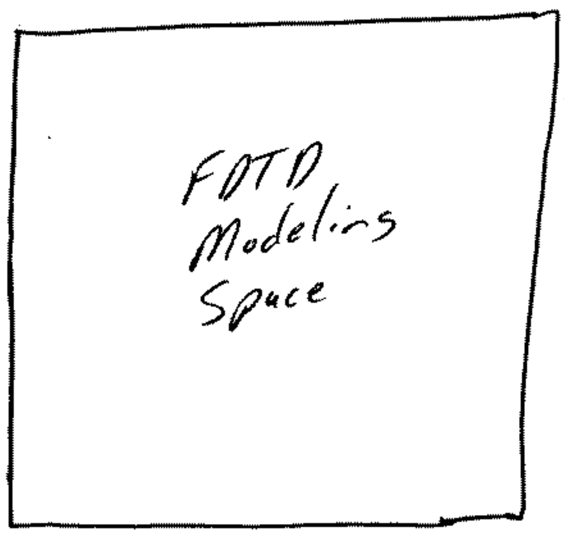
Chapter 6 Analytical Absorbing Boundary Conditions

6.1 Intro

- For open problems (i.e. problems where waves could keep propagating indefinitely), we need a means of truncating the FDTD grid/lattice due to finite computational resources.



Absorbing Boundary Condition (ABC)
or
Radiation Boundary Condition (RBC)

→ want only outward propagating waves thru an ABC or RBC (no reflections / $|\Gamma| \sim 0$)

→ problem is that what to do w/ field components on edges of the FDTD modeling space where we don't have the needed components for central difference updates.

6.1 cont.

→ the best ABCs can now yield $|\Gamma| \sim 10^{-4}$ to 10^{-6} (or -80dB to -120dB) so a dynamic range of ~ 70dB+ is achievable. (dynamic range is also limited by dispersion, ...)

→ Analytical ABCs were the first to be developed, but have largely been replaced by perfectly matched layer (PML) ABC (next chapter).

→ ABCs – some "material" absorbs or dissipates outward propagating waves

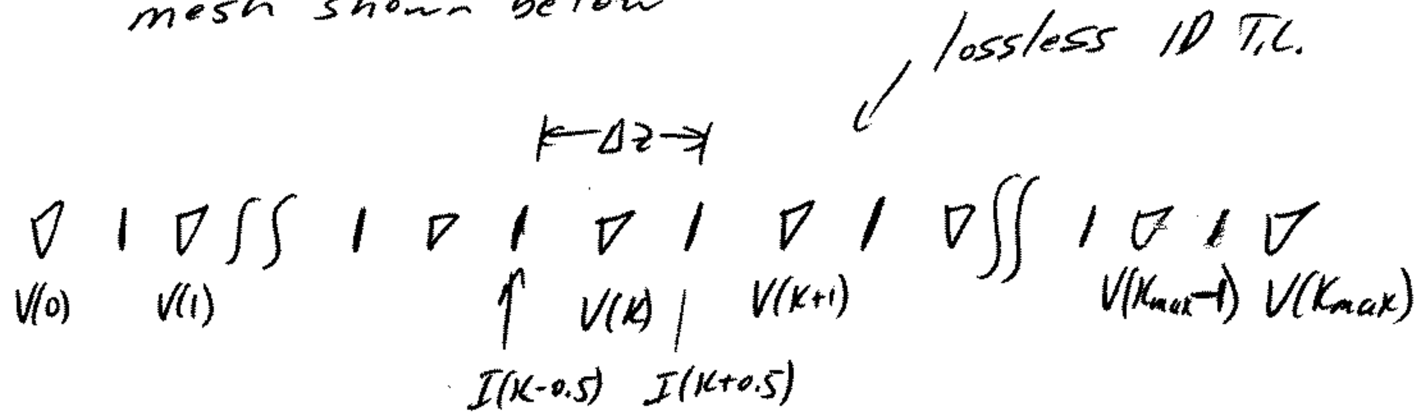
→ RBCs – simulate the outward propagating wave continuing to propagate past the boundary

$\Downarrow$

We'll just use the term ABC regardless of underlying technique.

Simple 1D ABC

- consider the 1D transmission line FDTD mesh shown below



where

$$V^{n+1}(K) = V^{n}(K) - Z_c\left(\frac{v_p \Delta t}{\Delta z}\right)\left[I^{n+0.5}(K+0.5) - I^{n-0.5}(K-0.5)\right]$$

$$I^{n+0.5}(K+0.5) = I^{n-0.5}(K+0.5) - \frac{1}{Z}\left(\frac{v_p \Delta t}{\Delta z}\right)\left[V^{n}(K+1) - V^{n}(K)\right]$$

→ Now, at $K=0$ + $K=K_{max}$ in the above lattice/grid, we do not have the needed current values to update $V(0)$ + $V(K_{max})$ respectively

→ Note, the Courant stability factor $S = \frac{v_p \Delta t}{\Delta z}$ defines how many spatial steps occur/are traveled per time step $S\Delta z = v_p \Delta t$

ex. $S=1$ $\quad (1)\Delta z = \Delta z = v_p \Delta t$ ← Travel $1\Delta z$ per Δt

$S=0.7$ $\quad 0.7\Delta z = v_p \Delta t$ ← Travel $0.7\Delta z$ per Δt

Travel $0.5\Delta z$ per Δt

$S=0.5$ $\quad 0.5\Delta z = v_p \Delta t \Rightarrow \Delta z = v_p(2\Delta t)$

→ Using this knowledge and the knowledge that waves must propagate in the $\pm z$-directions, we can say that the voltages at the two ends of the T.L. are related to those inside the T.L. by the Courant stability factor

e.g. At $z=0$, with $S=1$, outward wave must be in $-z$-dir

$$V^{n+1}(0) = V^{n}(1)$$ since it takes the wave a single Δt to travel a single Δz

However, if $S=0.5$

$$V^{n+1}(0) = V^{n-1}(1)$$ since it takes a wave <u>two</u> Δt to travel a single Δz

Simple 1D ABC cont.

What about $V(K_{max})$? Outward wave must be going in $+z$-direction, so $V(K_{max})$ depends on $V(K_{max}-1)$

e.g. $S=1$

$$V^{n+1}(K_{max}) = V^{n}(K_{max}-1)$$

e.g. $S=1/3$

$$V^{n+1}(K_{max}) = V^{n-2}(K_{max}-1)$$

This result extends to TEM waves in 2D grids (Hint!).

$\Rightarrow$ Makes it look as if the wave just keeps propagating off the mesh.

$\Rightarrow$ Works best if $S = 1/n$, n = integer

$\Rightarrow$ Interpolation possible, but not as accurate

FDTD ABCs cont.

We'll examine the Merewether algorithm for an RBC to illustrate the technique.

→ First, we assume the field near the boundary S enclosing our problem space can be approximated by or as an outwardly propagating spherical wave with the functional form

$$\frac{f(t - R/v_p)}{R}$$

v_p – velocity of wave

R – distance from source

ex.

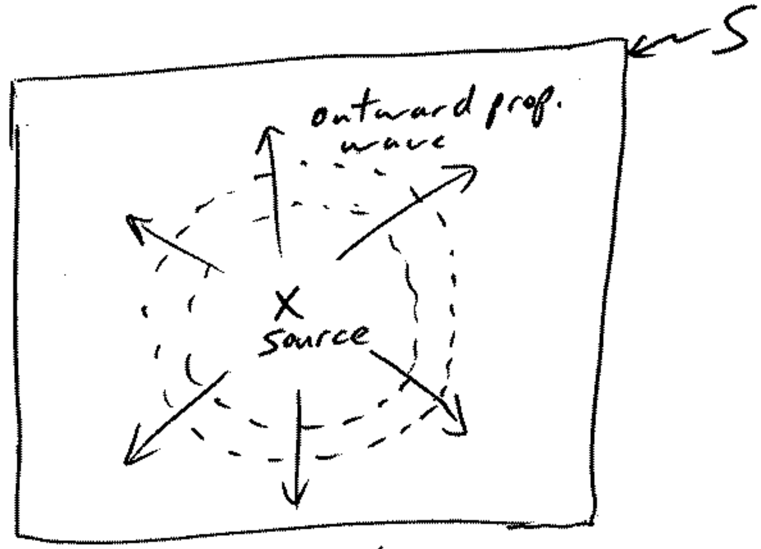


Fig. 1

→ Good approx. if you have a single/multiple source(s) near middle of modeling space and are in the far-field $\left(\sim R > \frac{2D^2}{\lambda}\right)$

FDTD ABCs cont.

→ Second, we'll limit ourselves to a 2D FDTD case (TM_z case)

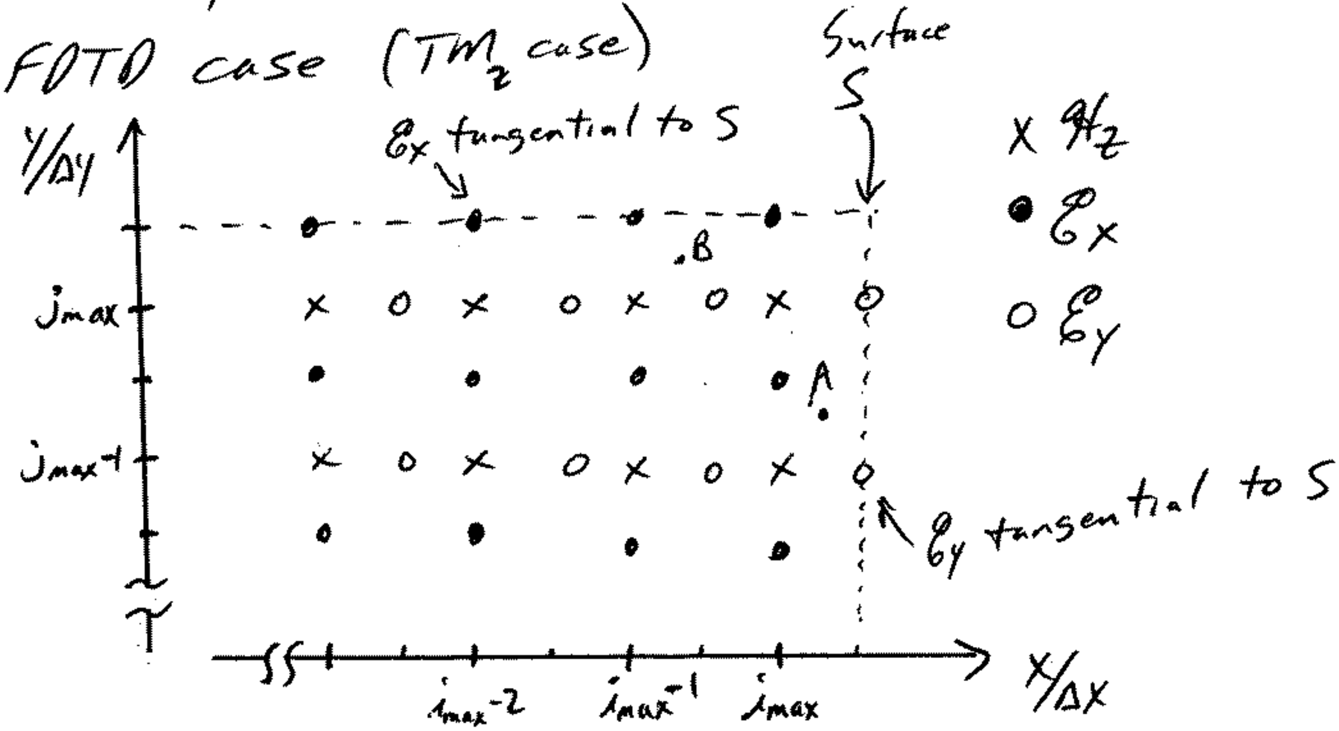


Fig. 2

→ Now, we'll want to predict the value of $\mathscr{E}_x$ or $\mathscr{E}_y$ that will arrive at the boundary S at time $t=(n+1)\Delta t$ using values of $\mathscr{E}_x$ or $\mathscr{E}_y$ at time $n\Delta t$.

From $\dfrac{f(t - R/v_p)}{R}$, we get

$$E_x^{n+1}(i, j_{max}+0.5) = \left(\frac{R - v_p \Delta t}{R}\right) E_x^n(\text{point } B)$$

and

$$E_y^{n+1}(i_{max}+0.5, j) = \left(\frac{R - v_p \Delta t}{R}\right) E_y^n(\text{point } A)$$

FDTD ABCs cont.

The points A + B are illustrated in the close-up figures shown below.

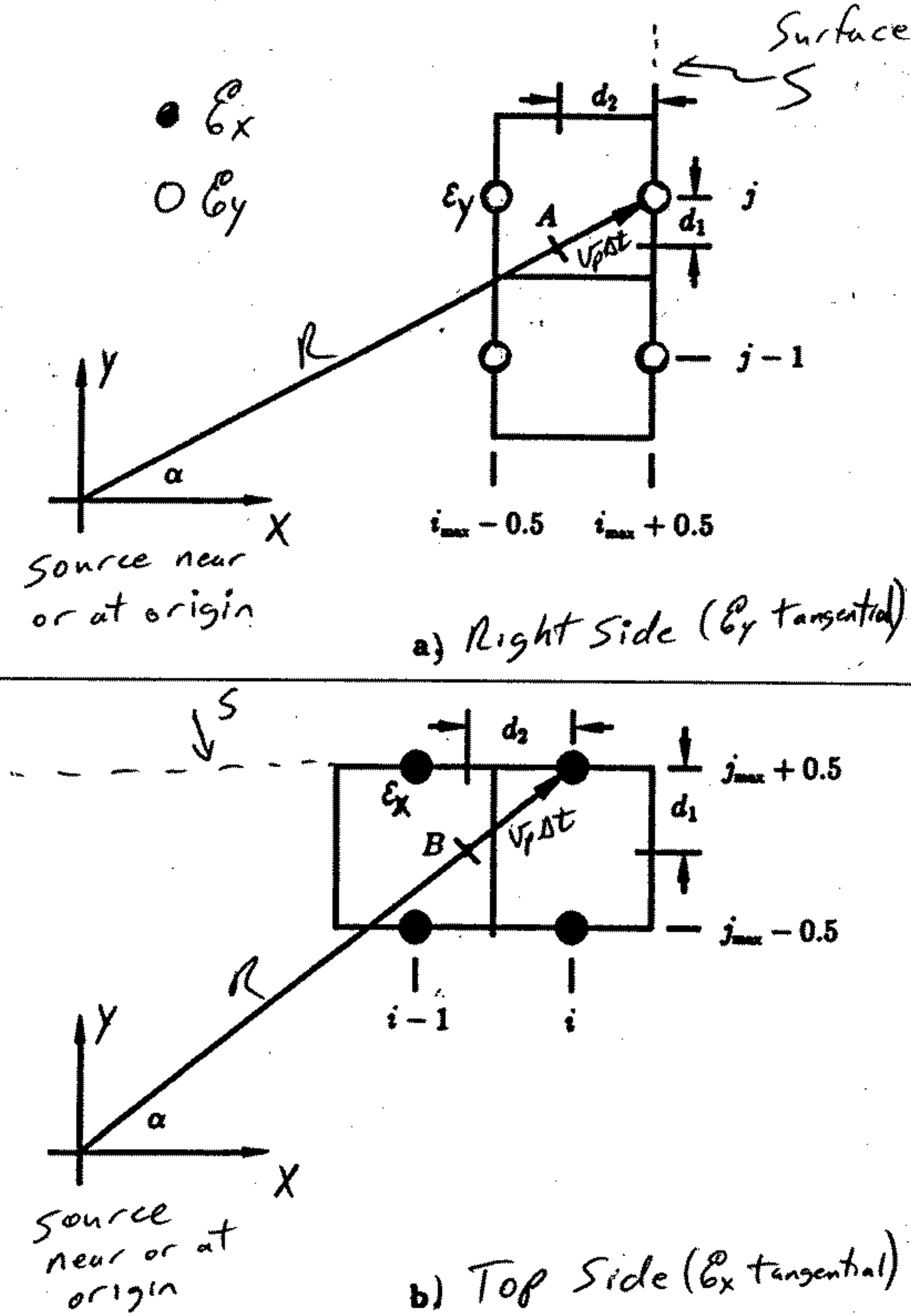


Fig. 3

On the right side (Fig. 3a)), we see that, in general, no $\mathcal{E}_y$ value will be located at point A. Therefore, we will use bilinear interpolation to calculate $\mathcal{E}_y(A)$ from available (in the grid) adjacent values.

$$\boxed{\begin{aligned} E_y^n(A) \simeq & \left(1-\frac{d_1}{\Delta y}\right)\left(1-\frac{d_2}{\Delta x}\right) E_y^n(i_{max}+0.5, j) \\ & + \left(1-\frac{d_1}{\Delta y}\right)\left(\frac{d_2}{\Delta x}\right) E_y^n(i_{max}-0.5, j) \\ & + \left(\frac{d_1}{\Delta y}\right)\left(\frac{d_2}{\Delta x}\right) E_y^n(i_{max}-0.5, j-1) \\ & + \left(\frac{d_1}{\Delta y}\right)\left(1-\frac{d_2}{\Delta x}\right) E_y^n(i_{max}+0.5, j-1) \end{aligned}}$$

where $d_1 = v_p \Delta t \sin\alpha$, $d_2 = v_p \Delta t \cos\alpha$, and α is the angle (wrt x) from the origin to point A as shown. The distance R from the origin is easily calculated using Pythogorean's Thm @ each $E_y^{n+1}(i_{max}+0.5, j)$ location as

$$R = \sqrt{\left[(i_{max}+0.5)\Delta x\right]^2 + (j\,\Delta y)^2}.$$

The angle $\alpha = \tan^{-1}\left(\frac{j\,\Delta y}{(i_{max}+0.5)\,\Delta x}\right).$

Similarly, on the top boundary of S (see Fig 3 b)), we see that, in general, no $\mathcal{E}_x$ value exists at point B. Again, using bilinear interpolation, we can calculate

$$\boxed{\begin{aligned} E_x^n(B) \simeq & \left(1-\frac{d_1}{\Delta y}\right)\left(1-\frac{d_2}{\Delta x}\right) E_x^n(i, j_{max}+0.5) \\ & + \left(1-\frac{d_1}{\Delta y}\right)\left(\frac{d_2}{\Delta x}\right) E_x^n(i-1, j_{max}+0.5) \\ & + \left(\frac{d_1}{\Delta y}\right)\left(\frac{d_2}{\Delta x}\right) E_x^n(i-1, j_{max}-0.5) \\ & + \left(\frac{d_1}{\Delta y}\right)\left(1-\frac{d_2}{\Delta x}\right) E_x^n(i, j_{max}-0.5) \end{aligned}}$$

where $d_1 = v_p \Delta t \sin\alpha$, $d_2 = v_p \Delta t \cos\alpha$, and α is the angle wrt the x-axis to point B from the origin as shown. Along the top surface of S at the $E_x^{n+1}(i, j_{max}+0.5)$ locations the distance R from the origin is

$$R = \sqrt{(i\Delta x)^2 + [(j_{max}+0.5)\Delta y]^2}$$

and the angle $\alpha = \tan^{-1}\left(\frac{(j_{max}+0.5)\Delta y}{i\,\Delta x}\right)$