Homework 5
EE691-82 Applied EM- FDTD Method, Spring 2012
Wednesday, March 14, 2012

Given an input signal $x(t) = \begin{cases} 10\sin\left(\frac{2\pi}{T}t\right) & 0 \le t \le T/2 \\ 0 & \text{elsewhere} \end{cases}$.

1) Determine the Fourier transform $X(f)$ of $x(t)$ (will be a function of T). Use the Fourier transform pair definition

$$v(t) = \int_{-\infty}^{\infty} V(f) e^{+j2\pi ft} df \quad \leftrightarrow \quad \mathcal{F}\{v(t)\} = V(f) = \int_{-\infty}^{\infty} v(t) e^{-j2\pi ft} dt .$$

2) For T = 10 μs, plot $|X|$ versus the normalized frequency $f_{\text{norm}} = f / f_T$ where $f_T = 1/T$ for $0 \le f_{\text{norm}} \le 10$. At what <u>normalized</u> frequency f_{peak} does the peak value of $|X|$ occur? At what <u>normalized</u> frequency $f_{\text{null\_1}}$ does the first null (i.e., $|X| = 0$) occur?

3) Using the result of 1), determine $|X(f)|_{\max}$ with T = 10 μs. Then, find the frequency $f_{\max}$ such that $|X(f_{\max})| / |X(f)|_{\max} \le 0.5*10^{-3}$ for all $f \ge f_{\max}$. Then, determine the spatial step-size Δ for N_λ = 10 in free space. With stability guaranteed, determine the maximum temporal step-sizes $\Delta t_{\max} = \Delta t_{1D}$, Δt_{2D}, and Δt_{3D} for 1-dimensional, square 2-dimensional, and cubic 3-dimensional FDTD lattices, respectively.

4) For the input signal $x(t)$ given in 1), determine $x(t = \Delta t)$ using for the cases where $\Delta t_{\max} = \Delta t_{1D}$, Δt_{2D}, & Δt_{3D} from 3). Then, find $|x(t = \Delta t_{\max})| / |x(t)|_{\max}$ for all three cases (express in both unitless and decibels forms).

Show **<u>all</u>** work for full credit.
Due Wednesday, March 21, 2012