Chapter 8 Near-to-Far-Field Transformation

→ use NTFF for Near-to-Far-Field

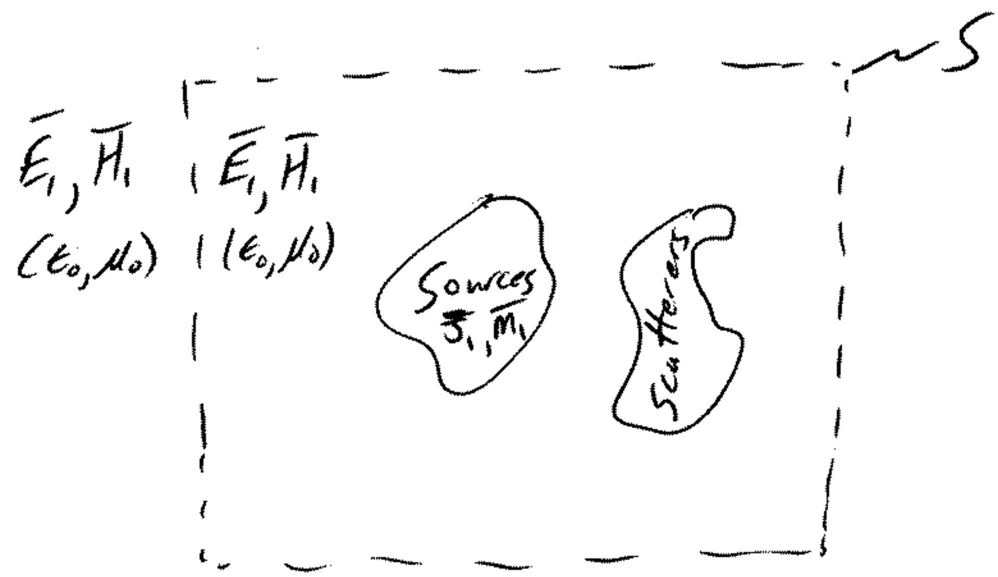
→ Do NOT need to extend FDTD modeling space to the far-field, but can transform near-field data to obtain far-field results

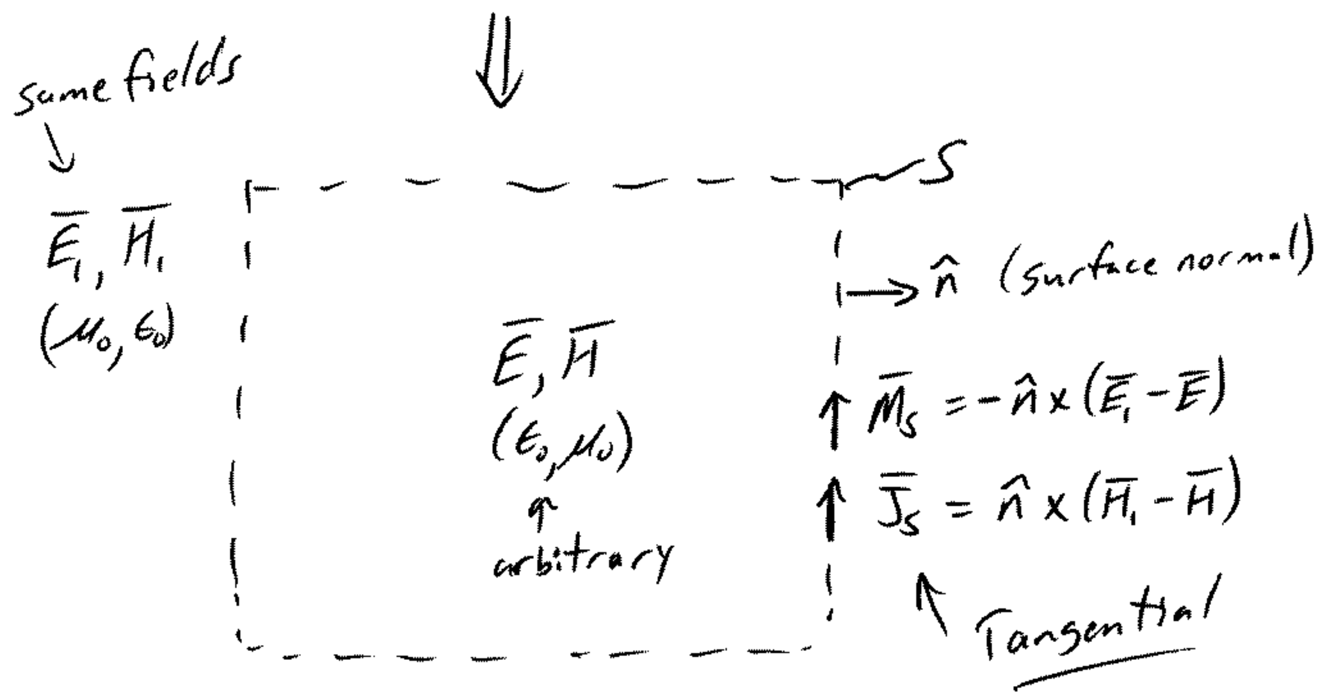
→ often done in experimental situations where measurements in the near-field are used to compute far-field results via a NTFF transformation.

→ we'll only consider the time-domain NTFF transformations

8.4 Surface Equivalence Theorem

→ NTFF transforms based on the premise that if we know the tangential field components on a closed contour/surface that we can find the fields external to that surface w/out knowing what is going on in the interior

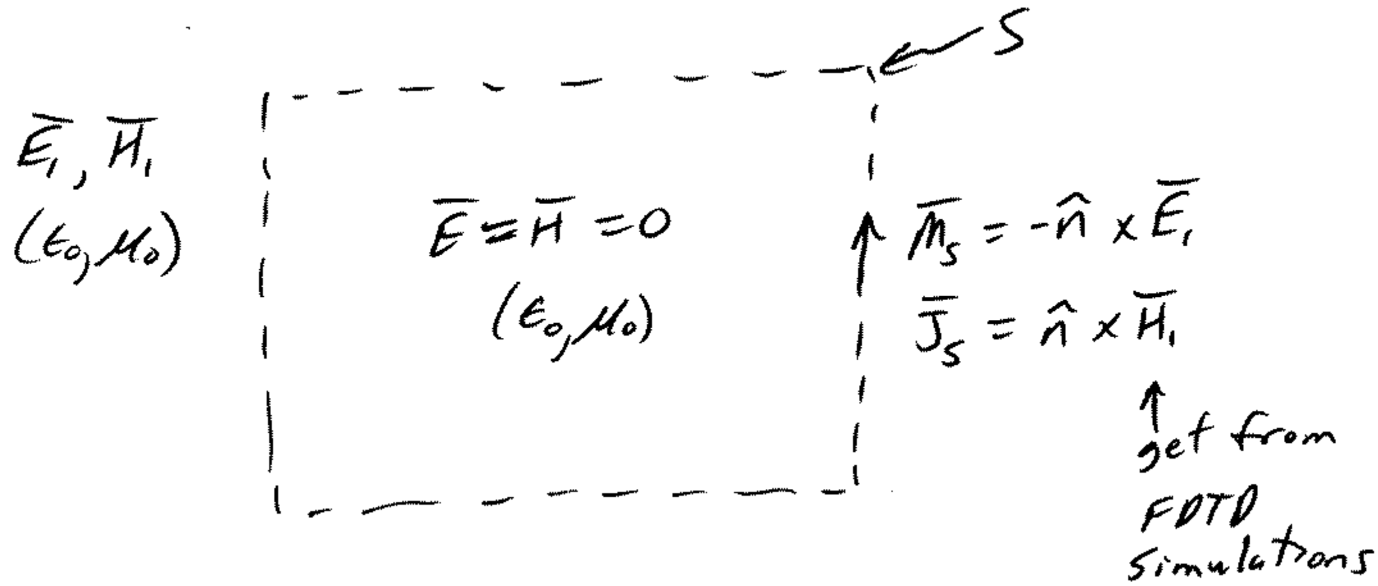




8.4 cont.

Note that the electric $\bar{J}_S$ + magnetic $\bar{M}_S$ surface current densities do **NOT** physically exist, but are selected so that $\bar{E}_1$ + $\bar{H}_1$ outside S are the *same*.

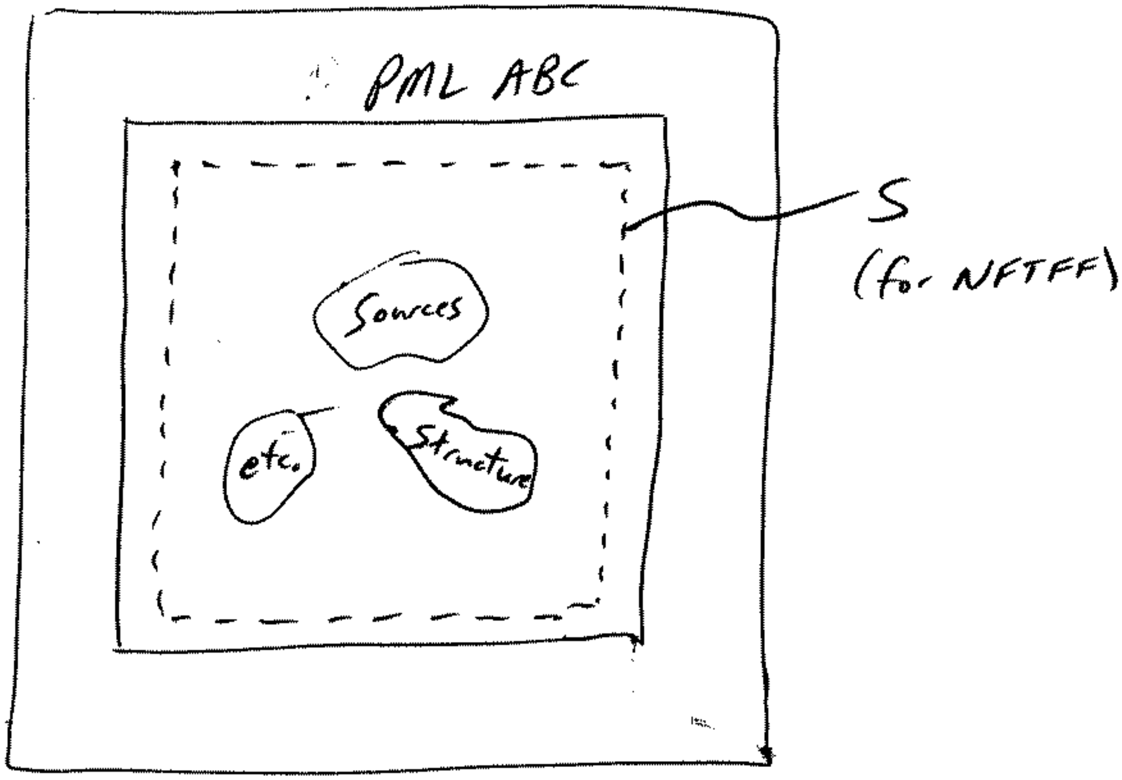
Next, choose $\bar{E}$ + $\bar{H}$ to be zero inside S (still want $\bar{E}_1$ + $\bar{H}_1$ outside S). Then, our equivalent problem is



→ These $\bar{M}_S$ + $\bar{J}_S$ can be used to calculate the far-fields outside of S regardless of what sources +/or objects/structures are inside S

8.4 cont.

Typical modeling situation



8.6 Time-Domain Near-to-Far-Field Transformation

→ while we could store the tangential $\bar{E}$ + $\bar{H}$ vs time for all points on the surface S, + then calculate $\bar{\mathcal{M}}_s$ + $\bar{\mathcal{J}}_s$, + finally calculate $\bar{\mathcal{E}}_{FF}(t)$ + $\bar{\mathcal{H}}_{FF}(t)$, it is more convenient to do the necessary calculations "on-the-fly" (i.e., part of regular time loop in the FDTD algorithm) keeping a running sum of the contributions of $\bar{E}$ + $\bar{H}$ on the surface S at each time-step.

→ Start w/ Luebbers, Kunz, Schneider, + Hunsberger approach (give out paper) that draws on background EM material covered in Balanis' Advanced Engineering Electromagnetics + Antenna Theory texts (time-harmonic/phasor case)

In the far-field, the magnetic vector potential is

$$\bar{A} = \frac{\mu_0}{4\pi} \iint_S \bar{J}_s \frac{e^{-jkR}}{R} ds' \simeq \frac{\mu_0 e^{-jkr}}{4\pi r} \bar{N}$$

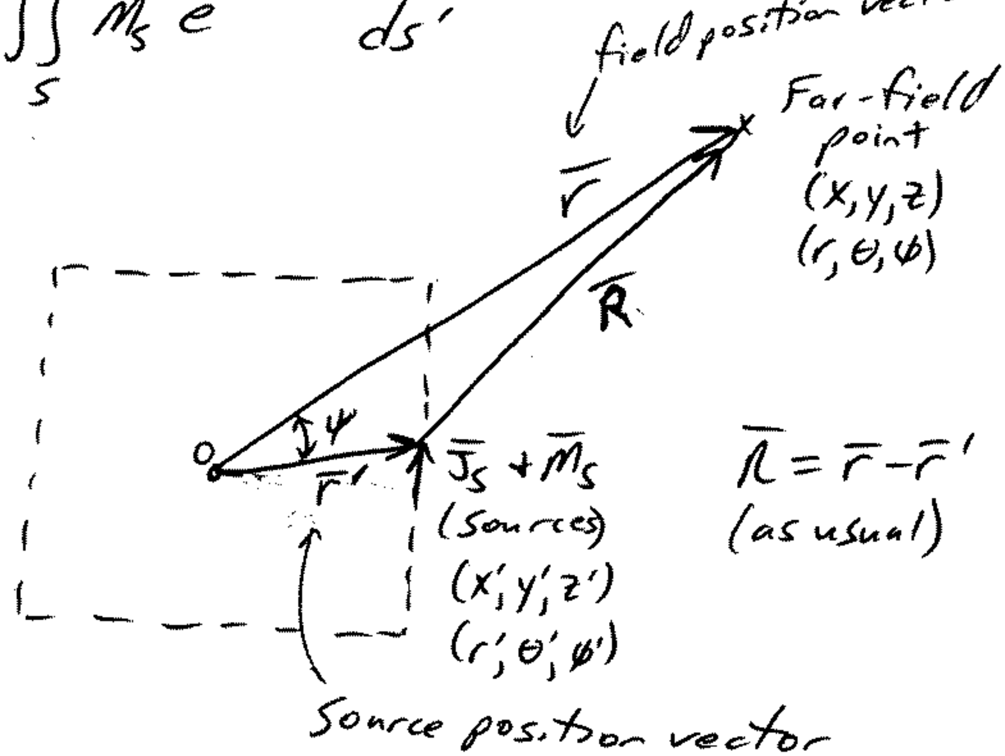
where
$$\bar{N} = \iint_S \bar{J}_s e^{jkr'\cos\psi} ds'$$

8.6 cont.

Similarly, the electric vector potential is

$$\overline{F} = \frac{\epsilon_o}{4\pi} \iint_S \overline{M}_s \frac{e^{-jkR}}{R} ds' \simeq \frac{\epsilon_o e^{-jkr}}{4\pi r} \overline{L}$$

where $\overline{L} = \iint_S \overline{M}_s e^{jkr'\cos\psi} ds'$



$\overline{R} = \overline{r} - \overline{r}'$ (as usual)

$\psi \equiv$ angle between $\overline{r}$ + $\overline{r}'$

$r'\cos\psi = \overline{r}' \cdot \hat{r}$ ← $\hat{a}_r = \hat{a}_x \sin\theta\cos\phi + \hat{a}_y \sin\theta\sin\phi + \hat{a}_z\cos\theta$

(by defn of dot product)

Balanis has shown that, in the far-field:

$$E_\theta = -j\omega A_\theta - j\omega\eta_o F_\phi = -j\omega \frac{\mu_o e^{-jkr}}{4\pi r} N_\theta - j\omega\eta_o \frac{\epsilon_o e^{-jkr}}{4\pi r} L_\phi$$

$$= \frac{-jke^{-jkr}}{4\pi r}\left(L_\phi + \eta_o N_\theta\right) = \frac{-je^{-jkr}}{2\lambda_o r}\left(L_\phi + \eta_o N_\theta\right) \tag{8.38}$$

8.6 cont.

$$E_\phi = -j\omega A_\phi + j\omega \eta_0 F_\theta = -j\omega \frac{\mu_0 e^{-jkr}}{4\pi r} N_\phi + j\omega \eta_0 \frac{\epsilon_0 e^{-jkr}}{4\pi r} L_\theta$$

$$= \frac{+jk e^{-jkr}}{4\pi r}\left(L_\theta - \eta_0 N_\phi\right) = \frac{j e^{-jkr}}{2\lambda_0 r}\left(L_\theta - \eta_0 N_\phi\right) \quad (8.39)$$

If the magnetic fields are also desired

$$H_\theta = j\frac{\omega}{\eta_0} A_\phi - j\omega F_\theta = \frac{-E_\phi}{\eta_0}$$

$$H_\phi = -j\frac{\omega}{\eta_0} A_\theta - j\omega F_\phi = \frac{E_\theta}{\eta_0}$$

Compute E_θ & E_ϕ & get H_θ & H_ϕ by these relations

$E_r = H_r = 0$ in far-field

Next, define some new vector potentials

$$\bar{W} = \frac{j e^{-jkr}}{2\lambda_0 r} \bar{N} \quad (8.40) \quad \& \quad \bar{U} = \frac{j e^{-jkr}}{2\lambda_0 r} \bar{L} \quad (8.41)$$

which allows us to write

$$E_\theta = -\eta_0 W_\theta - U_\phi \quad (8.42) \quad \& \quad E_\phi = -\eta_0 W_\phi + U_\theta \quad (8.43)$$

Now, it is time to head back to the time-domain using inverse Fourier Transforms

$$\mathcal{F}^{-1}\{\bar{W}\} = \bar{\mathcal{W}}$$

$$\frac{\partial (x(t))}{\partial t} \longleftrightarrow j\omega X(\omega)$$

8.6 cont.

The inverse Fourier Transform of $\bar{W} + \bar{U}$ yields

$$\bar{\mathcal{W}}(\bar{r},t) = \frac{1}{4\pi rc}\frac{\partial}{\partial t}\left[\iint_S \bar{\mathcal{J}}_s\left(t - \frac{r - \bar{r}'\cdot\hat{r}}{c}\right)ds'\right] \quad (8.44)$$

$$\bar{\mathcal{U}}(\bar{r},t) = \frac{1}{4\pi rc}\frac{\partial}{\partial t}\left[\iint_S \bar{\mathcal{M}}_s\left(t - \frac{r - \bar{r}'\cdot\hat{r}}{c}\right)ds'\right] \quad (8.45)$$

and

$$\mathcal{E}_\theta(\bar{r},t) = -\eta_0 \mathcal{W}_\theta(\bar{r},t) - \mathcal{U}_\phi(\bar{r},t) \quad (8.46)$$

$$\mathcal{E}_\phi(\bar{r},t) = -\eta_0 \mathcal{W}_\phi(\bar{r},t) + \mathcal{U}_\theta(\bar{r},t) \quad (8.47)$$

$$\mathcal{H}_\theta(\bar{r},t) = \frac{-\mathcal{E}_\phi(\bar{r},t)}{\eta_0}$$

$$\mathcal{H}_\phi(\bar{r},t) = \frac{\mathcal{E}_\theta(\bar{r},t)}{\eta_0}$$

Now, the $\frac{r - \bar{r}'\cdot\hat{r}}{c}$ term represents the time delay in getting from some point on surface S to the far-field point

$$\tau_d = \frac{r - \bar{r}'\cdot\hat{r}}{c} = \frac{r - r'\cos\psi}{c}$$

Also, from our Surface Equivalence Theorem material

$$\overline{\mathcal{M}}_s = \mathcal{F}^{-1}\{\overline{M}_s = -\hat{n}\times\overline{E}_1\} = -\hat{n}\times\overline{\mathcal{E}}_1(\bar{r}',t) \leftarrow \text{No time-delay}$$

$$\overline{\mathcal{J}}_s = \mathcal{F}^{-1}\{\overline{J}_s = \hat{n}\times\overline{H}_1\} = \hat{n}\times\mathcal{H}_1(\bar{r}',t)$$

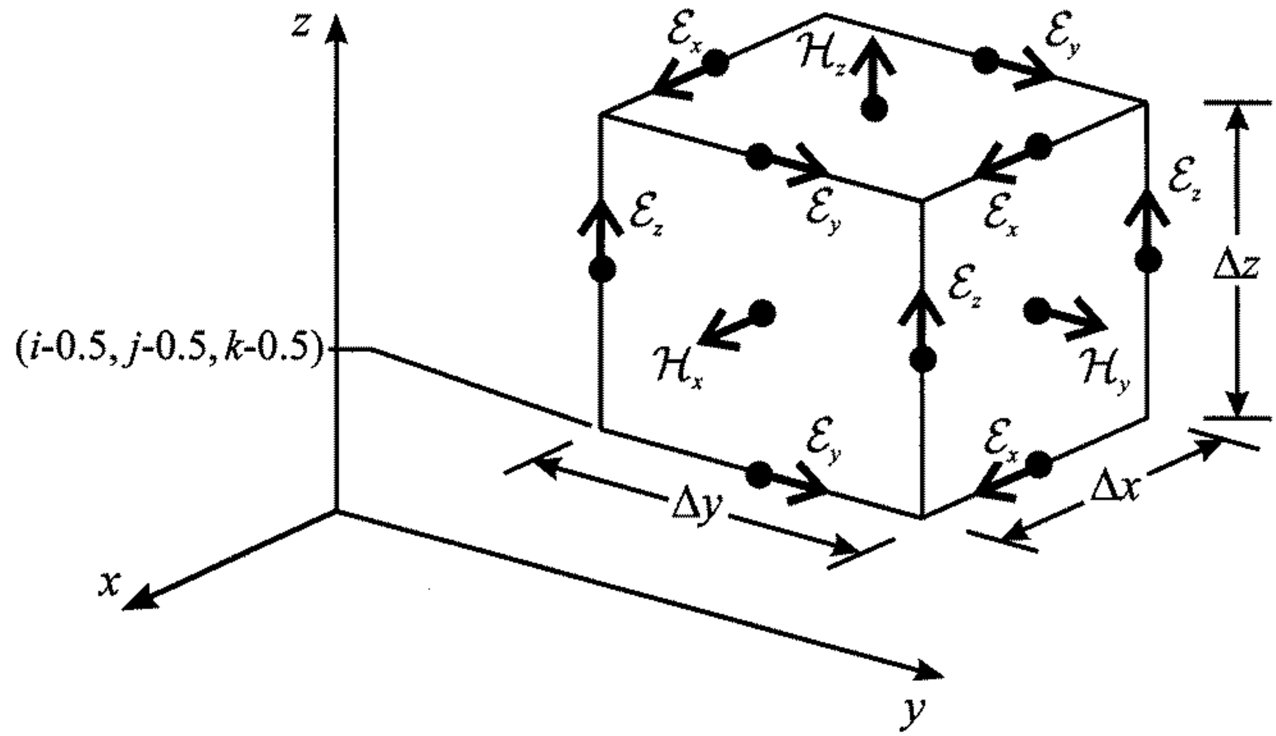
The big challenge is to implement (8.44) + (8.45) on the surface S (see figure on next page)

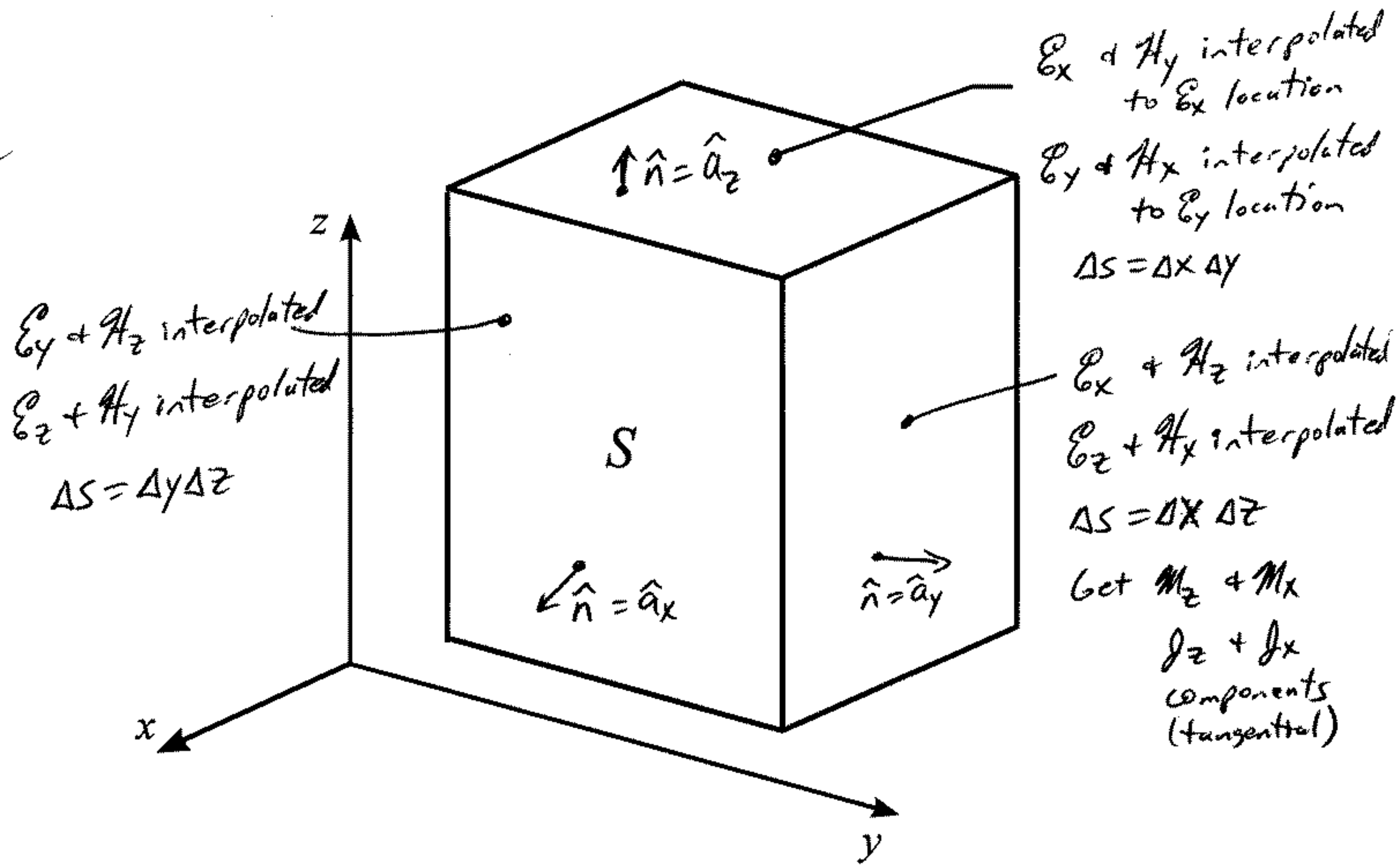
→ Need to evaluate (8.44) + (8.45) on each of six sides of S in Cartesian coordinates (i.e. $W_x(\bar{r},t)$, $W_y(\bar{r},t)$, $W_z(\bar{r},t)$, $U_x(\bar{r},t)$, $U_y(\bar{r},t)$, + $U_z(\bar{r},t)$) at <u>each time-step</u> (keep a running sum)

→ Convert from Cartesian to spherical coordinates after finishing time-loop

part of selected field point

i.e. Use
$$A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$$
$$A_\phi = -A_x\sin\phi + A_y\cos\phi$$





Surface S for 3D Cartesian FDTD model has six sides
(e.g. Top at $z = z_{\text{top}}$, $x_{\text{front}} \le x \le x_{\text{back}}$, $y_{\text{left}} \le y \le y_{\text{right}}$)

8.6 cont.

To illustrate the process, let's look at the right side of S ($y' = y_0$, $x_{Back} \leq x' \leq x_{FRONT}$, $z_{Bott} \leq z' \leq z_{Top}$ + $\bar{r}' = x'\hat{a}_x + y_0\hat{a}_y + z'\hat{a}_z$)

First, what equivalent electric + magnetic surface currents are there?

$\mathcal{E}_x$: $\bar{M}_{\mathcal{E}_x} = -\hat{a}_y \times \mathcal{E}_x \hat{a}_x = \hat{a}_z \mathcal{E}_x(i, j+0.5, k+0.5)$

$\mathcal{H}_z$: $\bar{J}_{\mathcal{H}_z} = \hat{a}_y \times \mathcal{H}_z \hat{a}_z = \hat{a}_x \mathcal{H}_z$ ← interpolated left/right to $\mathcal{E}_x$ location

$$= \hat{a}_x \left(\frac{\mathcal{H}_z(i, j+1, k+0.5) + \mathcal{H}_z(i, j+k+0.5)}{2} \right)$$

$\mathcal{E}_z$: $\bar{M}_{\mathcal{E}_z} = -\hat{a}_y \times \mathcal{E}_z \hat{a}_z = -\hat{a}_x \mathcal{E}_z(i+0.5, j+0.5, k)$

$\mathcal{H}_x$: $\bar{J}_{\mathcal{H}_x} = \hat{a}_y \times \mathcal{H}_x \hat{a}_x = -\hat{a}_z \mathcal{H}_x$ ← interpolated left/right to $\mathcal{E}_z$ location

$$= -\hat{a}_z \left(\frac{\mathcal{H}_x(i+0.5, j+1, k) + \mathcal{H}_x(i+0.5, j, k)}{2} \right)$$

Now on the right face $ds' = dx'dz' \simeq \Delta x \Delta z$

⇒ Be careful w/ field components on edges of S (e.g. $\frac{\Delta x}{2}\Delta z$ or $\Delta x \frac{\Delta z}{2}$ might be appropriate)

$\mathcal{E}_z$ + $\mathcal{H}_x$ interp. on edge w/ front + Back

$\mathcal{E}_x$ + $\mathcal{H}_z$ interp. on edge w/ Top + Bottom

8.6 cont.

Following some $\mathcal{E}_x$ components contribution (assume somewhere in middle of right face of S) we see that it will contribute to $\bar{U}(\bar{r},t)$ as it generates $\bar{\mathcal{M}}_{\mathcal{E}_x} = \hat{a}_z \mathcal{E}_x$. Specifically,

$$\Delta \bar{U} = \frac{1}{4\pi r c} \frac{\partial}{\partial t}\left[\hat{a}_z \mathcal{E}_x(i, j+0.5, k+0.5)\, \Delta x \Delta z\right]$$

$$= \hat{a}_z \frac{\Delta x \Delta z}{4\pi r c} \frac{\partial \mathcal{E}_x(i, j+0.5, k+0.5)}{\partial t}$$

$$\Delta U_z = \frac{\Delta x \Delta z}{4\pi r c} \frac{\partial \mathcal{E}_x(i, j+0.5, k+0.5)}{\partial t}$$

Finish discretizing by doing a central-difference approx. to $\frac{\partial}{\partial t}$ about time $t = (n+\frac{1}{2})\Delta t$

$$\Delta U_z = \frac{\Delta x \Delta z}{4\pi r c} \frac{\mathcal{E}_x^{n+1}(\,) - \mathcal{E}_x^{n}(\,)}{\Delta t}$$

Now, we need to "credit" this contribution to the overall $U_z(\bar{r},t)$ function. To do so, we realize that U_z is saved at discrete times $t = n\Delta t$ and that we have a time delay going from the $\mathcal{E}_x$ location to the far-field. In terms of Δt, the delay is

$$f = \frac{\tau_d}{\Delta t} = \frac{r - r'\cos\psi}{c\Delta t}$$

← put in terms of $\mathcal{E}_x$ location

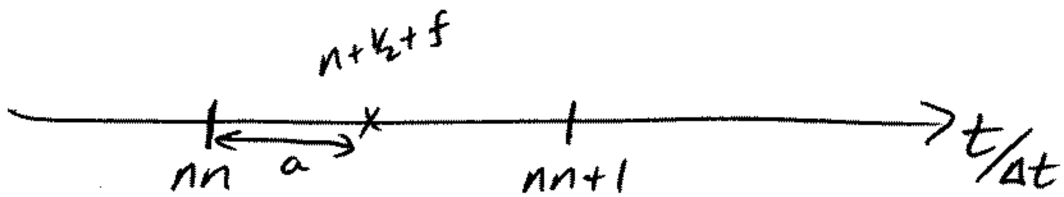
8.6 cont.

Here $\bar{r}' = x'\hat{a}_x + y_o\hat{a}_y + z'\hat{a}_z$ becomes

$$\bar{r}'_{o\,\mathcal{E}_x} = i\Delta x\,\hat{a}_x + (j+0.5)\Delta y\,\hat{a}_y + (k+0.5)\Delta z\,\hat{a}_z$$

$$r'\cos\psi = i\Delta x\sin\theta\cos\phi + (j+0.5)\Delta y\sin\theta\sin\phi + (k+0.5)\Delta z\cos\theta$$

So the time index to which ΔU_z must be credited is $n+\frac{1}{2}+f$. However, this sum is unlikely to be exactly an integer. Therefore, we split ΔU_z between the closest adjacent U_z temporal locations



where $nn = INT(n+\frac{1}{2}+f)$ ← rounds down to closest integer

$$a = (n+\tfrac{1}{2}+f) - nn \qquad 0 \le a \le 1$$

$$U_z^{nn}\Big|_{\bar{r}} = U_z^{nn}\Big|_{\bar{r}} + (1-a)\,\Delta U_z$$ ← smaller $a \Rightarrow$ bigger contrib.

$$U_z^{nn+1}\Big|_{\bar{r}} = \underbrace{U_z^{nn+1}\Big|_{\bar{r}}}_{\text{running sum}} + a\,\Delta U_z$$ ← bigger $a \Rightarrow$ bigger contrib.

8.6 cont.

Obviously, we need to ensure that the $U_z|_{\bar{r}}$ vector has enough memory locations M.

Let $s\Delta$ = Maximum diagonal distance on S (fractional Δ in max diagonal dist.)

& use the Courant Stability factor S_c to see we need

$$M = \underbrace{N_{max}}_{\text{timeloop max. index}} + \underbrace{\frac{s\Delta}{(\Delta/S_c)}}_{\text{\# of time steps per } \Delta} = N_{max} + \underbrace{\left(\frac{1}{S_c}\right)}_{\text{INT}} \underbrace{s}_{\text{fractional } \Delta \text{ in max diagonal}}$$

e.g. $N_{max} = 1000$

max diagonal $= 50\Delta$

$S = 0.5$

$$M = 1000 + \left(\frac{1}{0.5}\right)50 = 1100 \text{ allocutions}$$

Note that we have NOT specified at which values of n we start & stop the indices (depends on value of r selected)

→ most of the time, $n_{min} = \text{INT}\left(\text{Min}(f)\right)$

can be used or start @ $n=1$ by subtracting n_{min}

8.6 cont.

A similar process can be used for the other contributions to $\bar{U}$ and for the contributions to $\bar{W}$.

Notes:

1) $\bar{H}$ components are being interpolated onto S

2) $\bar{H}$ components are shifted by $\Delta t/2$ relative to $\bar{E}$ components

3) Careful w/ signs for equiv. currents.

After running your time loop, the six Cartesian components $W_x, W_y, W_z, U_x, U_y,$ + U_z (at $\bar{r}$ + vs n/t_n) need to be used ~~converted~~ to find $W_\theta, W_\phi, U_\theta,$ + U_ϕ by standard vector conversions

$$W_\theta(\bar{r},t) = W_x \cos\theta \cos\phi + W_y \cos\theta \sin\phi - W_z \sin\theta$$

$$W_\phi(\bar{r},t) = -W_x \sin\phi + W_y \cos\phi$$

$$U_\theta(\bar{r},t) = U_x \cos\theta \cos\phi + U_y \cos\theta \sin\phi - U_z \sin\theta$$

$$U_\phi(\bar{r},t) = -U_x \sin\phi + U_y \cos\phi$$

which can then be used to calculate

E_θ, E_ϕ, H_θ, H_ϕ in the far-field.