Chapter 5 Incident Wave Source Conditions

5.1 Introduction

→ Need to "drive" the FDTD model

→ ideally source will be accurate + use minimal computational resources.

Types of Sources:

1) Hard
2) Additive
3) Additive one-way
4) Current source (i.e., $\bar{J}$ + $\bar{m}$)
5) Plane wave
6) waveguide
7) magnetic frill

⋮

Hard Sources

→ set variable(s) (e.g. voltage, electric field component(s), ...) equal to a specific time function/waveform

→ variable(s) do **NOT** depend on anything w/in the FDTD model (other waves can not pass thru the fixed variable(s))

→ launches outward propagating traveling waves in the lattice.

→ sometimes the hard source is applied for a limited duration and then the variable is allowed to resume its normal update equation

→ Examples & discussion given in sections 5.2 thru 5.3

→ This is the type of source we used in the 1D Transmission Line material

ex. $E_x^n(K) = 8\sin(0.1\pi n\Delta t)$ → doesn't depend on surrounding magnetic fields

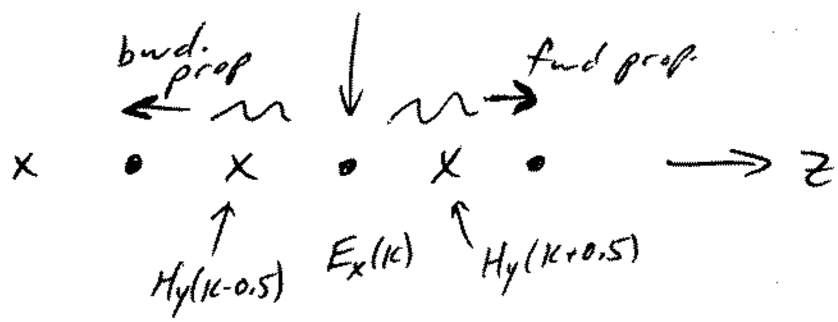
Additive Sources

→ use regular update equation for a variable or variable(s) w/ additional "add-on" term

→ variable(s) do depend on other variable(s) w/in FDTD model. So, other waves can pass on through the source variables.

→ also launches outward propagating traveling waves in the FDTD lattice.

ex. $$E_x^{n+1}(k) = C1 * E_x^{n}(k) + C2\left[H_y^{n+0.5}(k+0.5) - H_y^{n+0.5}(k-0.5)\right] + 8\sin(0.1\pi n \Delta t)$$



Additive One-way Sources

→ use regular additive source for a variable or variables

→ then, adjacent to these sources, subtract out the source (works best when S selected so that it takes the wave(s) an integer # of time steps to propagate a distance Δ).

ex. $$E_x^{n+1}(k) = C1 * E_x^{n}(k) + C2 * \left[H_y^{n+0.5}(k+0.5) - H_y^{n+0.5}(k-0.5)\right] + 8\sin(0.1\pi n \Delta t)$$

$$E_x^{n+1}(k-1) = C1 * E_x^{n}(k-1) + C2 * \left[H_y^{n+0.5}(k-0.5) - H_y^{n+0.5}(k-1.5)\right] - 8\sin\left[0.1\pi (n-1)\Delta t\right]$$

$C_S = 1$ (one spatial step per Δt)

No bwd! prop. $\xrightarrow{\sim}$ fwd prop.

o x • x • x o x

$E_x(k-1)$ $E_x(k)$

⇒ This foreshadows a type of absorbing boundary condition

5.4 $\bar{J}$ + $\bar{M}$ Current Sources in Three Dimensions

→ $\bar{J}$ co-located w/ corresponding electric field components but $\Delta t/2$ offset in time. (see Chap 3, eqn (3.29))

→ $\bar{M}$ co-located w/ corresponding magnetic field components but $\Delta t/2$ offset in time. (see Chap. 3, eq (3.30))

→ can be/are sources of electric/magnetic charge which has implications as far as divergence is concerned (think Gauss' law/static fields)

→ w/ charges capacitance of the FDTD lattice becomes an issue (5.4.4) for $\bar{J}$

→ and, inductance of the lattice (see section 5.4.5) for $\bar{M}$

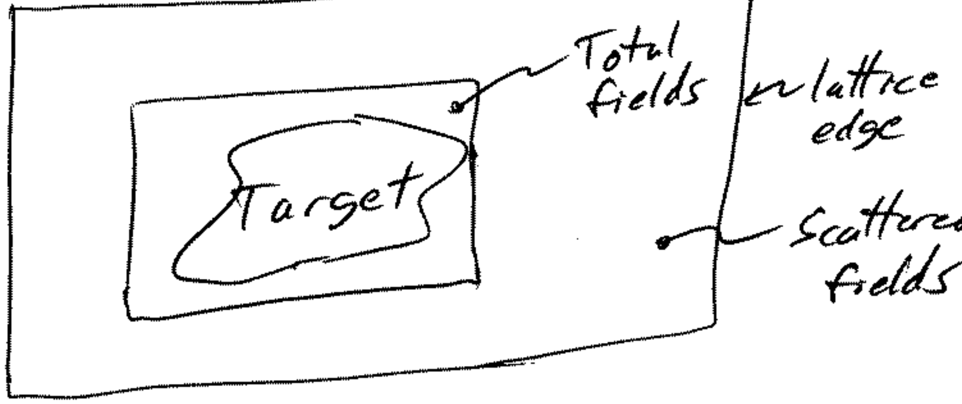
→ also makes modeling lumped-element capacitors + inductors more difficult (see section 5.4.6)

5.5 The Plane-Wave Source Condition

→ one of the earliest applications of FDTD was RCS (radar cross section) problems for defense

→ one approach is to define <u>all</u> $\bar{E}$ + $\bar{H}$ components in lattice for a given plane wave and then let time go forward to observe scattered fields (expensive computationally + has problems at boundaries of lattice)

→ Section 5.6-5.9 discusses a combined total-field/scattered-field approach



→ Section 5.10 discusses a <u>scattered-field Formulation</u> for plane waves (usually separate <u>incident</u> $\bar{E}$ + $\bar{H}$ from scattered $\bar{E}$ + $\bar{H}$ components)

→ section 5.11 discusses <u>Waveguide Source Conditions</u>

* CST has this as an option

Practical considerations

1) Selecting $\Delta x, \Delta y, \Delta z$ or Δ (square/cubic lattices)

2) Selecting Δt

3) Turning source(s) on/off

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1)
* In earlier chapters (e.g. 2 + 4), we've discussed how $N_\lambda = \dfrac{\lambda}{\Delta_i}$ (grid sampling resolution) impacts $\tilde{k}$ and $\tilde{v}_p$.
* General rule for N_λ is that bigger is better. ($N_\lambda \geq 8$ is generally used); trade-off is computer memory is limited/finite
* A key concern here is that the N_λ condition must be met for <u>all</u> frequencies of interest. This leads us to consider Fourier series for periodic signals and the Fourier transform for time-limited signals/aperiodic signals.

selecting Δ cont.

* For practical signals, $X(\omega) = \mathcal{F}\{x(t)\}$ approaches zero as $\omega \to \infty$. Therefore, we can set a condition/specification for the highest frequency of interest ω_{max} as

$$\frac{|X(\omega_{max})|}{|X(\omega)|_{maximum}} = \text{arbitrary specification}$$

Common choices are 10^{-3} (-60 dB down)

10^{-4} (-80 dB down) } better

10^{-5} (-100 dB down) }

* The wavelength corresponding to ω_{max} is the minimum wavelength λ_{min} for the <u>entire</u> lattice

$$\lambda_{min} = \frac{v_p}{\left(\frac{\omega_{max}}{2\pi}\right)} = \frac{v_p}{f_{max}}$$

smallest in lattice

* $N_\lambda = \dfrac{\lambda_{min}}{\Delta} \Rightarrow \boxed{\Delta = \dfrac{\lambda_{min}}{N_\lambda}}$

↑ selection

Selecting Δ cont.

* Another concern for Δ is the actual FDTD model/structure dimensions which can tweak Δ after $\Delta = \frac{d_{min}}{N_d}$ calculated

2) Selecting Δt

⇒ Δt is selected based on practical (e.g. absorbing boundary conditions) considerations and (of paramount concern) <u>stability</u> per the Courant Stability factor S (see chapters 2+4)

1D $\quad S \le 1$

2D $\quad S \le 1/\sqrt{2} = 0.707 \quad$ square lattice

3D $\quad S \le \frac{1}{\sqrt{3}} = 0.577 \quad$ cubic lattice

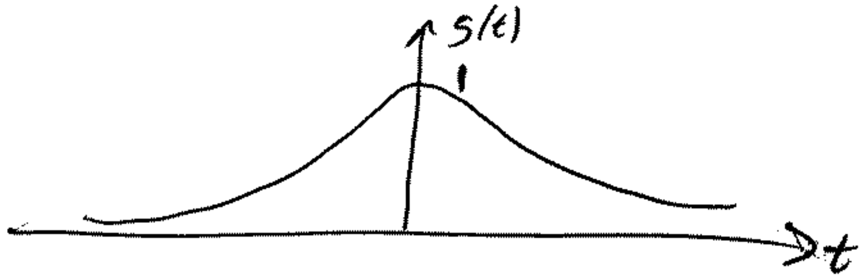
where $S = \frac{v_p \Delta t}{\Delta} \Rightarrow \underline{\underline{\Delta t = \left(\frac{\Delta}{v_p}\right) S}}$

3) Turning Source(s) ON/OFF

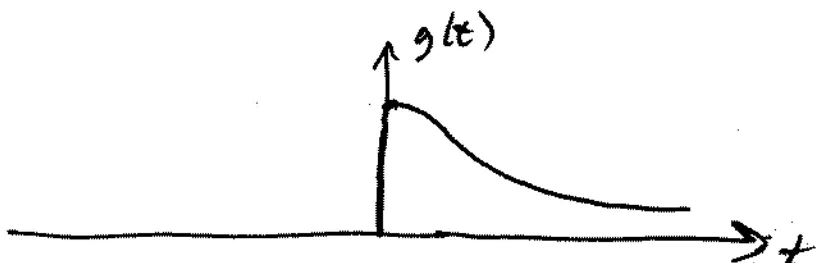
→ Due to the discrete nature of the FDTD method, the source goes from an initial steady-state value (usually zero) to some different value → like a little/large step function (potentially large freq content)

→ remember from Fourier theory that sharp edges require infinite frequency content to model

ex. Gaussian pulse $g(t) = e^{-\frac{1}{2}(t^2/\tau^2)}$



if we started at $t=0$



↑ infinite freq. content w/ substantial energy.

3) Turning Source(s) ON/OFF cont.

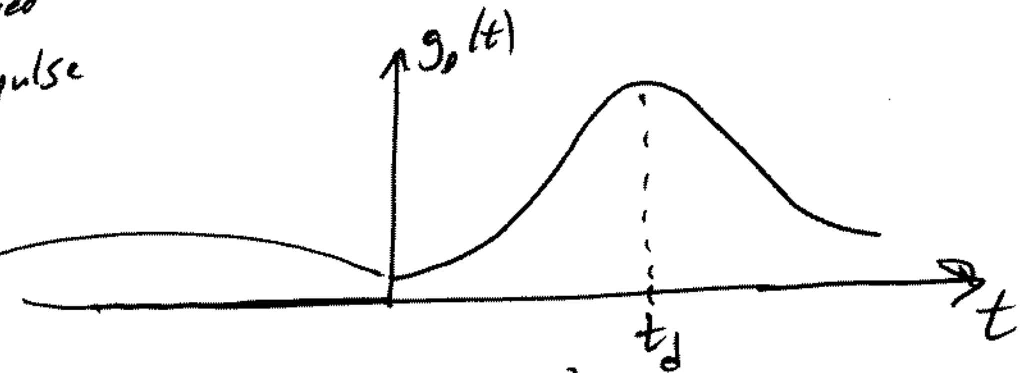
→ To avoid problems, turn signals on gradually (i.e. start signal near zero) by appropriate signal selection (avoid square edged waveforms) and time-shifting so that

$$\frac{|x(t_{shift})|}{|x(t)|_{max}} \le \text{some arbitrary \#}$$

↳ common choices are 10^{-4} (−80 dB) + 10^{-5} (−100 dB)

ex. $g_p(t) = g(t - t_d) = e^{-\frac{1}{2}\left(\frac{t-t_d}{\tau}\right)^2}$

time-delayed Gaussian pulse



$g_p(0) = g(-t_d) = e^{-\frac{1}{2} t_d^2/\tau^2} = 10^{-4}$ or 10^{-5}

↳ solve for t_d