EE691 Applied EM- FDTD Method (Spring 2012)
Computer Project 1- 1D Scalar Wave Equation

Overview

In Chapter 2 of the text [1], we discussed applying the finite-difference time-domain (FDTD) method to the one-dimensional (1D) scalar wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. For the free-space 1D grid shown in Figure 1, input, at spatial location i = 0, a single, positive, unit-amplitude, sinusoidal (T = 100 ms) half-cycle signal/pulse (see Figure 2). Scale the grid so that the signal/pulse is 50 spatial steps in the grid. Select $I_{\max} = 250$.

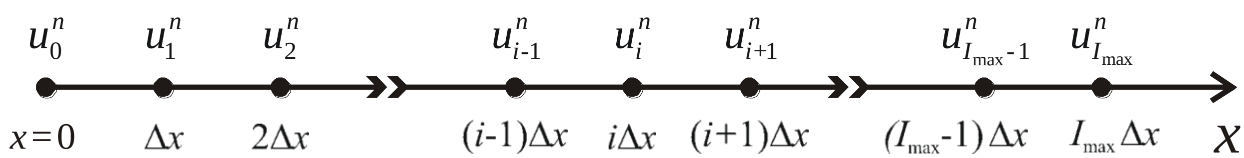

Figure 1 Grid for 1D scalar wave equation

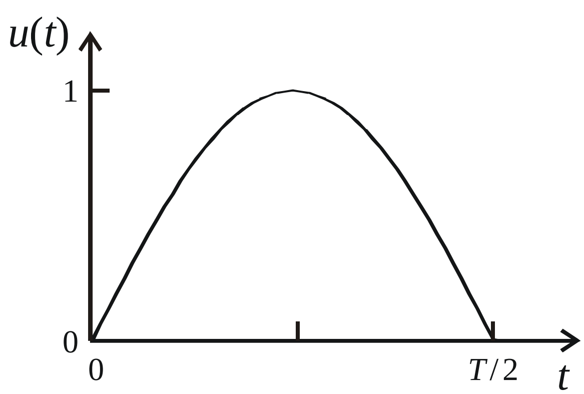

Figure 2 Input signal/pulse for 1D scalar wave equation

Tasks

You may use the programming language(s) and/or mathematics package(s) of your choice for the numerical calculations and plotting of results.

1) Write the appropriate mathematical equation to define a single, positive, unit-amplitude, sinusoidal half-cycle signal/pulse that is 50 spatial steps wide at u_0^n in terms of the variables i, n, Δx, Δt, and S (as applicable).

2) Write an FDTD program that will model the 1D scalar grid shown in Figure 1 for the input signal/pulse found in step 1). Include a code/program listing(s) in the report as an appendix. The report should include figure(s) detailing the overall mesh set-up (with corresponding variables and indices). Clearly identify and define important variables in a comment block at the beginning of the code/program. Make copious use of comment statements to explain what code/program blocks are doing.

3) Run your program for Courant stability factors S = 1, 0.99, and 0.5 to create plots similar to those shown in Figures 2.3 and 2.4 of the text [1]. In each case, if no dispersion occurred, the leading edge of the signal/pulse should be at $i = 200$. The horizontal (spatial) axis should be for $0 \leq i \leq 250$. In the plot legends, include S, the applicable time & spatial step sizes as well as the time index where each trace in plot was 'snapshot'. Discuss the results both qualitatively and quantitatively. Where possible, compare modeling results with results expected from theory.

4) Run your program with a Courant stability factor of $S = 1.0002$ to create plots similar to those shown in Figure 2.6 of the text [1]. I.e., show snapshots of the signal/pulse at $n = 200$, 210, and 220 with one plot having a horizontal axis of $0 \leq i \leq 250$ while the other has a horizontal axis of $0 \leq i \leq 20$. What are the applicable time & spatial step sizes for $S = 1.0002$? Discuss the results both qualitatively and quantitatively. Where possible, compare modeling results with results expected from theory. For example, does the instability have the expected period/wavelength?

<u>Report</u>

Write a report covering your results. The report should be as short as possible, while remaining complete in its description of your work. Begin with an overview followed by a discussion of the problem. Then proceed with a description of what you did for this project, and a presentation and discussion of your results. End with your conclusions and their significance.

General format: Cover page, Introduction, Body, Conclusions/Summary, & Appendice(s). Use 12 point Times New Roman font with 1.25/1.5 line spacing. The report must be totally produced on a computer using, among other software, a word processor, an equation editor, and a data plotting package.

<u>Due Monday February 13, 2012</u>