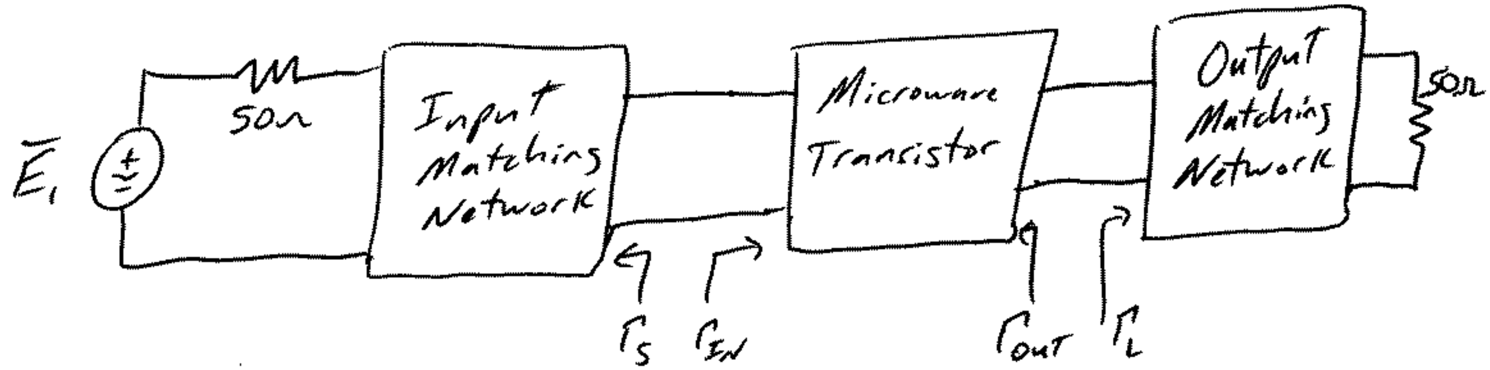
3.6 Simultaneous Conjugate Match: Bilateral Case

→ Now $S_{12} \neq 0$ ← Possible for unconditionally stable 2-ports.

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}$$



for maximum transducer power gain $G_{T,max}$

we need $\Gamma_S = \Gamma_{IN}^*$ and $\Gamma_L = \Gamma_{OUT}^*$

i.e., simultaneous complex conjugate match.

Using this requirement and the above Γ_{in} & Γ_{OUT} equations, leads to the desired Γ_S + Γ_L values for $G_{T,max}$

$$\Gamma_{MS} = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

and

$$\Gamma_{ML} = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

[Note: Use the "−" solutions for Γ_{MS} + Γ_{ML} for an unconditionally stable 2-port network.]

3.6 cont.

where $B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$\Delta = S_{11} S_{22} - S_{21} S_{12}$$

$$C_1 = S_{11} - \Delta S_{22}^*$$

$$C_2 = S_{22} - \Delta S_{11}^*$$

Appendix E covers how we choose between the the $\pm$ solutions for Γ_{MS} + Γ_{ML}.

For Γ_{MS}, if $\left|\frac{B_1}{2C_1}\right| > 1$ + $B_1 > 0$ → "−" gives $|\Gamma_{MS}| < 1$ → "+" gives $|\Gamma_{MS}| > 1$

if $\left|\frac{B_1}{2C_1}\right| > 1$ + $B_1 < 0$ → "+" gives $|\Gamma_{MS}| < 1$ → "−" gives $|\Gamma_{MS}| > 1$

For Γ_{ML}, if $\left|\frac{B_2}{2C_2}\right| > 1$ + $B_2 > 0$ → "−" gives $|\Gamma_{ML}| < 1$ → "+" gives $|\Gamma_{ML}| > 1$

if $\left|\frac{B_2}{2C_2}\right| > 1$ + $B_2 < 0$ → "+" gives $|\Gamma_{ML}| < 1$ → "−" gives $|\Gamma_{ML}| > 1$

From a practical standpoint, it may be easiest just to compute both the $\pm$ solutions and see where they fall on the Γ_S or Γ_L planes.

3.6 cont.

These conditions can be related to

$$K = \frac{1-|S_{11}|^2-|S_{22}|^2-|\Delta|^2}{2|S_{12}S_{21}|}$$

If $K>1$ and $B_1>0$ $\Rightarrow$ use "−" solutions.

If $K<-1$ $\Rightarrow$ No simultaneous conjugate match.
or $K<1$

$\Downarrow$

For $|\Gamma_{MS}|<1$ (i.e. $Re(Z_S)>0$) and $|\Gamma_{ML}|<1$ (i.e. $Re(Z_L)>0$)

↳ $K>1$ and $|\Delta|<1$

Note: $|\Delta|<1$ implies $B_1>0$ and $B_2>0$ $\Rightarrow$ use "−" sol'n

What about potentially unstable problems?

↳ use G_P and/or G_A per next section.

w/ $\Gamma_S=\Gamma_{MS}=\Gamma_{IN}^*$ and $\Gamma_L=\Gamma_{ML}=\Gamma_{OUT}^*$

$$G_{T,max} = \frac{1}{1-|\Gamma_{MS}|^2}|S_{21}|^2\frac{1-|\Gamma_{ML}|^2}{|1-S_{22}\Gamma_{ML}|^2} = \frac{|S_{21}|}{|S_{12}|}\left(K-\sqrt{K^2-1}\right)$$

Note: Need $K>1$

+

$$G_{T,max} = G_{P,max} = G_{A,max}$$

3.6 cont.

From the last eq'n, we see that $G_{T,max}$ is largest when $K=1$. Call it the maximum stable gain

$$G_{MSG} = \frac{|S_{21}|}{|S_{12}|}.$$

If a transistor is potentially unstable (i.e. $K<1$), the ratio $\frac{|S_{21}|}{|S_{12}|}$ is used as a figure of merit.

what if $K>1$ but $|\Delta|>1$?

Then, the "+" solutions to Γ_{MS} and Γ_{ML} yield the minimum value $G_{T,min}$ of the transducer gain and the input & output VSWR = 1.

$$G_{T,min} = \frac{|S_{21}|}{|S_{12}|}\left(K+\sqrt{K^2-1}\right)$$

What if we do NOT want $G_{T,max}$?

→ A constant gain circle approach is possible, but not practical (very iterative as Γ_{in} is a function of Γ_L which makes the G_S' function dependent on G_L.

3.6 cont.

→ A better set of approaches for bilateral transistors is described in Section 3.7.

Some possibilities

1. Unconditionally stable bilateral transistor
   a) design for $G_{T,max}$
   b) design for desired operating power gain G_P
   c) design for desired available power gain G_A
2. Potentially unstable bilateral transistor
   ↳ b) or c) from above.

Simultaneous Conjugate Match: Bilateral Case example

1) For the MLN2037F NPN silicon RF power transistor at 1 GHz:

$$Z0 := 50 \quad \Omega$$

$$S11 := 0.8128 \cdot e^{j \cdot 156 \cdot \frac{\pi}{180}} \qquad S12 := 0.0944 \cdot e^{j \cdot 63 \cdot \frac{\pi}{180}}$$

$$S21 := 1.81 \cdot e^{j \cdot 55 \cdot \frac{\pi}{180}} \qquad S22 := 0.3801 \cdot e^{j \cdot -136 \cdot \frac{\pi}{180}}$$

$$\Delta := S11 \cdot S22 - S12 \cdot S21 \qquad \boxed{|\Delta| = 0.37328} \qquad \underline{< 1, \text{ good}}$$

$$K := \frac{1 - (|S11|)^2 - (|S22|)^2 + (|\Delta|)^2}{2\,|S12 \cdot S21|} \qquad \boxed{K = 0.978} \qquad \underline{< 1, \text{ bad}}$$

Stability Conditions are NOT met at 1 GHz.

What happens to the load and source reflection coefficient equations?

$$B1 := 1 + (|S11|)^2 - (|S22|)^2 - (|\Delta|)^2 \qquad B1 = 1.377$$

$$B2 := 1 + (|S22|)^2 - (|S11|)^2 - (|\Delta|)^2 \qquad B2 = 0.344$$

$$C1 := S11 - \Delta \cdot \overline{S22} \qquad |C1| = 0.689 \qquad \arg(C1) \cdot \frac{180}{\pi} = 161.4 \quad \text{deg}$$

$$C2 := S22 - \Delta \cdot \overline{S11} \qquad |C2| = 0.176 \qquad \arg(C2) \cdot \frac{180}{\pi} = -84.6 \quad \text{deg}$$

$$\Gamma Ms\_plus := \frac{B1 + \sqrt{B1^2 - 4 \cdot (|C1|)^2}}{2 \cdot C1} \qquad \boxed{|\Gamma Ms\_plus| = 1}$$

$$\boxed{\arg(\Gamma Ms\_plus) \cdot \frac{180}{\pi} = -158.4} \quad \text{deg}$$

$$\Gamma Ms\_minus := \frac{B1 - \sqrt{B1^2 - 4 \cdot (|C1|)^2}}{2 \cdot C1} \qquad \boxed{|\Gamma Ms\_minus| = 1}$$

$$\boxed{\arg(\Gamma Ms\_minus) \cdot \frac{180}{\pi} = -164.3} \quad \text{deg}$$

$$\Gamma ML\_plus := \frac{B2 + \sqrt{B2^2 - 4\cdot(|C2|)^2}}{2\cdot C2}$$

$$\boxed{|\Gamma ML\_plus| = 1}$$

$$\boxed{\arg(\Gamma ML\_plus)\cdot\frac{180}{\pi} = 96.3} \quad \text{deg}$$

$$\Gamma ML\_minus := \frac{B2 - \sqrt{B2^2 - 4\cdot(|C2|)^2}}{2\cdot C2}$$

$$\boxed{|\Gamma ML\_minus| = 1}$$

$$\boxed{\arg(\Gamma ML\_minus)\cdot\frac{180}{\pi} = 72.9} \quad \text{deg}$$

Obviously, these load and source reflection coefficients are NOT useful (all the power gets reflected!).

2) For a *modified* lower |S12| MLN2037F NPN silicon RF power transistor at 1 GHz where:

$$S12m := 0.08\cdot e^{j\cdot 63\cdot\frac{\pi}{180}}$$

$$\Delta m := S11\cdot S22 - S12m\cdot S21 \qquad \boxed{|\Delta m| = 0.35898} \quad \underline{< 1,\ \text{good}}$$

$$Km := \frac{1 - (|S11|)^2 - (|S22|)^2 + (|\Delta m|)^2}{2\,|S12m\cdot S21|} \qquad \boxed{Km = 1.1179} \quad \underline{> 1,\ \text{good}}$$

<u>Stability Conditions ARE met at 1 GHz.</u>

Now what happens to the load and source reflection coefficient equations?

$$B1m := 1 + (|S11|)^2 - (|S22|)^2 - (|\Delta m|)^2 \qquad B1m = 1.387$$

$$B2m := 1 + (|S22|)^2 - (|S11|)^2 - (|\Delta m|)^2 \qquad B2m = 0.355$$

$$C1m := S11 - \Delta m\cdot\overline{S22} \qquad |C1m| = 0.69 \qquad \arg(C1m)\cdot\frac{180}{\pi} = 160.5 \quad \text{deg}$$

$$C2m := S22 - \Delta m\cdot\overline{S11} \qquad |C2m| = 0.162 \qquad \arg(C2m)\cdot\frac{180}{\pi} = -90 \quad \text{deg}$$

$$\Gamma Msm\_plus := \frac{B1m + \sqrt{B1m^2 - 4\cdot(|C1m|)^2}}{2\cdot C1m}$$

$$\boxed{|\Gamma Msm\_plus| = 1.11037}$$

$$\boxed{\arg(\Gamma Msm\_plus)\cdot\frac{180}{\pi} = -160.5} \quad deg$$

$$Zsm\_plus := Z0\cdot\left(\frac{1 - \Gamma Msm\_plus}{1 + \Gamma Msm\_plus}\right)$$

$$Zsm\_plus = -83.695 + 265.945i \quad \Omega$$

$$\Gamma Msm\_minus := \frac{B1m - \sqrt{B1m^2 - 4\cdot(|C1m|)^2}}{2\cdot C1m}$$

$$\boxed{|\Gamma Msm\_minus| = 0.9006}$$

$$\boxed{\arg(\Gamma Msm\_minus)\cdot\frac{180}{\pi} = -160.5} \quad deg$$

$$Zsm\_minus := Z0\cdot\left(\frac{1 - \Gamma Msm\_minus}{1 + \Gamma Msm\_minus}\right)$$

$$\boxed{Zsm\_minus = 83.695 + 265.945i} \quad \Omega$$

$$\Gamma MLm\_plus := \frac{B2m + \sqrt{B2m^2 - 4\cdot(|C2m|)^2}}{2\cdot C2m}$$

$$\boxed{|\Gamma MLm\_plus| = 1.54164}$$

$$\boxed{\arg(\Gamma MLm\_plus)\cdot\frac{180}{\pi} = 90} \quad deg$$

$$ZLm\_plus := Z0\cdot\left(\frac{1 - \Gamma MLm\_plus}{1 + \Gamma MLm\_plus}\right)$$

$$ZLm\_plus = -20.39 - 45.667i \quad \Omega$$

$$\Gamma MLm\_minus := \frac{B2m - \sqrt{B2m^2 - 4\cdot(|C2m|)^2}}{2\cdot C2m}$$

$$\boxed{|\Gamma MLm\_minus| = 0.64866}$$

$$\boxed{\arg(\Gamma MLm\_minus)\cdot\frac{180}{\pi} = 90} \quad deg$$

$$ZLm\_minus := Z0\cdot\left(\frac{1 - \Gamma MLm\_minus}{1 + \Gamma MLm\_minus}\right)$$

$$\boxed{ZLm\_minus = 20.39 - 45.667i} \quad \Omega$$

Note: The "-" solutions are useful as predicted. However, the "+" solutions are not useful (negative real resistances for corresponding Z_L and Z_s.

What is the maximum transducer gain for the modified transistor?

$$GTmax\_m1 := \frac{1}{1-\left(\left|\Gamma Msm\_minus\right|\right)^2}\cdot\left(\left|S21\right|\right)^2\cdot\frac{\left[1-\left(\left|\Gamma MLm\_minus\right|\right)^2\right]}{\left(\left|1-S22\cdot\Gamma MLm\_minus\right|\right)^2}$$

OR

$$GTmax\_m2 := \frac{\left|S21\right|}{\left|S12m\right|}\cdot\left(Km-\sqrt{Km^2-1}\right)$$

$\boxed{GTmax\_m1 = 13.987}$ $\boxed{GTmax\_m2 = 13.987}$

$\boxed{10\cdot\log(GTmax\_m1) = 11.457}$ dB $\boxed{10\cdot\log(GTmax\_m2) = 11.457}$ dB

Same answer!

What is the maximum stable (transducer) gain for the modified transistor?

$$G\_MSG := \frac{\left|S21\right|}{\left|S12m\right|}$$

$\boxed{G\_MSG = 22.625}$ $\boxed{10\cdot\log(G\_MSG) = 13.546}$ dB

3.7 Operating and Available Power-Gain Circles

Operating Power-Gain Circles (G_P)

→ commonly used for bilateral ($S_{12} \neq 0$) transistors

→ G_P does NOT depend on Γ_S (or Z_S)

→ useful for both unconditionally stable and potentially unstable transistors

G_P for Unconditionally stable case

$$G_P = |S_{21}|^2 \frac{1-|\Gamma_L|^2}{\left(1-\left|\frac{S_{11}-\Delta\Gamma_L}{1-S_{22}\Gamma_L}\right|^2\right)|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 g_P$$

So $$g_P = \frac{G_P}{|S_{21}|^2} = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2 - |S_{11}-\Delta\Gamma_L|^2}$$

$$= \frac{1-|\Gamma_L|^2}{1-|S_{11}|^2+|\Gamma_L|^2(|S_{22}|^2-|\Delta|^2)-2\,\mathrm{Re}(\Gamma_L C_2)}$$

where $\Delta = S_{11}S_{22} - S_{12}S_{21}$ and $C_2 = S_{22} - \Delta S_{11}^*$

Both G_P and g_P are real numbers.

Appendix G shows that the equation defining circles of constant g_P on the Γ_L plane is

$$|\Gamma_L - C_P| = r_P$$

3.7 cont.

Center of circle $\equiv C_p = \dfrac{g_p C_2^*}{1+g_p(|S_{22}|^2-|\Delta|^2)}$ a complex #

radius of circle $\equiv r_p = \dfrac{\left[1-2K|S_{12}S_{21}|g_p+|S_{12}S_{21}|^2 g_p^2\right]^{1/2}}{|1+g_p(|S_{22}|^2-|\Delta|^2)|}$ Real #

Notes: $\measuredangle C_p$ is always equal to $\measuredangle C_2^*$
If r_p is imaginary, pick lower G_p.

At what value of Γ_L does the maximum G_p occur?

$\Rightarrow$ Maximum G_p occurs when $r_p = 0$

This leads to $g_{p,max} = \dfrac{1}{|S_{12}S_{21}|}\left(K-\sqrt{K^2-1}\right)$

and $G_{p,max} = |S_{21}|^2 g_{p,max} = \dfrac{|S_{21}|}{|S_{12}|}\left(K-\sqrt{K^2-1}\right)$

$= G_{T,max}$ (as expected!)

To get Γ_L, use the equation for C_p w/ $g_{p,max}$ yielding

$$\Gamma_L = \Gamma_{ML} = C_{p,max} = \frac{g_{p,max}\, C_2^*}{1+g_{p,max}(|S_{22}|^2-|\Delta|^2)}$$

Lowest gain? $g_p = G_p = 0$

Occurs when $|\Gamma_L| = 1$ (all power reflected from load)

3.7 cont.

Notes: * $\Gamma_{L,max} = \Gamma_{ML}$ occur where $g_{P,max} = \frac{G_{P,max}}{|S_{21}|^2}$

* Maximum output power when $\Gamma_S = \Gamma_{IN}^*$ where $P_{IN} = P_{AVS}$. Then, $\Gamma_S = \Gamma_{MS}$ and

$G_{P,max} = G_{T,max}$

Procedure:

1) Select a value of G_P (helps to calculate $G_{P,max} = G_{T,max}$).

2) Calculate corresponding g_P, C_P, + r_P. plot on Γ_L Smith chart.

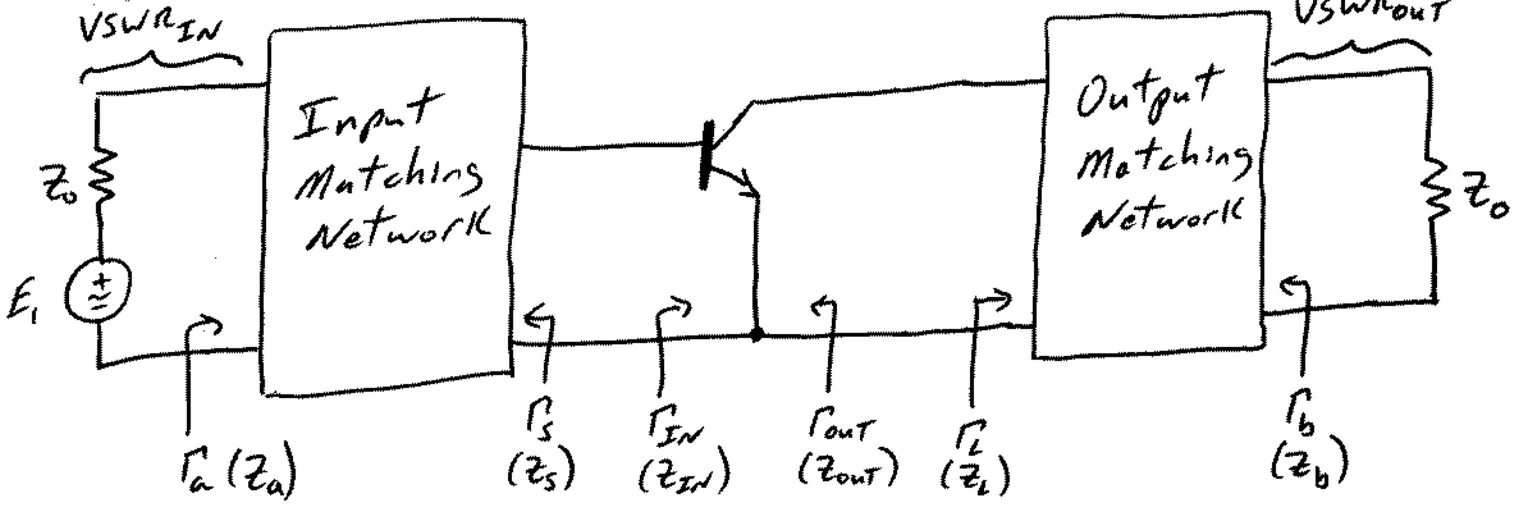
3) Select desired Γ_L on G_P circle

Note: For selected Γ_L, maximum output power is obtained when $\Gamma_S = \Gamma_{IN}^*$.
Then, this $G_P = G_T$.

depends on Γ_L

* Another consideration for selecting Γ_L is the output VSWR, $VSWR_{OUT}$. It is desirable to keep $VSWR_{OUT}$ as low as possible. The input VSWR, $VSWR_{IN} = 1$ w/ $\Gamma_S = \Gamma_{IN}^*$

3.7 cont.



From Section 2.8,

$$VSWR_{IN} = \frac{1+|\Gamma_a|}{1-|\Gamma_a|} \quad \text{where } |\Gamma_a| = \left|\frac{\Gamma_{IN}-\Gamma_S^*}{1-\Gamma_{IN}\Gamma_S}\right|$$

↳ For $\Gamma_S = \Gamma_{IN}^*$, $|\Gamma_a| = 0$ and $\underline{VSWR_{IN} = 1}$

and

$$VSWR_{OUT} = \frac{1+|\Gamma_b|}{1-|\Gamma_b|} \quad \text{where } |\Gamma_b| = \left|\frac{\Gamma_{OUT}-\Gamma_L^*}{1-\Gamma_{OUT}\Gamma_L}\right|$$

* If $VSWR_{OUT}$ is unacceptably high, choose another value of Γ_L on the G_p circle.

* Sometimes $\Gamma_S \neq \Gamma_{IN}^*$ and hence $VSWR_{IN} > 1$ must be accepted in order to lower $VSWR_{OUT}$.

Operating Power-Gain (GP) Circles for Unconditionally Stable case

For a modified MLN2037F (lower |S12|) RF power transistor at 1 GHz:

$Z0 := 50 \quad \Omega$

$S11 := 0.8128 \cdot e^{j \cdot 156 \cdot \frac{\pi}{180}}$ $\quad S12m := 0.08 \cdot e^{j \cdot 63 \cdot \frac{\pi}{180}}$

$S21 := 1.81 \cdot e^{j \cdot 55 \cdot \frac{\pi}{180}}$ $\quad S22 := 0.3801 \cdot e^{j \cdot -136 \cdot \frac{\pi}{180}}$

$\Delta := S11 \cdot S22 - S12m \cdot S21$ $\quad \boxed{|\Delta| = 0.35898}$ $\quad$ < 1, good

$K := \frac{1 - (|S11|)^2 - (|S22|)^2 + (|\Delta|)^2}{2\,|S12m \cdot S21|}$ $\quad \boxed{K = 1.1179}$ $\quad$ > 1, good

Stability Conditions ARE met at 1 GHz.

$C2 := S22 - \Delta \cdot \overline{S11}$ $\quad |C2| = 0.162$ $\quad arg(C2) \cdot \frac{180}{\pi} = -90$ deg

What is the maximum stable (transducer) gain for the modified transistor?

$G\_MSG := \frac{|S21|}{|S12m|}$ $\quad \boxed{G\_MSG = 22.625}$ $\quad \boxed{10 \cdot \log(G\_MSG) = 13.546}$ dB

Choose to plot GP for 10 dB.

$GP := 10^{\frac{10}{10}}$ $\quad GP = 10$

$gP := \frac{GP}{(|S21|)^2}$ $\quad gP = 3.052$

$CP := \frac{gP \cdot \overline{C2}}{1 + gP \cdot \left[(|S22|)^2 - (|\Delta|)^2\right]}$ $\quad \boxed{|CP| = 0.472}$ $\quad \boxed{arg(CP) \cdot \frac{180}{\pi} = 90}$ deg

$rP := \frac{\left[1 - 2 \cdot K \cdot |S12m \cdot S21| \cdot gP + (|S12m \cdot S21|)^2 \cdot gP^2\right]^{0.5}}{\left|1 + gP \cdot \left[(|S22|)^2 - (|\Delta|)^2\right]\right|}$ $\quad \boxed{rP = 0.434}$

Plot G_P = 10 dB circle on Γ_L Smith Chart.

Select Point A on the Smith Chart. Here the normalized impedance $z_L = 1.6 + j0.4$ corresponds to $\Gamma_L = 0.274 / 24.94^o$.

$zL := 1.6 + j\cdot 0.4$ $\quad \Gamma L := \dfrac{zL - 1}{zL + 1}$ $\quad \boxed{|\Gamma L| = 0.274}$ $\quad \boxed{\arg(\Gamma L)\cdot\dfrac{180}{\pi} = 24.944}$ deg

For maximum output power, select $\Gamma_S = \Gamma_{IN}^*$. Check input & output VSWR.

$\Gamma IN := S11 + \dfrac{S12m\cdot S21\cdot \Gamma L}{1 - S22\cdot \Gamma L}$ $\quad \boxed{|\Gamma IN| = 0.849}$ $\quad \boxed{\arg(\Gamma IN)\cdot\dfrac{180}{\pi} = 155.188}$ deg

$\Gamma S := \overline{\Gamma IN}$ $\quad \boxed{|\Gamma S| = 0.849}$ $\quad \boxed{\arg(\Gamma S)\cdot\dfrac{180}{\pi} = -155.188}$ deg

$\Gamma a\_mag := \left|\dfrac{\Gamma IN - \overline{\Gamma S}}{1 - \Gamma IN\cdot \Gamma S}\right|$ $\quad VSWR\_IN := \dfrac{1 + \Gamma a\_mag}{1 - \Gamma a\_mag}$

$\boxed{\Gamma a\_mag = 0}$ $\quad \boxed{VSWR\_IN = 1}$

$\Gamma OUT := S22 + \dfrac{S12m\cdot S21\cdot \Gamma S}{1 - S11\cdot \Gamma S}$ $\quad \boxed{|\Gamma OUT| = 0.496}$ $\quad \boxed{\arg(\Gamma OUT)\cdot\dfrac{180}{\pi} = -84.24}$ deg

$\Gamma b\_mag := \left|\dfrac{\Gamma OUT - \overline{\Gamma L}}{1 - \Gamma OUT\cdot \Gamma L}\right|$ $\quad VSWR\_OUT := \dfrac{1 + \Gamma b\_mag}{1 - \Gamma b\_mag}$

$\boxed{\Gamma b\_mag = 0.455}$ $\quad \boxed{VSWR\_OUT = 2.672}$

Check on gain values --> very good agreement.

$$GT := \frac{1 - (|\Gamma S|)^2}{(|1 - \Gamma IN\cdot \Gamma S|)^2}\cdot(|S21|)^2\cdot\frac{1 - (|\Gamma L|)^2}{(|1 - S22\cdot \Gamma L|)^2}$$

$|GT| = 9.997$

$\boxed{10\cdot\log(|GT|) = 9.999}$ dB

$$GP := \frac{1}{1 - (|\Gamma IN|)^2}\cdot(|S21|)^2\cdot\frac{1 - (|\Gamma L|)^2}{(|1 - S22\cdot \Gamma L|)^2}$$

$|GP| = 9.997$

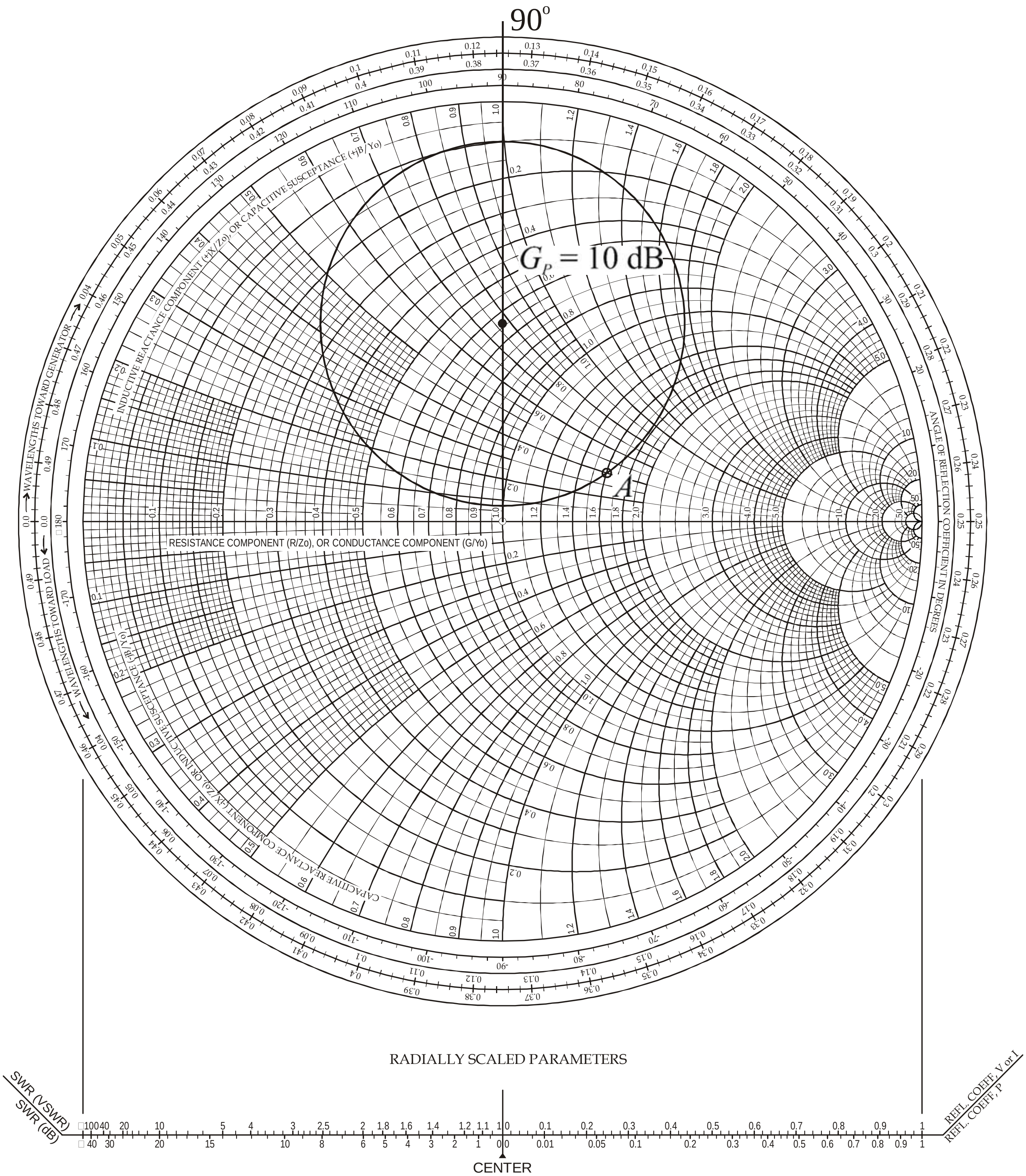
$\boxed{10\cdot\log(|GP|) = 9.999}$ dB

$$GA := \frac{1 - (|\Gamma S|)^2}{(|1 - S11\cdot \Gamma S|)^2}\cdot(|S21|)^2\cdot\frac{1}{1 - (|\Gamma OUT|)^2}$$

$|GA| = 12.612$

$\boxed{10\cdot\log(|GA|) = 11.008}$ dB

Γ_L Smith Chart



3.7 cont.

Potentially unstable bilateral Case G_p Circles

* G_p can be ∞ in this case
* As a rule, choose $G_p < G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$ to get good stability and acceptable $VSWR_{IN}$ + $VSWR_{OUT}$.

  why? When $G_p > G_{MSG}$, Γ_S +/or Γ_L get close to unstable regions and result in large $VSWR_{IN}$ + $VSWR_{OUT}$

Procedure

1) Select a value of G_p.
2) Calculate g_p, C_p, + r_p. Plot G_p circle on Γ_L Smith Chart.
3) Calculate and draw Output Stability Circle (see p. 218) on Γ_L Smith Chart
4) Select value of Γ_L on G_p circle that is in output stability stable region (not too close to output stability circle!).
5) Calculate $\Gamma_{IN} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$ for this Γ_L.
6) Calculate and draw Input Stability Circle (see p. 218) on Γ_S Smith Chart. Is $\Gamma_S = \Gamma_{IN}^*$ in the stable region? Yes, let $\Gamma_S = \Gamma_{IN}^*$.

3.7 cont.

What if $\Gamma_S = \Gamma_{IN}^*$ is in the unstable region or too close to input stability circle for comfort?

↳ Choose Γ_S arbitrarily considering resulting $VSWR_{IN}$ and output power.

What if $K > 1$ (good) and $|\Delta| > 1$ (bad)?

Then, there is a $G_{P,min}$ (similar to $G_{T,min}$) where

$$g_{p,min} = \frac{1}{|S_{12}S_{21}|}\left(K + \sqrt{K^2-1}\right)$$

and

$$G_{P,min} = \frac{|S_{21}|}{|S_{12}|}\left(K + \sqrt{K^2-1}\right).$$

This is the minimum G_P in the stable region when $K > 1$ and $|\Delta| > 1$. The corresponding Γ_L is

$$\Gamma_{L,min} = \frac{g_{p,min} C_2^*}{1 + g_{p,min}\left(|S_{22}|^2 - |\Delta|^2\right)} = \Gamma_{ML}$$

when "+" sign used

~~Note: G_P circles and the~~

Note: G_P circles and the Output Stability circle will intersect the outside edge ($|\Gamma_L| = 1$) of the Γ_L Smith Chart at the same points.

3.7 cont.

Available Power - Gain Circles (G_A)

* For a given G_A, we will find the values of Γ_S. $\Rightarrow$ G_A circle on Γ_S Smith Chart.
* For a given G_A + Γ_S, $\Gamma_L = \Gamma_{out}^*$ will yield the maximum output power and $G_A = G_T$.
* Later (Chapter 4), we will see that constant noise figure circles are also functions of Γ_S and plotted on Γ_S Smith Chart.

G_A circles - Unconditionally stable bilateral case

$$G_A = |S_{21}|^2 \frac{1-|\Gamma_S|^2}{\left(1-\left|\frac{S_{22}-\Delta\Gamma_S}{1-S_{11}\Gamma_S}\right|^2\right)|1-S_{11}\Gamma_S|^2} = |S_{21}|^2 g_a$$

So
$$g_a = \frac{G_A}{|S_{21}|^2} = \frac{1-|\Gamma_S|^2}{1-|S_{22}|^2+|\Gamma_S|^2(|S_{11}|^2-|\Delta|^2)-2\,\mathrm{Re}(\Gamma_S C_1)}$$

where $C_1 = S_{11} - \Delta S_{22}^*$.

The defining equation for the G_A circles is

$$|\Gamma_S - C_a| = r_a$$

3.7 cont.

$$\begin{array}{l}\text{Center of} \\ G_A \text{ circle}\end{array} \equiv C_a = \frac{g_a C_1^*}{1 + g_a(|S_{11}|^2 - |\Delta|^2)} \leftarrow \text{complex \#}$$

$$\begin{array}{l}\text{radius of} \\ G_A \text{ circle}\end{array} \equiv r_a = \frac{\left[1 - 2K|S_{12}S_{21}|g_a + |S_{12}S_{21}|^2 g_a^2\right]^{1/2}}{|1 + g_a(|S_{11}|^2 - |\Delta|^2)|}$$

$\nwarrow$ real #

Procedure

1) Select a value of G_A (helps to calculate $G_{A,max} = G_{T,max}$).

2) Calculate corresponding g_a, C_a, + r_a. Plot on Γ_S Smith Chart.

3) Select desired Γ_S on G_A circle.
   - * Remember for a given Γ_S, maximum output power is obtained when $\Gamma_L = \Gamma_{OUT}^*$ which also gives $VSWR_{OUT} = 1$
   - * May want to check what $VSWR_{IN}$ results from Γ_S selection. It may be unacceptably high. If so, choose another Γ_S on G_A circle.

3.7 cont.

G_A circles – Potentially unstable bilateral case

* G_A can be ∞ in this case.
* Again, as a rule, choose $G_A < G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$ to get good stability and acceptable $VSWR_{IN}$ + $VSWR_{OUT}$

Procedure

1) Select a value of G_A.
2) Calculate corresponding g_a, C_a, + r_a. Plot G_A circle on Γ_S Smith Chart.
3) Calculate and draw Input Stability Circle (see p. 218) on Γ_S Smith Chart.
4) Select value of Γ_S on G_A circle that is in the stable region (not too close to input stability circle!).
5) Calculate $\Gamma_{OUT} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}$ for this Γ_S.
6) Calculate and draw Output Stability Circle (see p. 218) on Γ_L Smith Chart. Is $\Gamma_L = \Gamma_{OUT}^*$ in the stable region? Yes, let $\Gamma_L = \Gamma_{OUT}^*$ to get maximum output power.

3.7 cont.

What if $\Gamma_L = \Gamma_{OUT}^*$ is in the unstable region (or too close to output stability circle for comfort)?

↳ Choose Γ_L arbitrarily considering resulting $VSWR_{OUT}$ and output power.