EE691 Applied EM- FDTD Method (Spring 2012)
Computer Project 4- 2D Absorbing Boundary Conditions

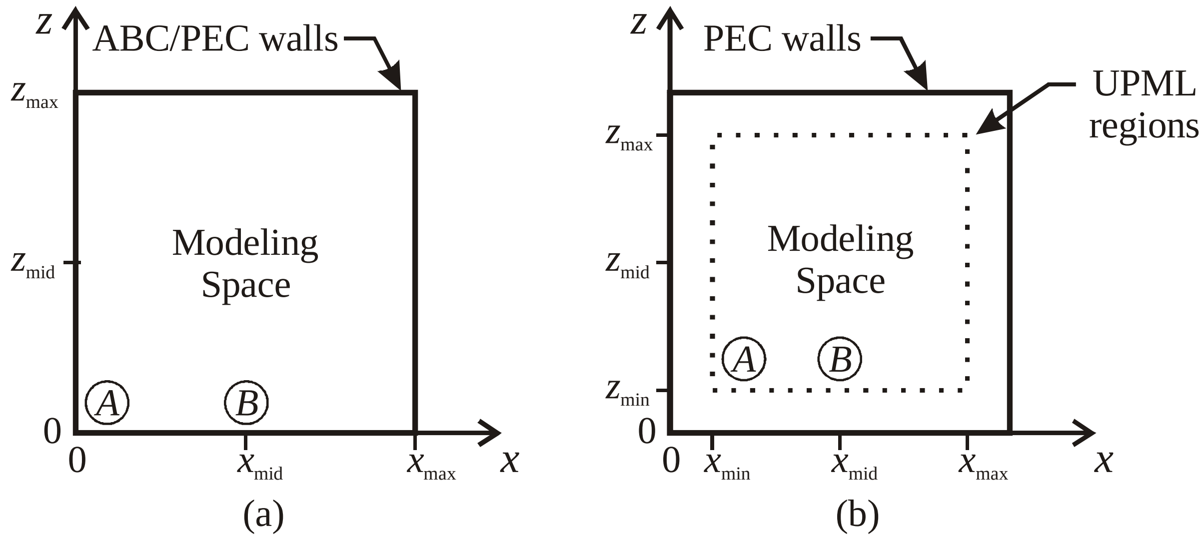

Figure 1 Problem Geometries with (a) analytic ABC or PEC walls and (b) with UPML regions.

Overview

In this project, a TE_y mode ($\mathcal{E}_x$, $\mathcal{E}_z$, and $\mathcal{H}_y$ field components) will be excited in a two-dimensional (2D) lossless dielectric (ε, μ_0) region (i.e., modeling space). Then, the performance of some absorbing boundary conditions (ABCs) will be studied. In Figure 1a, the modeling space is surrounded by an analytic ABC or perfect electrical conductor (PEC) walls (used for comparison). In Figure 1b, the modeling space is shown surrounded by a uniaxial perfectly matched layer (UPML) ABC. The FDTD grid will start and terminate on $\mathcal{E}_z$ components in the *x*-direction and $\mathcal{E}_x$ components in the *z*-direction. The velocity of light in the dielectric is $v_p = 2.5 \times 10^8$ m/s.

The excitation for the problem will be applied to both of the $\mathcal{E}_x$ components closest to the middle of the modeling region (location denoted by indices I_{mid} and K_{mid}) using additive source(s). A time-delayed unit-amplitude differentiated Gaussian pulse

$$\mathcal{E}_{\text{DFG}}(t) = -\frac{\sqrt{e}\;\, t-\tau_d}{\tau_p}\, e^{-0.5\left(\frac{t-\tau_d}{\tau_p}\right)^2} \quad \text{(V/m)}$$

where τ_p is the characteristic time and τ_d is the time-delay will be used for the excitation. The peak in the frequency spectrum of this signal is $f_{\text{pk}} = \dfrac{1}{2\pi\tau_p}$. The values of f_{pk}, τ_d / τ_p, $\Delta = \Delta x = \Delta z$, S and Δt selected for the modeling are given in Table 1.

Table 1 FDTD Modeling Values

Quantity	Value
f_{pk}	**2 GHz**
τ_d / τ_p	**5**
Δ	**2.5 mm**
S	**0.5**
Δt	**5 ps**

Tasks

You may use the programming language(s) and/or mathematics package(s) of your choice for the numerical calculations and plotting results.

1) Draw a figure detailing the overall spatial grid set-up for the problem shown in Figure 1a, including the source node(s), with PEC walls as it will be implemented in your FDTD code. Label field variables and indices.

2) In your report list the governing differential equations and the FDTD update equations needed to model the field components for a TE_y mode both with and without the UPML. Use the Yee cell grid/lattice and indices shown in Figure 1 of the Chapter 3 notes. The update equations should be in standard form and put in terms of the characteristic impedance, Courant stability factor, and phase velocity.

3) Write an FDTD program that will model the problem shown in Figure 1a with PEC walls (no ABC). Make a list of relevant variables in a comment block at the beginning of the program. Make copious use of comment statements to explain what program blocks are doing. Attach program listing as Appendix A.

4) Run the program of part3) for an $80\Delta \times 80\Delta$ grid. Adjacent to the PEC walls, save E_x at point $A(I_{mid} - 37, K_{mid} - 37)$ and at point $B\,(I_{mid}, K_{mid} - 37)$ versus time for $0 \le t \le 80\tau_p$. Next, run the program for a $500\Delta \times 500\Delta$ grid and save the E_x at the same locations relative to the source(s). This (i.e., $500\Delta \times 500\Delta$ grid) will serve as the reference solution. On two graphs (one each for points A and B), plot E_x for each case (two traces) versus normalized time t/τ_p. On the plots, using vertical dashed lines (with the times labeled) indicate the normalized "clear" times, i.e., earliest times when the leading edge of reflection(s) from the PEC walls arrive at points A and B for each grid. Comment on differences.

5) Draw a figure detailing the overall spatial grid set-up for the problem shown in Figure 1a, including the source node(s), with Merewether ABC walls as it will be implemented in your

FDTD code. List update equations and equations to calculate variables necessary to implement the Merewether ABC. Label variables and indices.

6) Write an FDTD program that will model the problem shown in Figure 1a with a Merewether ABC. Make a list of relevant variables in a comment block at the beginning of the program. Make copious use of comment statements to explain what program blocks are doing. Attach program listing as Appendix B.

7) Run the program of part 6) for an $80\Delta \times 80\Delta$ grid. Adjacent to the PEC walls, save E_x at point $A(I_{mid} - 37, K_{mid} - 37)$ and at point B $(I_{mid}, K_{mid} - 37)$ versus time for $0 \le t \le 80\tau_p$. On two graphs (one each for points A and B), plot E_x for the Merewether ABC plus your reference solution from part 4) versus normalized time t/τ_p. Use vertical lines (with the times labeled) to indicate the "clear" times, i.e., earliest times when the leading edge of reflection(s) from the PEC walls can arrive at the observation points. Comment on results.

8) Draw a figure detailing the overall spatial mesh set-up for the problem shown in Figure 1b, including the source node(s), UPML interfaces, and PEC walls as it will be implemented in your FDTD code. Label field variables and indices.

9) Write an FDTD program that will model the problem shown in Figure 1b. Specify whether the UPML updates are implemented everywhere or just outside the modeling region. Within the UPML regions, use polynomial grading for σ_x and σ_z, and let $\kappa_x = \kappa_z = 1$. The user should be able to specify the UPML thickness d in terms of integer increments of Δ (i.e., $M = d/\Delta$), and the order of the polynomial grading m. Make a list of relevant variables in a comment block at the beginning of the program. Make copious use of comment statements to explain what program blocks are doing. Attach program listing as Appendix C.

10) Run the program of part 9) with a 5Δ-thick, $m = 3$, UPML layer around an $80\Delta \times 80\Delta$ modeling region ($90\Delta \times 90\Delta$ grid overall). Save E_x at points A and B versus time for $0 \le t \le 80\tau_p$. Next, run the program with a 10Δ-thick, $m = 3$, UPML layer ($100\Delta \times 100\Delta$ grid overall). Again, save E_x at points A and B versus time for $0 \le t \le 80\tau_p$. What are $\sigma_{x,opt}$ and $\sigma_{z,opt}$ in each case? On two graphs (one each for points A and B), plot E_x for each UPML case plus your reference solution from part 4) versus normalized time t/τ_p. Use vertical lines (with the normalized times labeled) to indicate the "clear" times, i.e., earliest times when a leading edge(s) of reflection(s) from the PEC walls for can arrive at the observation points. Comment on results.

11) Next, on a two graphs (one each for points *A* and *B*), calculate and plot the relative errors [see (7.135) in text] of E_x for the Merewether ABC and the two UPML ABCs with respect to the reference solution versus normalized time t/τ_p up to the "clear" time at each point. Use a log scale for the vertical axis [see Fig. 7.3 in text]. Comment on results.

12) (Optional/ extra credit) On a two graphs (one each for points *A* and *B*), calculate and plot the

$$\text{Reflection Error (dB)} = 20\log_{10}\left[\left(\left|E^k_{x,\text{UPML}}\right| - \left|E^k_{x,\text{ref}}\right|\right)\right]$$

where $\left|E^k_{x,\text{UPML}}\right|$ and $\left|E^k_{x,\text{ref}}\right|$ are the discrete Fourier transforms of E^n_x for the ABCs and for the reference case versus normalized frequency f/f_{pk} for $0 \le f/f_{\text{pk}} \le 5$. For the discrete Fourier transforms, use only the data up to the "clear" time for the reference case at each point. Comment on results.

Report

Write a report covering your results. The report should be as short as possible, while remaining complete in its description of your work. Begin with an overview followed by a discussion of the problem. Then proceed with a description of what you did for this project, and a presentation and discussion of your results. End with your conclusions and their significance.

General format: Cover page, Introduction, Body, Conclusions/Summary, & Appendice(s). Use 12 point Times New Roman font with 1.25/1.5 line spacing. The report must be totally produced on a computer using, among other software, a word processor, an equation editor, and a data plotting package.

Due Wednesday, April 4, 2012.