*}{|1 - S_{11}C_a|^2 - r_a^2|S_{11}|^2}$$ ← center (complex #)

$$r_{OUT} = \frac{r_a|S_{12}S_{21}|}{\left||1 - S_{11}C_a|^2 - r_a^2|S_{11}|^2\right|}$$ ← radius (real #)

Then, on the Γ_L plane, we define the circle for $\Gamma_L = \Gamma_{OUT}^*$ as $|\Gamma_L - C_0| = r_0$

where $C_0 = C_{OUT}^*$ and $r_0 = r_{OUT}$.

A similar process to take $\Gamma_{IN} = \dfrac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L}$ and $\Gamma_L = \dfrac{S_{11} - \Gamma_{IN}}{\Delta - S_{22}\Gamma_{IN}}$ (also related by bilinear trans.)

and map circles of constant G_P on the Γ_L plane into circles on the Γ_{IN} plane and in turn onto circles on the Γ_S plane where $\Gamma_S = \Gamma_{IN}^*$.

For a G_P circle, $|\Gamma_L - C_P| = r_P$, where

$$C_P = \frac{g_P C_2^*}{1 + g_P(|S_{22}|^2 - |\Delta|^2)}$$

$$r_P = \frac{[1 - 2K|S_{12}S_{21}|g_P + |S_{12}S_{21}|^2 g_P^2]^{1/2}}{|1 + g_P(|S_{22}|^2 - |\Delta|^2)|}$$

w/ $g_P = \dfrac{G_P}{|S_{21}|^2}$ and $C_2 = S_{22} - \Delta S_{11}^*$

maps to a circle on the Γ_{IN} plane $|\Gamma_{IN} - C_{IN}| = r_{IN}$

where

$$C_{IN} = \frac{(1 - S_{22}C_P)^*(S_{11} - \Delta C_P) - r_P^2 \Delta S_{22}^*}{|1 - S_{22}C_P|^2 - r_P^2|S_{22}|^2}$$ ~ center (complex #)

$$r_{IN} = \frac{r_P|S_{12}S_{21}|}{\left||1 - S_{22}C_P|^2 - r_P^2|S_{22}|^2\right|}$$ ← radius (real #)

3.8 cont.

Then, on the Γ_S plane, we define the circle for $\Gamma_S = \Gamma_{IN}^*$ as $|\Gamma_S - C_i| = r_i$

where $C_i = C_{IN}^*$ and $r_i = r_{IN}$.

These mappings of circles are useful for analyzing the trade-offs between gains (e.g., G_A and G_P), stability, and VSWRs.

In this example, we will map the G_P = 10 dB circle to the Γ_S plane, keeping our $VSWR_{IN}$ =1.666 circle for an unconditionally stable, modified MLN2037F RF power transistor at 1 GHz.

$Z0 := 50 \quad \Omega$

$S11 := 0.8128 \cdot e^{j \cdot 156 \cdot \frac{\pi}{180}}$ $\qquad S12m := 0.08 \cdot e^{j \cdot 63 \cdot \frac{\pi}{180}}$

$S21 := 1.81 \cdot e^{j \cdot 55 \cdot \frac{\pi}{180}}$ $\qquad S22 := 0.3801 \cdot e^{j \cdot -136 \cdot \frac{\pi}{180}}$

$\Delta := S11 \cdot S22 - S12m \cdot S21$ $\qquad \boxed{|\Delta| = 0.35898}$ $\qquad$ < 1, good

$$K := \frac{1 - (|S11|)^2 - (|S22|)^2 + (|\Delta|)^2}{2\,|S12m \cdot S21|}$$ $\qquad \boxed{K = 1.1179}$ $\qquad$ > 1, good

Stability Conditions ARE met at 1 GHz.

$C2 := S22 - \Delta \cdot \overline{S11}$ $\qquad |C2| = 0.162$ $\qquad arg(C2) \cdot \frac{180}{\pi} = -90$ deg

$G\_MSG := \frac{|S21|}{|S12m|}$ $\qquad \boxed{G\_MSG = 22.625}$ $\qquad \boxed{10 \cdot \log(G\_MSG) = 13.546}$ dB

Choose to plot GP for 10 dB. $\qquad GP := 10^{\frac{10}{10}}$ $\qquad GP = 10$

$gP := \frac{GP}{(|S21|)^2}$ $\qquad gP = 3.052$

$$CP := \frac{gP \cdot \overline{C2}}{1 + gP \cdot \left[(|S22|)^2 - (|\Delta|)^2\right]}$$ $\qquad \boxed{|CP| = 0.472}$ $\qquad \boxed{arg(CP) \cdot \frac{180}{\pi} = 90}$ deg

$$rP := \frac{\left[1 - 2 \cdot K \cdot |S12m \cdot S21| \cdot gP + (|S12m \cdot S21|)^2 \cdot gP^2\right]^{0.5}}{\left|1 + gP \cdot \left[(|S22|)^2 - (|\Delta|)^2\right]\right|}$$ $\qquad \boxed{rP = 0.434}$

CP & rP are for the Γ_L plane.

First, map the circle of Γ_L values where GP = 10 dB to the Γ_{IN} plane.

$$CIN := \frac{\overline{(1 - S22 \cdot CP)} \cdot (S11 - \Delta \cdot CP) - rP^2 \cdot \Delta \cdot \overline{S22}}{(|1 - S22 \cdot CP|)^2 - rP^2 \cdot (|S22|)^2}$$

$$rIN := \frac{rP \cdot |S12m \cdot S21|}{\left|(|1 - S22 \cdot CP|)^2 - rP^2 \cdot (|S22|)^2\right|}$$

$|CIN| = 0.87157$ $\quad$ $\arg(CIN) \cdot \frac{180}{\pi} = 160.53137$ deg $\quad$ $rIN = 0.083262$

Now, map the circle on the Γ_{IN} plane to the $\Gamma_S = \Gamma_{IN}{}^*$ plane and plot on the Γ_S Smith Chart.

$Ci := \overline{CIN}$ $\quad$ $ri := rIN$

$|Ci| = 0.87157$ $\quad$ $\arg(Ci) \cdot \frac{180}{\pi} = -160.53137$ deg $\quad$ $ri = 0.083262$

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Select Point A on the Γ_L Smith Chart. Here the normalized impedance $z_L = 1.6 + j0.4$ corresponds to $\Gamma_L = 0.274 / 24.94^o$.

$zL := 1.6 + j \cdot 0.4$ $\quad \Gamma L := \frac{zL - 1}{zL + 1}$ $\quad |\Gamma L| = 0.274$ $\quad \arg(\Gamma L) \cdot \frac{180}{\pi} = 24.944$ deg

For maximum output power, select $\Gamma_S = \Gamma_{IN}^*$. Check input & output VSWR.

$\Gamma IN := S11 + \frac{S12m \cdot S21 \cdot \Gamma L}{1 - S22 \cdot \Gamma L}$ $\quad |\Gamma IN| = 0.849$ $\quad \arg(\Gamma IN) \cdot \frac{180}{\pi} = 155.188$ deg

$\Gamma S := \overline{\Gamma IN}$ $\quad |\Gamma S| = 0.849$ $\quad \arg(\Gamma S) \cdot \frac{180}{\pi} = -155.188$ deg

$\Gamma a\_mag := \left| \frac{\Gamma IN - \overline{\Gamma S}}{1 - \Gamma IN \cdot \Gamma S} \right|$ $\quad VSWR\_IN := \frac{1 + \Gamma a\_mag}{1 - \Gamma a\_mag}$ $\quad VSWR\_IN = 1$

$\Gamma OUT := S22 + \frac{S12m \cdot S21 \cdot \Gamma S}{1 - S11 \cdot \Gamma S}$ $\quad |\Gamma OUT| = 0.496$ $\quad \arg(\Gamma OUT) \cdot \frac{180}{\pi} = -84.24$ deg

$\Gamma b\_mag := \left| \frac{\Gamma OUT - \overline{\Gamma L}}{1 - \Gamma OUT \cdot \Gamma L} \right|$ $\quad VSWR\_OUT := \frac{1 + \Gamma b\_mag}{1 - \Gamma b\_mag}$ $\quad VSWR\_OUT = 2.672$

$\Gamma a\_1666 := \frac{1.666 - 1}{1.666 + 1}$ $\quad \Gamma a\_1666 = 0.2498$

$Cvi1666 := \frac{\overline{\Gamma IN} \cdot \left(1 - \Gamma a\_1666^2\right)}{1 - \Gamma a\_1666^2 \cdot (|\Gamma IN|)^2}$ $\quad rvi1666 := \frac{\Gamma a\_1666 \cdot \left[1 - (|\Gamma IN|)^2\right]}{1 - \Gamma a\_1666^2 \cdot (|\Gamma IN|)^2}$

$|Cvi1666| = 0.83354$ $\quad \arg(Cvi1666) \cdot \frac{180}{\pi} = -155.188$ deg $\quad rvi1666 = 0.07302$

Plot $VSWR_{IN} = 1.666$ circle on Γ_S Smith Chart.

Γ_S Smith Chart