What about stability for unilateral ($S_{12} = 0$) two-port networks?

Here, $\Gamma_{IN} = S_{11}$ & $\Gamma_{OUT} = S_{22}$

and $K \to \infty$ & $\Delta = S_{11} S_{22}$

which leads to the requirement that

$|S_{11}| < 1$ and $|S_{22}| < 1$ for

a unilateral two-port to be unconditionally stable

What if Γ_L and/or Γ_S selection results in $|\Gamma_{IN}| > 1$ (Z_{in} has negative resistance) and/or $|\Gamma_{OUT}| > 1$ (Z_{OUT} has negative resistance)?

We can still achieve an unconditionally stable two-port by resistively loading the input and/or output to make $Re(Z_{in}) > 0$ and/or $Re(Z_{OUT}) > 0$.

Cost? Lower gain, Lower efficiency, worse noise figure, worse VSWR — Not acceptable for Narrowband designs, may be acceptable for broadband applications

3.4 Constant-Gain Circles: Unilateral Case

Unilateral means $S_{12} \simeq 0$, $\Gamma_{IN} = S_{11}$, & $\Gamma_{OUT} = S_{22}$.

Then, the transducer power gain G_T becomes

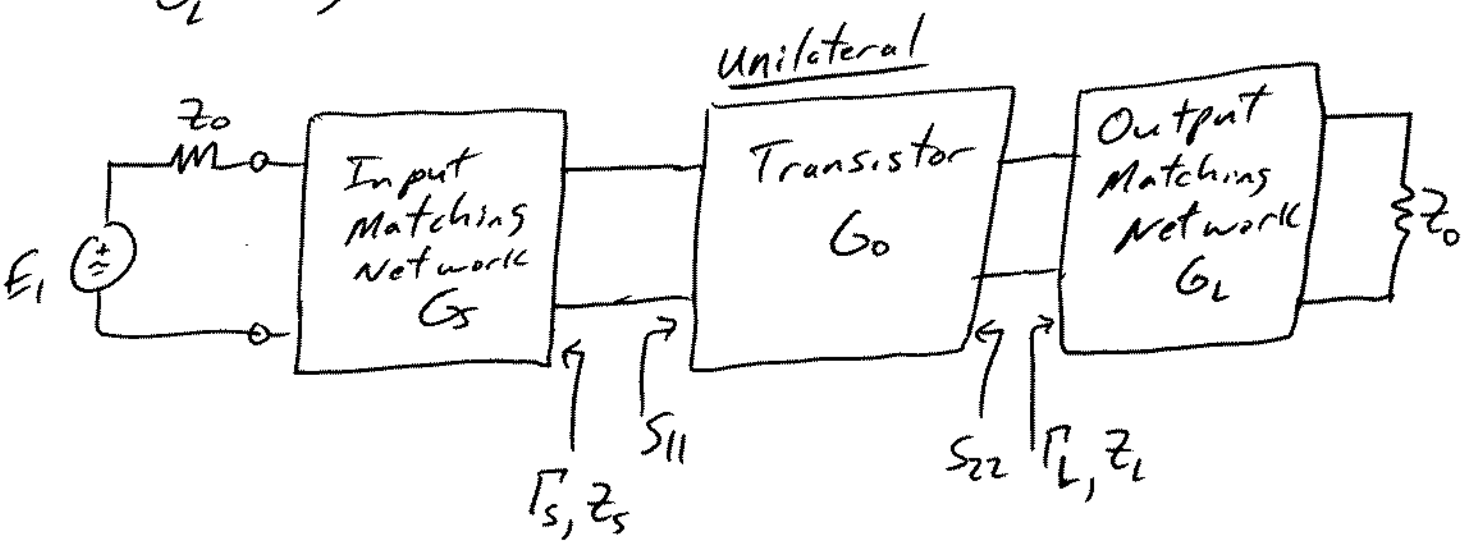
$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} \, |S_{21}|^2 \, \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$= G_S \, G_0 \, G_L$$

G_S ~ gain/loss of input matching network

G_0 ~ gain of transistor

G_L ~ gain loss of output matching network



In decibels

$$G_{TU}(dB) = G_S(dB) + G_0(dB) + G_L(dB)$$

3.4 cont.

Maximum transducer power gain $G_{TU,max}$ for $|S_{11}|<1$ + $|S_{22}|<1$ occurs when

$$\Gamma_S = S_{11}^* \quad \text{and} \quad \Gamma_L = S_{22}^*$$

$$\hookrightarrow \quad G_{S max} = \frac{1}{1-|S_{11}|^2}$$

$$G_{L,max} = \frac{1}{1-|S_{22}|^2}$$

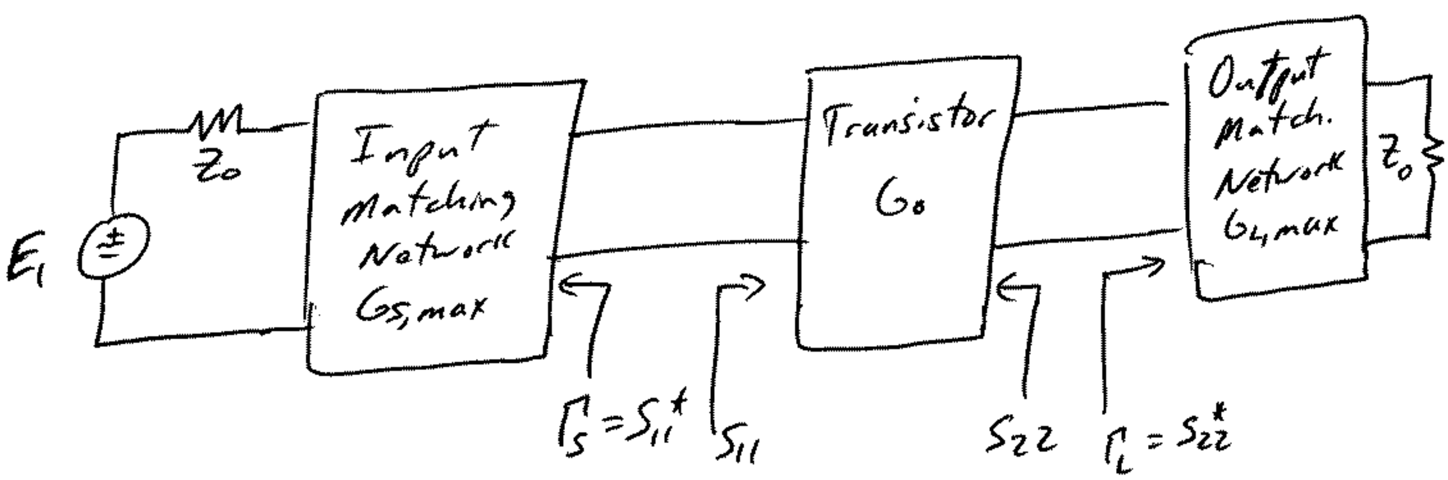
and

$$G_{TU,max} = G_{S max}\, G_0\, G_{L max}$$

$$= \frac{1}{1-|S_{11}|^2}\,|S_{21}|^2\,\frac{1}{1-|S_{22}|^2}$$

Further, since $\Gamma_S = S_{11}^* = \Gamma_{IN}^*$ and $\Gamma_L = S_{22}^* = \Gamma_{OUT}^*$ for the unilateral case

$$G_{TU,max} = G_{PU,max} = G_{AU,max}$$



3.4 cont.

Unconditionally Stable Case, $|S_{ii}|<1$

Let's consider $G_i = \dfrac{1-|\Gamma_i|^2}{|1-S_{ii}\Gamma_i|^2}$

where $i=S$ w/ $ii=11$ (Input Matching network)
$i=L$ w/ $ii=22$ (Output Matching network)

We have seen that $G_{i,max} = \dfrac{1}{1-|S_{ii}|^2}$
when $\Gamma_i = S_{ii}^*$ (optimum termination).

$G_{i,min} = 0$ when $|\Gamma_i| = 1$ (everything reflected; on outside edge of Smith Cht)

Therefore, $0 \le G_i \le G_{i,max}$

Now, in an actual design, achieving $\Gamma_i = S_{ii}^*$ might be difficult or undesirable for another reason. Therefore, it is of great interest to know the values of Γ_i which result in constant values of G_i (less than $G_{i,max}$) & it turns out that these Γ_i fall on circles on the Smith Chart, e.g., constant G_S circles about S_{11}^* and constant G_L circles about S_{22}^*

3.4 cont.

Normalized gain factor $\equiv g_i = \dfrac{G_i}{G_{i,max}} = G_i\left(1-|S_{ii}|^2\right)$ ← you pick

$$= \frac{1-|\Gamma_i|^2}{|1-S_{ii}\Gamma_i|^2}\left(1-|S_{ii}|^2\right)$$

where $0 \le g_i \le 1$.

The constant g_i circle equation is

$$|\Gamma_i - C_{g_i}| = r_{g_i}$$

where

Circle center $\equiv C_{g_i} = \dfrac{g_i S_{ii}^*}{1-|S_{ii}|^2(1-g_i)}$ ← complex # on Γ_i plane

Circle radius $\equiv r_{g_i} = \dfrac{\sqrt{1-g_i}\,\left(1-|S_{ii}|^2\right)}{1-|S_{ii}|^2(1-g_i)}$

Note: $\measuredangle C_{g_i} = \measuredangle S_{ii}^*$

Note! $G_i = 1 = 0\,dB$ or $g_{i,0dB} = 1-|S_{ii}|^2$

circle always passes thru $\Gamma_i = 0$ (center of Smith Chart)

and $r_{g_{i,0dB}} = |C_{g_{i,0dB}}| = \dfrac{|S_{ii}|}{1+|S_{ii}|^2}$

3.4 cont.

Typical Procedure for drawing G_i circles on an impedance Smith Chart

1) Plot S_{ii}^* on Smith Chart and calculate

$$G_{i,max} = \frac{1}{1-|S_{ii}|^2} \qquad G_{i,max}(dB) = 10\log_{10} G_{i,max}$$

2) Select values of G_i # in the range $0 \le G_i \le G_{i,max}$ where you wish to draw circles of Γ_i which result in constant G_i. Then, calculate corresponding normalized gains $g_i = \frac{G_i}{G_{i,max}}$.

3) Calculate $C_{g_i} = \dfrac{g_i S_{ii}^*}{1-|S_{ii}|^2(1-g_i)}$

for each g_i

4) Calculate $r_{g_i} = \dfrac{\sqrt{1-g_i}\,(1-|S_{ii}|^2)}{1-|S_{ii}|^2(1-g_i)}$

for each g_i

5) Plot circles on Smith Chart(s)

Note: $G_i\ (dB) = 10\log_{10} G_i$

So $G_i = 10^{\frac{G_i(dB)}{10}}$

Example- Let's base our unilateral constant gain circles example on an Advanced Semiconductor, Inc. MLN2037F NPN silicon RF power transistor operating at $1\,\text{GHz}$.

Using the datasheet, the S parameters at 1 GHz are:

$$S_{11} = 0.8128\angle 156°, \quad S_{12} \approx 0 \text{ (assumed)}$$

$$S_{21} = 1.81\angle 55°, \quad S_{22} = 0.3801\angle -136°$$

****

Input Matching Network- circles of constant G_S on Γ_S Smith Chart

1) Plot $S_{11}^* = 0.8128\angle -156°$ on Γ_S Smith Chart (see following page) where

$$G_{S,\max} = \frac{1}{1-|S_{11}|^2} = \frac{1}{1-0.8128^2}$$
$$= 2.9467566 = 4.6934\text{ dB}$$

2) Let's plot $G_S = 3, 2, 1, 0, \text{and -1 dB}$. To simplify the process, use MathCad (see following pages) to compute the corresponding G_S and g_S. The results are summarized in Table 1.

3) Calculate corresponding C_{g_S} using MathCad. The results are summarized in Table 1.

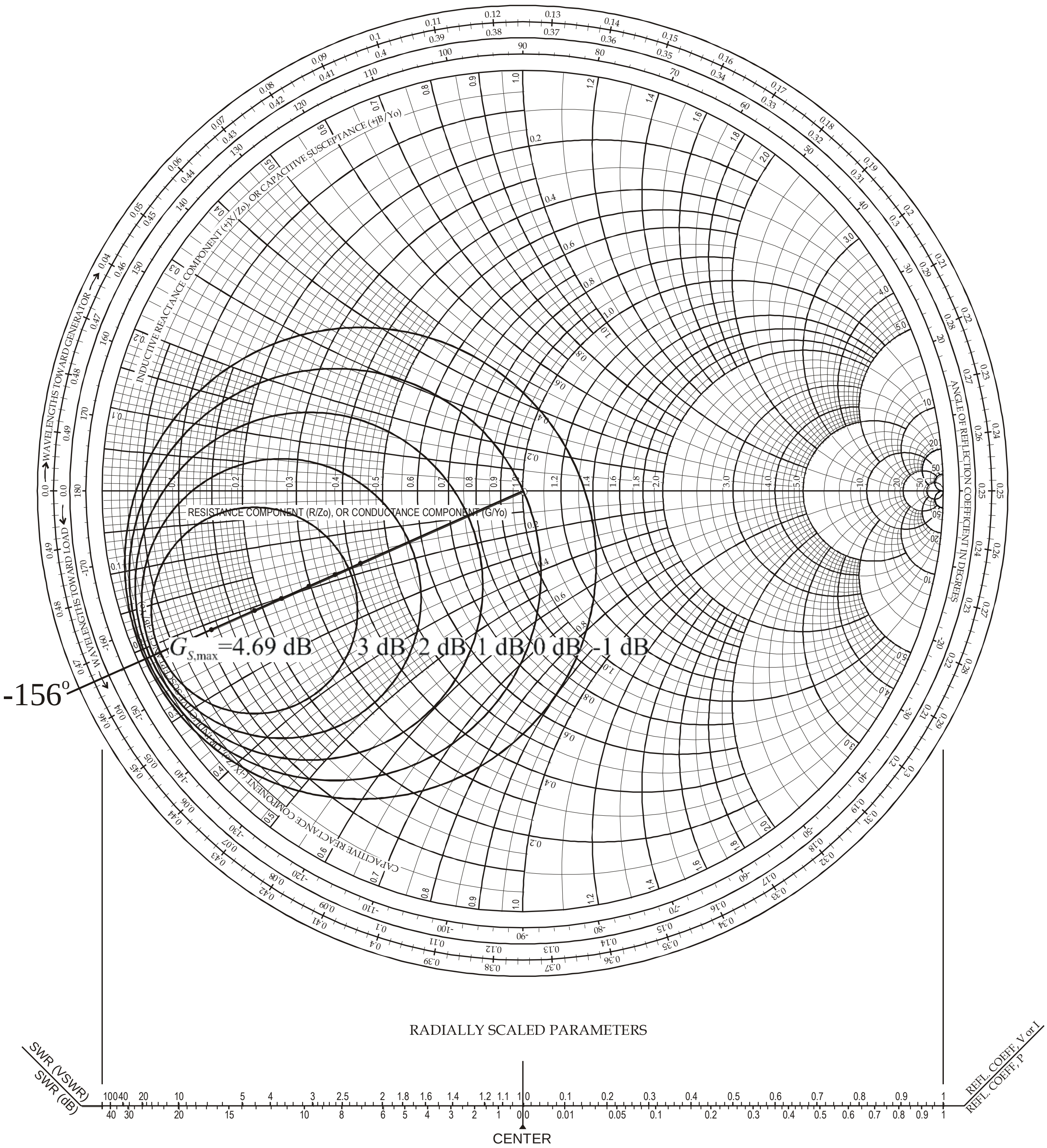
4) Calculate corresponding r_{g_S} using MathCad. The results are summarized in Table 1.

Table 1 Circles of constant G_S information

G_S (dB)	3	2	1	0	-1
G_S	1.9953	1.5849	1.2589	1	0.7943
g_S	0.6771	0.53784	0.42722	0.33936	0.26956
$\lvert C_{g_S} \rvert$	0.6996	0.6293	0.5586	0.4894	0.4234
$\angle C_{g_S}$	$-156°$	$-156°$	$-156°$	$-156°$	$-156°$
r_{g_S}	0.24513	0.3321	0.41318	0.48945	0.56052

5) Plot Γ_S circles for constant G_S (dB) on Smith Chart (used CorelDraw).

Γ_S Smith Chart



<u>Output Matching Network</u>- circles of constant G_L on Γ_L Smith Chart

1) Plot $S_{22}^* = 0.3801\angle 136°$ on Γ_S Smith Chart (see following page) where

$$G_{L,\max} = \frac{1}{1-|S_{22}|^2} = \frac{1}{1-0.3801^2}$$
$$= 1.16887 = 0.67768 \text{ dB}$$

2) Let's plot $G_L = 0, \text{-1}, \text{-2}, \text{and -3 dB}$. To simplify the process, use MathCad (see following pages) to compute the corresponding G_L and g_L. The results are summarized in Table 2.

3) Calculate corresponding C_{g_L} using MathCad. The results are summarized in Table 2.

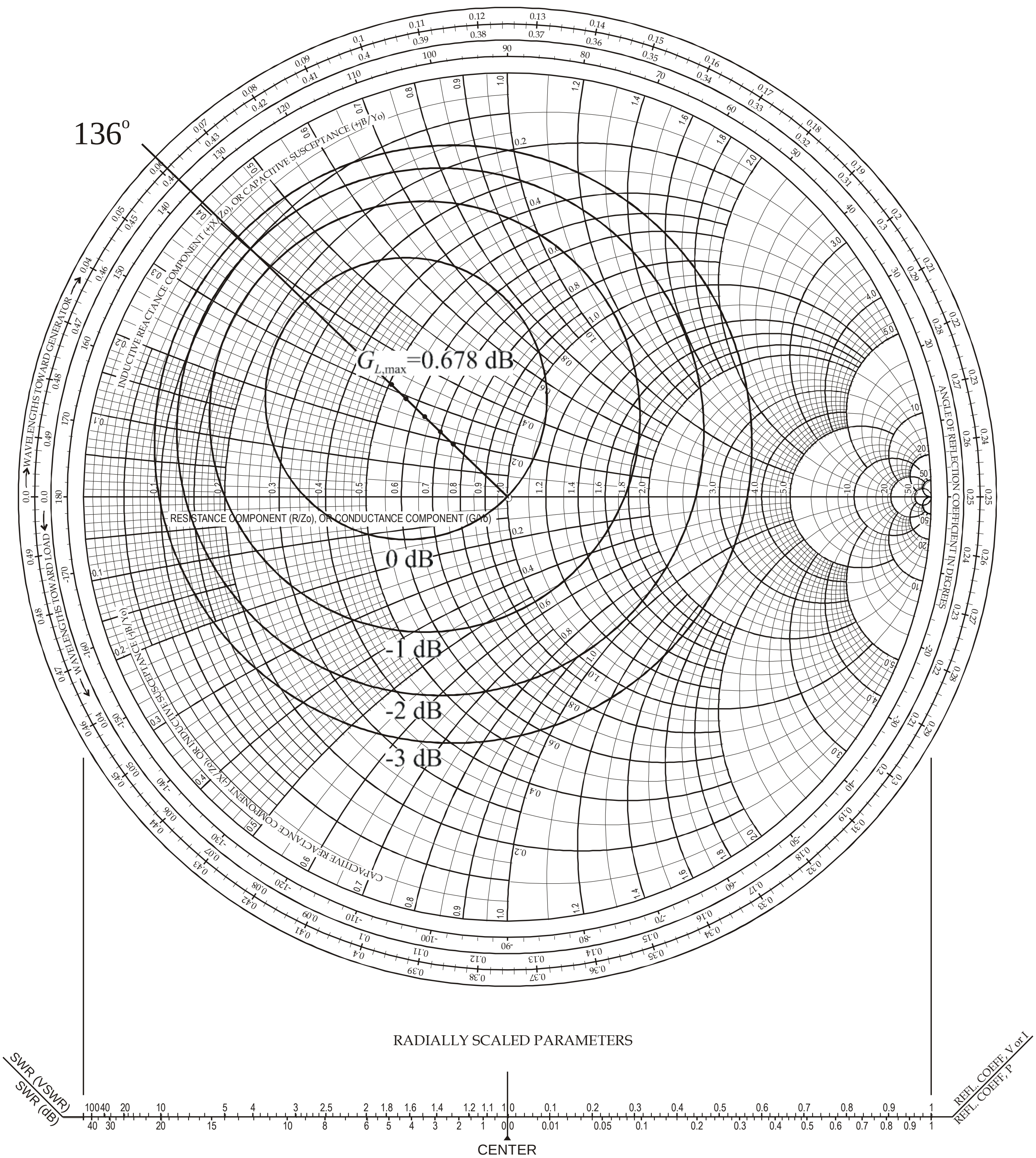
4) Calculate corresponding r_{g_L} using MathCad. The results are summarized in Table 2.

Table 2 Circles of constant G_L information

G_L (dB)	0	-1	-2	-3
G_L	1	0.7943	0.631	0.5012
g_L	0.85552	0.67957	0.5398	0.4288
$\lvert C_{g_L} \rvert$	0.33212	0.2708	0.2198	0.1776
$\angle C_{gL}$	136°	136°	136°	136°
r_{g_L}	0.33212	0.50779	0.62171	0.70476

5) Plot Γ_L circles for constant G_L (dB) on Smith Chart (used CorelDraw).

Γ_L Smith Chart



Unilateral gain circles example for MLN2037F at 1 GHz

$$S11 := 0.8128 \cdot e^{j \cdot 156 \cdot \frac{\pi}{180}} \qquad S12 := 0$$

$$S21 := 1.81 \cdot e^{j \cdot 55 \cdot \frac{\pi}{180}} \qquad S22 := 0.3801 \cdot e^{j \cdot -136 \cdot \frac{\pi}{180}}$$

<u>First, calculate GS circle information</u>

$$GSmax := \frac{1}{1 - (|S11|)^2} \qquad GSmaxdB := 10 \cdot \log(GSmax) \qquad GSmax = 2.94676$$

$$GSmaxdB = 4.69344 \ \ \text{dB}$$

$$GS\_3dB := 10^{\frac{3}{10}} \qquad GS\_3dB = 1.9953 \qquad gS\_3dB := \frac{GS\_3dB}{GSmax} \qquad gS\_3dB = 0.6771$$

$$GS\_2dB := 10^{\frac{2}{10}} \qquad GS\_2dB = 1.5849 \qquad gS\_2dB := \frac{GS\_2dB}{GSmax} \qquad gS\_2dB = 0.53784$$

$$GS\_1dB := 10^{\frac{1}{10}} \qquad GS\_1dB = 1.2589 \qquad gS\_1dB := \frac{GS\_1dB}{GSmax} \qquad gS\_1dB = 0.42722$$

$$GS\_0dB := 10^{\frac{0}{10}} \qquad GS\_0dB = 1 \qquad gS\_0dB := \frac{GS\_0dB}{GSmax} \qquad gS\_0dB = 0.33936$$

$$GS\_m1dB := 10^{\frac{-1}{10}} \qquad GS\_m1dB = 0.7943 \qquad gS\_m1dB := \frac{GS\_m1dB}{GSmax} \qquad gS\_m1dB = 0.26956$$

$$CgS3dB := \frac{gS\_3dB \cdot \overline{S11}}{1 - (|S11|)^2 \cdot (1 - gS\_3dB)}$$

$$|CgS3dB| = 0.6996$$

$$arg(CgS3dB) \cdot \frac{180}{\pi} = -156$$

$$rgS3dB := \frac{\sqrt{1 - gS\_3dB} \cdot \left[1 - (|S11|)^2\right]}{1 - (|S11|)^2 \cdot (1 - gS\_3dB)}$$

$$rgS3dB = 0.24513$$

$$CgS2dB := \frac{gS\_2dB \cdot \overline{S11}}{1 - (|S11|)^2 \cdot (1 - gS\_2dB)}$$

$$|CgS2dB| = 0.6293$$

$$arg(CgS2dB) \cdot \frac{180}{\pi} = -156$$

$$rgS2dB := \frac{\sqrt{1 - gS\_2dB} \cdot \left[1 - (|S11|)^2\right]}{1 - (|S11|)^2 \cdot (1 - gS\_2dB)}$$

$$rgS2dB = 0.3321$$

$$CgS1dB := \frac{gS\_1dB \cdot \overline{S11}}{1 - (|S11|)^2 \cdot (1 - gS\_1dB)}$$

$$|CgS1dB| = 0.5586$$

$$arg(CgS1dB) \cdot \frac{180}{\pi} = -156$$

$$rgS1dB := \frac{\sqrt{1 - gS\_1dB} \cdot \left[1 - (|S11|)^2\right]}{1 - (|S11|)^2 \cdot (1 - gS\_1dB)}$$

$$rgS1dB = 0.41318$$

$$CgS0dB := \frac{gS\_0dB \cdot \overline{S11}}{1 - (|S11|)^2 \cdot (1 - gS\_0dB)}$$

$$|CgS0dB| = 0.4894$$

$$arg(CgS0dB) \cdot \frac{180}{\pi} = -156$$

$$rgS0dB := \frac{\sqrt{1 - gS\_0dB} \cdot \left[1 - (|S11|)^2\right]}{1 - (|S11|)^2 \cdot (1 - gS\_0dB)}$$

$$rgS0dB = 0.48945$$

$$CgSm1dB := \frac{gS\_m1dB \cdot \overline{S11}}{1 - (|S11|)^2 \cdot (1 - gS\_m1dB)}$$

$$|CgSm1dB| = 0.4234$$

$$arg(CgSm1dB) \cdot \frac{180}{\pi} = -156$$

$$rgSm1dB := \frac{\sqrt{1 - gS\_m1dB} \cdot \left[1 - (|S11|)^2\right]}{1 - (|S11|)^2 \cdot (1 - gS\_m1dB)}$$

$$rgSm1dB = 0.56052$$

Next, calculate GL circle information

$$GLmax := \frac{1}{1-(|S22|)^2} \qquad GLmaxdB := 10\cdot\log(GLmax) \qquad GLmax = 1.16887$$

$$GLmaxdB = 0.67768 \ \ \mathrm{dB}$$

$$GL\_0dB := 10^{\frac{0}{10}} \qquad GL\_0dB = 1 \qquad gL\_0dB := \frac{GL\_0dB}{GLmax} \qquad gL\_0dB = 0.85552$$

$$GL\_m1dB := 10^{\frac{-1}{10}} \qquad GL\_m1dB = 0.7943 \qquad gL\_m1dB := \frac{GL\_m1dB}{GLmax} \qquad gL\_m1dB = 0.67957$$

$$GL\_m2dB := 10^{\frac{-2}{10}} \qquad GL\_m2dB = 0.631 \qquad gL\_m2dB := \frac{GL\_m2dB}{GLmax} \qquad gL\_m2dB = 0.5398$$

$$GL\_m3dB := 10^{\frac{-3}{10}} \qquad GL\_m3dB = 0.5012 \qquad gL\_m3dB := \frac{GL\_m3dB}{GLmax} \qquad gL\_m3dB = 0.4288$$

$$CgL0dB := \frac{gL\_0dB \cdot \overline{S22}}{1 - (|S22|)^2 \cdot (1 - gL\_0dB)}$$

$$rgL0dB := \frac{\sqrt{1 - gL\_0dB} \cdot \left[1 - (|S22|)^2\right]}{1 - (|S22|)^2 \cdot (1 - gL\_0dB)}$$

$$|CgL0dB| = 0.3321$$

$$\arg(CgL0dB) \cdot \frac{180}{\pi} = 136$$

$$rgL0dB = 0.33212$$

$$CgLm1dB := \frac{gL\_m1dB \cdot \overline{S22}}{1 - (|S22|)^2 \cdot (1 - gL\_m1dB)}$$

$$rgLm1dB := \frac{\sqrt{1 - gL\_m1dB} \cdot \left[1 - (|S22|)^2\right]}{1 - (|S22|)^2 \cdot (1 - gL\_m1dB)}$$

$$|CgLm1dB| = 0.2708$$

$$\arg(CgLm1dB) \cdot \frac{180}{\pi} = 136$$

$$rgLm1dB = 0.50779$$

$$CgLm2dB := \frac{gL\_m2dB \cdot \overline{S22}}{1 - (|S22|)^2 \cdot (1 - gL\_m2dB)}$$

$$rgLm2dB := \frac{\sqrt{1 - gL\_m2dB} \cdot \left[1 - (|S22|)^2\right]}{1 - (|S22|)^2 \cdot (1 - gL\_m2dB)}$$

$$|CgLm2dB| = 0.2198$$

$$\arg(CgLm2dB) \cdot \frac{180}{\pi} = 136$$

$$rgLm2dB = 0.62171$$

$$CgLm3dB := \frac{gL\_m3dB \cdot \overline{S22}}{1 - (|S22|)^2 \cdot (1 - gL\_m3dB)}$$

$$rgLm3dB := \frac{\sqrt{1 - gL\_m3dB} \cdot \left[1 - (|S22|)^2\right]}{1 - (|S22|)^2 \cdot (1 - gL\_m3dB)}$$

$$|CgLm3dB| = 0.1776$$

$$\arg(CgLm3dB) \cdot \frac{180}{\pi} = 136$$

$$rgLm3dB = 0.70476$$

3.4 cont.

Potentially Unstable Case, $|S_{ii}| > 1$

* Still unilateral ($S_{12} \simeq 0$)
* In practice, this would be unusual for a transistor.

When $|S_{ii}| > 1$, it is possible for

$$G_i = \frac{1 - |\Gamma_i|^2}{|1 - S_{ii}\Gamma_i|^2} \longrightarrow \infty \quad \text{when } \Gamma_{i,c} = \frac{1}{S_{ii}}$$

critical Γ_i

Note: $i = S$ w/ $ii = 11$ (Input)
$i = L$ w/ $ii = 22$ (Output)

This implies that $Re\{Z_{i,c}\}$ or positive resistance exactly cancels the negative $Re\{Z_{ii}\}$

where $Z_{i,c} = Z_0 \dfrac{1 + \Gamma_{i,c}}{1 - \Gamma_{i,c}}$ + $Z_{ii} = Z_0 \dfrac{1 + S_{ii}}{1 - S_{ii}}$

$\Downarrow$

Zero input/output loop resistance!

Plots

1) Obviously, $G_{i,max} = \infty$ and occurs at $\Gamma_{i,c} = \frac{1}{S_{ii}}$ and this point is along the $\measuredangle S_{ii}^*$ line

2) Select values of G_i where you wish to draw circles of Γ_i which result in constant G_i.

$$G_i = 10^{G_i(dB)/10}$$

3.4 cont.

2) cont. calculate corresponding normalized gain factor $g_i = G_i \left(1-|S_{ii}|^2\right) = \frac{1-|\Gamma_i|^2}{|1-S_{ii}\Gamma_i|^2}\left(1-|S_{ii}|^2\right)$

Note! g_i can now be a negative number!

3) Calculate $C_{g_i} = \frac{g_i S_{ii}^*}{1-|S_{ii}|^2(1-g_i)}$ for each g_i

4) Calculate $r_{g_i} = \frac{\sqrt{1-g_i}\left(1-|S_{ii}|^2\right)}{1-|S_{ii}|^2(1-g_i)}$ for each g_i

5) Plot circles on Γ_i Smith Chart(s)

How do I know if a particular Γ_i is in the stable region?

$$\text{Need} \begin{cases} Re\{Z_S\} > |Re(Z_{IN})| & \text{and/or} \\ Re\{Z_L\} > |Re(Z_{OUT})| \end{cases}$$

where $Z_S = Z_0 \frac{1+\Gamma_S}{1-\Gamma_S}$ and $Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L}$

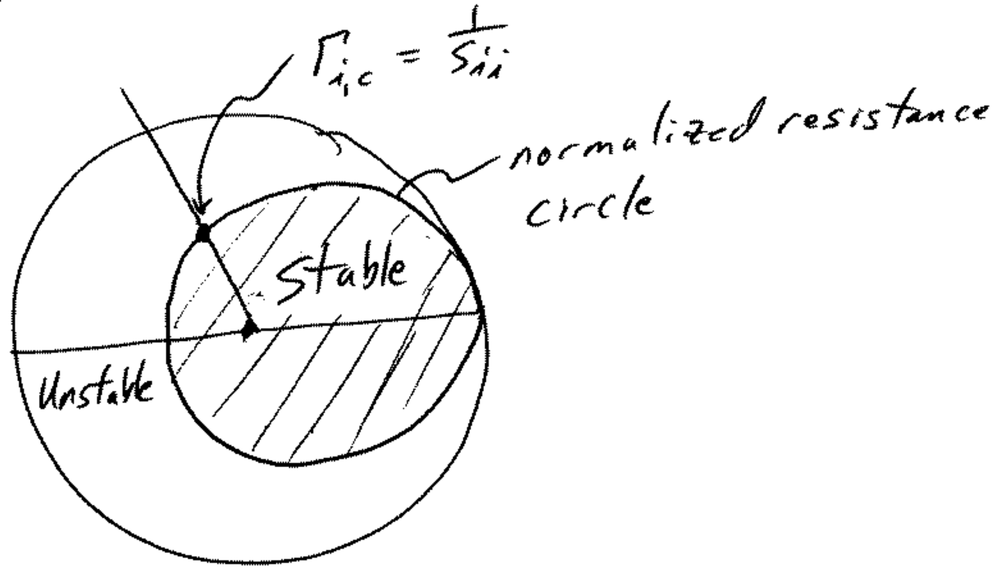
$Z_{IN} = Z_0 \frac{1+S_{11}}{1-S_{11}}$ and $Z_{OUT} = Z_0 \frac{1+S_{22}}{1-S_{22}}$

$\Rightarrow$ For plotting on Smith Chart, divide by Z_0.

3.4 cont.

Rather than doing complicated calculations, the normalized resistance circle passing thru $\Gamma_{i,c} = \frac{1}{S_{ii}}$ defines the boundary between the stable (all larger normalized resistances) and unstable regions on the Smith Chart

e.g.



3.5 Unilateral Figure of Merit

What if $S_{12} \neq 0$, but we wish to assume (i.e., set $S_{12} = 0$) so we can take advantage of all the simplifications and constant gain circles? How can we tell how good our results will be in this case?

We'll compare the magnitude ratio of G_T w/ the unilateral G_{TU} (ideally equal to 1)

$$\frac{G_T}{G_{TU}} = \frac{1}{|1 - X|^2}$$

where

$$X = \frac{S_{12} S_{21} \Gamma_S \Gamma_L}{(1 - S_{11}\Gamma_S)(1 - S_{22}\Gamma_L)} \quad \leftarrow \text{complex \#}$$

This ratio is bounded as

$$\frac{1}{(1 + |X|)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1 - |X|)^2}$$

One case of particular interest is $G_{TU,max}$ which occurs when $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$

3.5 cont.

For $G_{TU,max}$

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU,max}} < \frac{1}{(1-U)^2}$$

where $$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} \equiv \text{Unilateral Figure of Merit}$$

Easier to calculate than X

3.5 cont.

Example – Let's revisit our earlier examples w/ the MLN2037F NPN Silicon RF power transistor operating at 1GHz w/ $\Gamma_S = 0.8\angle -150°$ and $\Gamma_L = 0.3\angle 140°$

and
$$S_{11} = 0.8128\angle 156° \qquad S_{12} = 0.0944\angle 63°$$
$$S_{21} = 1.81\angle 55° \qquad S_{22} = 0.3801\angle -136°$$

Here
$$X = \frac{(0.0944\angle 63°)(1.81\angle 55°)(0.8\angle -150°)(0.3\angle 140°)}{[1-(0.8128\angle 156°)(0.8\angle -150°)][1-(0.3801\angle -136°)(0.3\angle 140°)]}$$

$$= 0.12859607\angle 119.403205°$$

and
$$\frac{1}{(1+0.1286)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-0.1286)^2}$$

$$0.7850963 < \frac{G_T}{G_{TU}} < 1.316924786$$

$$-1.05077\,dB < \frac{G_T}{G_{TU}} < 1.19561\,dB$$

Max. Error is $\sim \pm 1.1\,dB$
(Not too good)

3.5 cont.

Example cont. -

What if we let $\Gamma_S = S_{11}^* = 0.8128\angle{-156°}$

and $\Gamma_L = S_{22}^* = 0.3801\angle{+136°}$ {the $G_{TU,max}$ case)?

Now
$$U = \frac{(0.0944)(1.81)(0.8128)(0.3801)}{(1-0.8128^2)(1-0.3801^2)}$$

$$U = 0.181821068$$

$$\frac{1}{(1+0.18182)^2} < \frac{G_T}{G_{TU,max}} < \frac{1}{(1-0.18182)^2}$$

$$0.71597283 < \frac{G_T}{G_{TU,max}} < 1.4938377$$

$$-1.451\,dB < \frac{G_T}{G_{TU,max}} < 1.743\,dB$$

Max. error is worse under this assumption.

$\Rightarrow$ All in all, the unilateral assumption for the MLN2037F @ 1GHz is pretty poor.

Unilateral Figure of Merit example for MLN2037F at 1 GHz

$$S11 := 0.8128 \cdot e^{j \cdot 156 \cdot \frac{\pi}{180}} \qquad S12 := 0.0944 \cdot e^{j \cdot 63 \cdot \frac{\pi}{180}}$$

$$S21 := 1.81 \cdot e^{j \cdot 55 \cdot \frac{\pi}{180}} \qquad S22 := 0.3801 \cdot e^{j \cdot -136 \cdot \frac{\pi}{180}}$$

$$\Gamma S := 0.8 \cdot e^{j \cdot -150 \cdot \frac{\pi}{180}} \qquad \Gamma L := 0.3 \cdot e^{j \cdot 140 \cdot \frac{\pi}{180}}$$

$$X := \frac{S12 \cdot S21 \cdot \Gamma S \cdot \Gamma L}{(1 - S11 \cdot \Gamma S)(1 - S22 \cdot \Gamma L)} \qquad |X| = 0.128596 \qquad \arg(X) \cdot \frac{180}{\pi} = 119.4032$$

$$\frac{1}{(1 + |X|)^2} = 0.7851 \qquad \frac{1}{(1 - |X|)^2} = 1.31692$$

$$10 \cdot \log\left[\frac{1}{(1 + |X|)^2}\right] = -1.05077 \quad \text{dB} \qquad 10 \cdot \log\left[\frac{1}{(1 - |X|)^2}\right] = 1.19561 \quad \text{dB}$$

$$U := \frac{|S12| \cdot |S21| \cdot |S11| \cdot |S22|}{\left[1 - (|S11|)^2\right]\left[1 - (|S22|)^2\right]} \qquad \boxed{U = 0.181821}$$

$$\frac{1}{(1 + U)^2} = 0.71597 \qquad \frac{1}{(1 - U)^2} = 1.49384$$

$$10 \cdot \log\left[\frac{1}{(1 + U)^2}\right] = -1.45103 \quad \text{dB} \qquad 10 \cdot \log\left[\frac{1}{(1 - U)^2}\right] = 1.74303 \quad \text{dB}$$