EE692 Applied EM- FDTD Method
One-Dimensional Transmission Lines Notes- Lecture 3

FDTD Modeling of Parallel/Series RLC Loads in Parallel and Series

As a another step toward modeling transmission line circuits beyond individual lumped elements [1], this lecture will present the FDTD update equations necessary to implement parallel or series RLC loads placed in parallel or series with the transmission line [2-3]. Figure 1 shows the parallel or series RLC loads placed in parallel (left) and series (right) in an incremental section of the 1-D transmission line. These RLC loads are located at some arbitrary point in the transmission line. Later, they will be used as part of a voltage source or as a terminating loads. Some results will be shown to demonstrate the accuracy and validity of the update equations by comparison with analytic results.

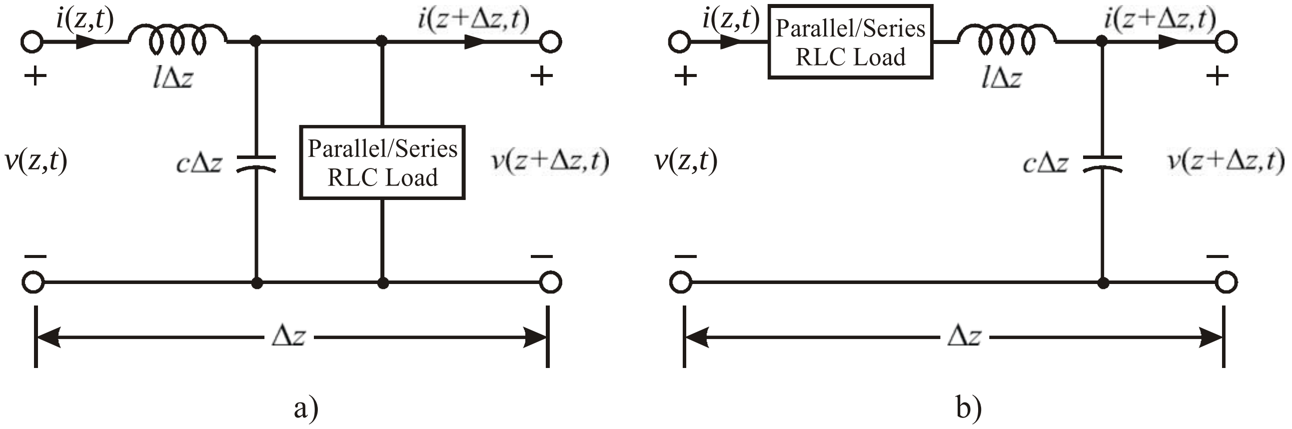
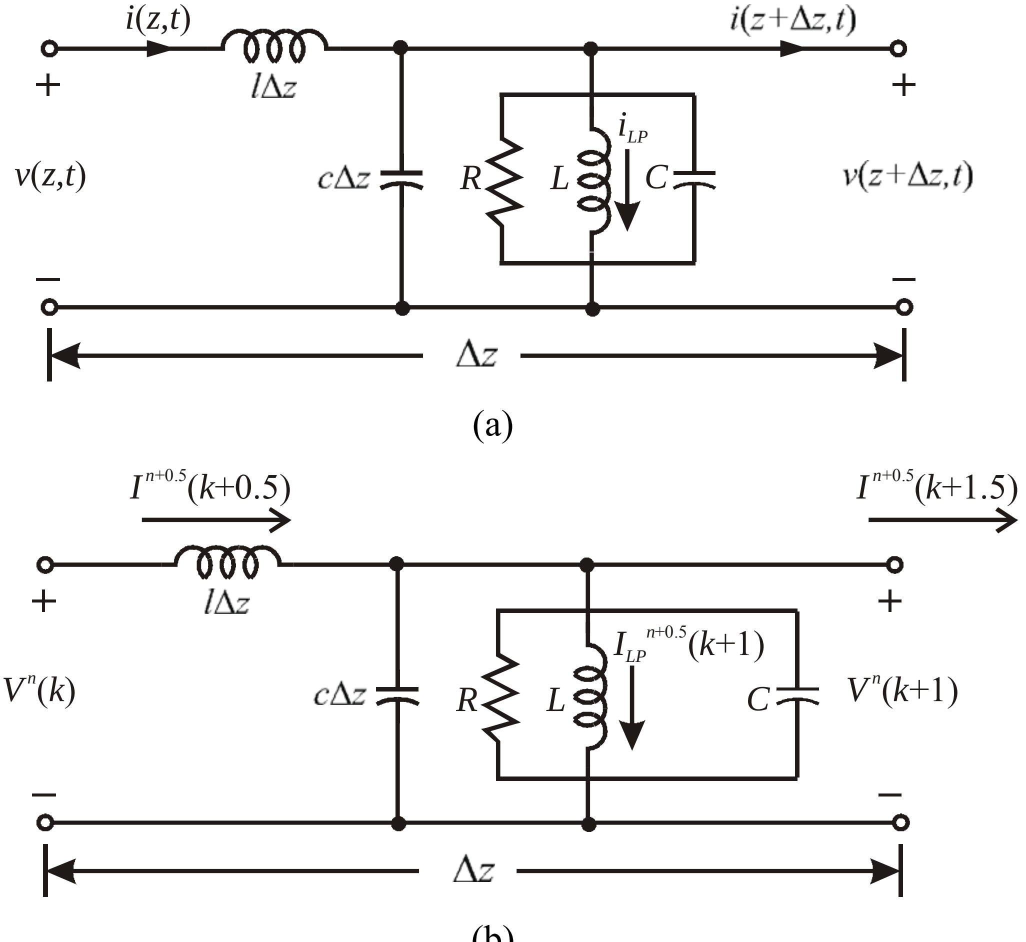


Figure 1 Incremental section of lossless 1D transmission line with parallel or series RLC loads in a) parallel and b) series.

Parallel RLC Load in Parallel

To begin, a circuit model for an incremental section (Δz) of lossless transmission line with a parallel lumped element RLC load in parallel is shown in Figure 2a. Figure 2b shows this incremental section after making the FDTD discretizations. In Figure 2, *l* and *c* are the inductance and capacitance per unit length, respectively, and *R*, *L*, and *C* are the lumped resistor, inductor, and capacitor respectively.



(a)

(b)

Figure 2 (a) Incremental and (b) discretized incremental section of a lossless 1D transmission line with a lumped element parallel RLC load in parallel.

Applying Kirchoff's Current Law (KCL) to the top right node of Figure 2a yields

$$i(z,t) - i(z+\Delta z,t) - c\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - C\frac{\partial v(z+\Delta z,t)}{\partial t} - \frac{v(z+\Delta z,t)}{R} - i_{LP} = 0$$

where i_{LP} is defined by the integral equation

$$i_{LP}(z+\Delta z,t)=\frac{1}{L}\int_{t_0}^{t} v(z+\Delta z,t)\,\partial t+i_{LP}(z+\Delta z,t_0)\ .$$

Applying Kirchoff's Voltage Law (KVL) clockwise around the outside loop yields

$$v(z+\Delta z,t)-v(z,t)+l\Delta z\frac{\partial i(z,t)}{\partial t}=0\ .$$

To save time, the intermediate step of letting Δz→0 and finding the derivatives with respect to *z* will be skipped. Instead, we will directly discretize the KCL and parallel inductor current equations about position $z=(k+1)\Delta z$ and time $t=(n+0.5)\Delta t$ to get

$$I^{n+0.5}(k+0.5)-I^{n+0.5}(k+1.5)-\left(c\Delta z+C_p\right)\left[\frac{V^{n+1}(k+1)-V^{n}(k+1)}{\Delta t}\right]$$
$$-\frac{V^{n+0.5}(k+1)}{R}-I_{LP}^{n+0.5}(k+1)=0$$

and

$$I_{LP}{}^{n+0.5}(k+1)=\frac{1}{L_p}\left[V^{n}(k+1)\Delta t\right]+I_{LP}{}^{n-0.5}(k+1)\ .$$

Again, a problem arises with the term dealing with the current through the parallel resistor *R*. Specifically, the voltage across *R* is needed at time $t=(n+0.5)\Delta t$, a point not included in the temporal grid. To approximate this voltage, the simple average

$$V^{n+0.5}(k+1)\approx\frac{V^{n+1}(k+1)+V^{n}(k+1)}{2}$$

is used yielding

$$I^{n+0.5}(k+0.5)-I^{n+0.5}(k+1.5)-\left(c\Delta z+C_p\right)\left[\frac{V^{n+1}(k+1)-V^{n}(k+1)}{\Delta t}\right]$$
$$-\left[\frac{V^{n+1}(k+1)+V^{n}(k+1)}{2R}\right]-I_{LP}^{n+0.5}(k+1)=0\ .$$

The KVL equation is discretized about position $z=(k+0.5)\Delta z$ and time $t=n\Delta t$, to get

$$V^{n}(k+1)-V^{n}(k)+l\Delta z\left[\frac{I^{n+0.5}(k+0.5)-I^{n-0.5}(k+0.5)}{\Delta t}\right]=0.$$

These equations, when re-arranged and simplified, yield the update equations

$$\boxed{I^{n+0.5}(k+0.5)=I^{n-0.5}(k+0.5)-\frac{1}{Z_C}\left(\frac{v_p\Delta t}{\Delta z}\right)\left[V^n(k+1)-V^n(k)\right]},$$

$$\boxed{I_{LP}^{n+0.5}(k+1)=I_{LP}^{n-0.5}(k+1)+\frac{\Delta t}{L}V^n(k+1)},$$

and

$$\boxed{\begin{aligned}V^{n+1}(k+1)&=A_1V^n(k+1)-A_2Z_C\left(\frac{v_p\Delta t}{\Delta z}\right)\left[I^{n+0.5}(k+1.5)-I^{n+0.5}(k+0.5)\right]\\&\quad-A_2Z_C\left(\frac{v_p\Delta t}{\Delta z}\right)I_{LP}{}^{n+0.5}(k+1)\end{aligned}}$$

where

$$\boxed{A_1=\frac{1-Z_C\left(\frac{v_p\Delta t}{\Delta z}\right)\left(\frac{1}{2R}-\frac{C}{\Delta t}\right)}{1+Z_C\left(\frac{v_p\Delta t}{\Delta z}\right)\left(\frac{1}{2R}+\frac{C}{\Delta t}\right)}}$$

and

$$\boxed{A_2=\frac{1}{1+Z_C\left(\frac{v_p\Delta t}{\Delta z}\right)\left(\frac{1}{2R}+\frac{C}{\Delta t}\right)}}.$$

Again, define the characteristic impedance $Z_C=\sqrt{\frac{l}{c}}$, phase velocity $v_p=\frac{1}{\sqrt{lc}}$, and Courant stability factor $S=\frac{v_p\Delta t}{\Delta z}$. This implies $l=\frac{Z_C}{v_p}$ and $c=\frac{1}{v_pZ_C}$. Note that the update equation for the current is unchanged from that for a 1D lossless transmission line. However, there is now an intermediate **auxiliary** equation to update the current through the parallel inductor L, and the voltage update equation has an additional term (when compared to that for a 1D lossless transmission line) and coefficients that account for the parallel RLC load in parallel with the lossless transmission line. Also, the parallel RLC load in parallel is located at $z=(k+1)\Delta z$ in the FDTD grid.

Series RLC Load in Parallel

Next, a circuit model for an incremental section (Δz) of lossless transmission line with a series lumped element RLC load in parallel is shown in Figure 3a. Figure 3b shows this incremental section after making the FDTD discretizations.

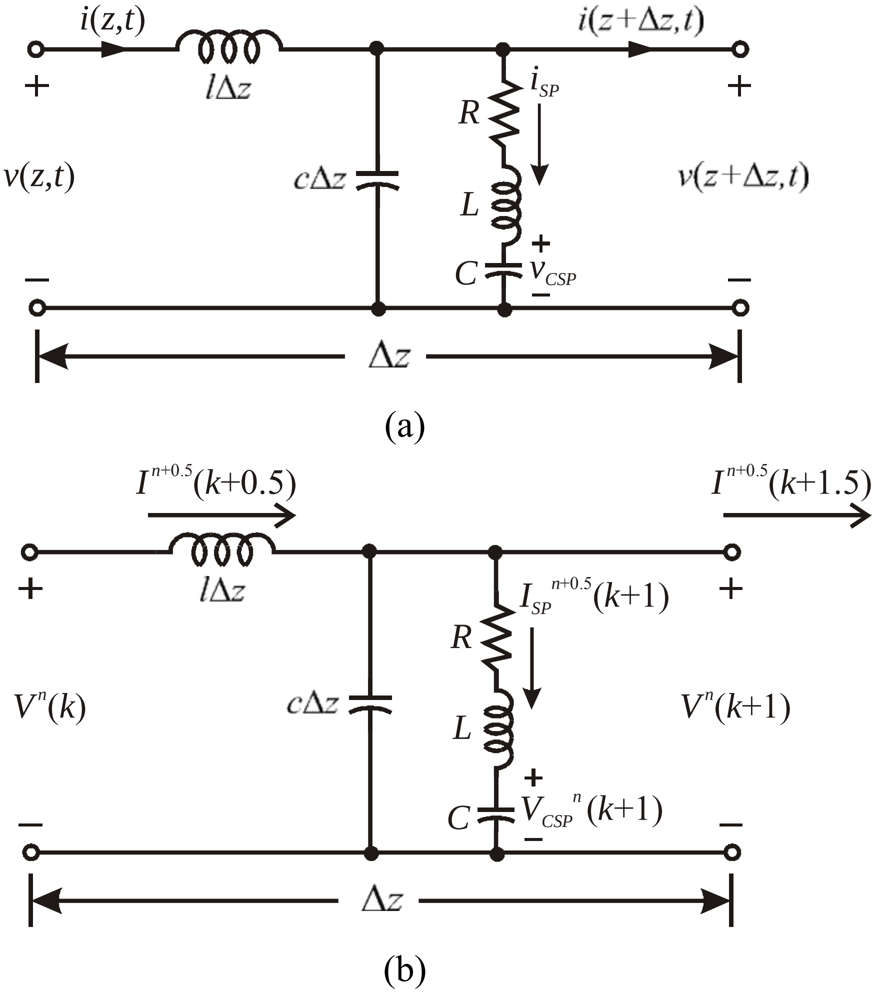


Figure 3 (a) Incremental and (b) discretized incremental section of lossless 1D transmission line with lumped element series RLC load in parallel.

Applying KCL to the top right node of Figure 3a yields

$$i(z,t)-i(z+\Delta z,t)-c\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t}-i_{SP}=0\ .$$

Applying KVL clockwise around the outside loop of Figure 3a yields

$$v(z+\Delta z,t)-v(z,t)+l\Delta z\frac{\partial i(z,t)}{\partial t}=0\ .$$

Applying KVL counterclockwise around the right-hand loop of the circuit of Figure 3 yields

$$i_{SP}R + L\frac{\partial i_{SP}}{\partial t} + v_{CSP} - v(z+\Delta z,t) = 0$$

where

$$v_{CSP}(z+\Delta z,t) = \frac{1}{C}\int_{t_0}^{t} i_{SP}(z+\Delta z,t)\partial t + v_{CSP}(z+\Delta z,t_0)\ .$$

To save time, the intermediate step of letting Δ*z*→0 and finding the derivatives with respect to *z* will be skipped. Instead, we will directly discretize the KCL equation about position $z=(k+1)\Delta z$ and time $t=(n+0.5)\Delta t$ to get

$$I^{n+0.5}(k+0.5) - I^{n+0.5}(k+1.5) - c\Delta z\left[\frac{V^{n+1}(k+1)-V^{n}(k+1)}{\Delta t}\right] - I_{SP}^{n+0.5}(k+1) = 0\ .$$

The KVL equation about the outer loop is discretized about position $z=(k+0.5)\Delta z$ and time $t=n\Delta t$, to get

$$V^{n}(k+1) - V^{n}(k) + l\Delta z\left[\frac{I^{n+0.5}(k+0.5)-I^{n-0.5}(k+0.5)}{\Delta t}\right] = 0.$$

The KVL equation about the righthand loop and the capacitor voltage equation are discretized about position $z=(k+1)\Delta z$ and time $t=n\Delta t$, to get

$$I_{SP}^{n}(k+1)R + L\left[\frac{I_{SP}^{n+0.5}(k+1)-I_{SP}^{n-0.5}(k+1)}{\Delta t}\right] + V_{CSP}^{n}(k+1) - V^{n}(k+1) = 0$$

and

$$V_{CSP}^{n}(k+1) = \frac{1}{C}\left[I_{SP}^{n-0.5}(k+1)\Delta t\right] + V_{CSP}^{n-1}(k+1).$$

A problem arises with the term dealing with the current i_{SP} being needed at integer as well as half-integer time steps in the temporal grid. To resolve this issue, the simple average

$$I_{SP}^{n}(k+1) \approx \frac{I_{SP}^{n+0.5}(k+1)+I_{SP}^{n-0.5}(k+1)}{2}$$

is used yielding

$$R\left[\frac{I_{SP}^{n+0.5}(k+1)+I_{SP}^{n-0.5}(k+1)}{2}\right]+L\left[\frac{I_{SP}^{n+0.5}(k+1)-I_{SP}^{n-0.5}(k+1)}{\Delta t}\right]+V_{CSP}^{n}(k+1) \\ -V^{n}(k+1)=0 .$$

These equations, when re-arranged and simplified, yield the update equations

$$\boxed{V_{CSP}{}^{n}(k+1)=V_{CSP}{}^{n-1}(k+1)+\frac{\Delta t}{C}I_{SP}^{n-0.5}(k+1)},$$

$$\boxed{I_{SP}{}^{n+0.5}(k+1)=\left[\frac{\frac{L}{\Delta t}-\frac{R}{2}}{\frac{L}{\Delta t}+\frac{R}{2}}\right]I_{SP}{}^{n-0.5}(k+1)+\left[\frac{1}{\frac{L}{\Delta t}+\frac{R}{2}}\right]\left[V^{n}(k+1)-V_{CSP}{}^{n}(k+1)\right]},$$

$$\boxed{I^{n+0.5}(k+0.5)=I^{n-0.5}(k+0.5)-\frac{1}{Z_c}\left(\frac{v_p\Delta t}{\Delta z}\right)\left[V^{n}(k+1)-V^{n}(k)\right]},$$

and

$$\boxed{\begin{aligned}V^{n+1}(k+1)=V^{n}(k+1)-Z_c\left(\frac{v_p\Delta t}{\Delta z}\right)\left[I^{n+0.5}(k+1.5)-I^{n+0.5}(k+0.5)\right]\\ -Z_c\left(\frac{v_p\Delta t}{\Delta z}\right)I_{SP}{}^{n+0.5}(k+1)\end{aligned}}.$$

Note that the current update equation is unchanged when compared to that for a 1D lossless transmission line. However, now two intermediate auxiliary variables and equations are needed. One for the current through the series RLC load (I_{SP}), and one for the voltage across the lumped capacitor in the series RLC load (V_{CSP}). Further, the voltage update equation has an additional term (when compared to that for a 1D lossless transmission line) that accounts for the series RLC load in parallel. Also, the series RLC load in parallel is located at $z=(k+1)\Delta z$ in the FDTD grid.

Series RLC Load in Series

Figure 4a shows a circuit model for an incremental section (Δz) of lossless transmission line with a series lumped element RLC load in series. Figure 4b shows this incremental section after making the FDTD discretizations.

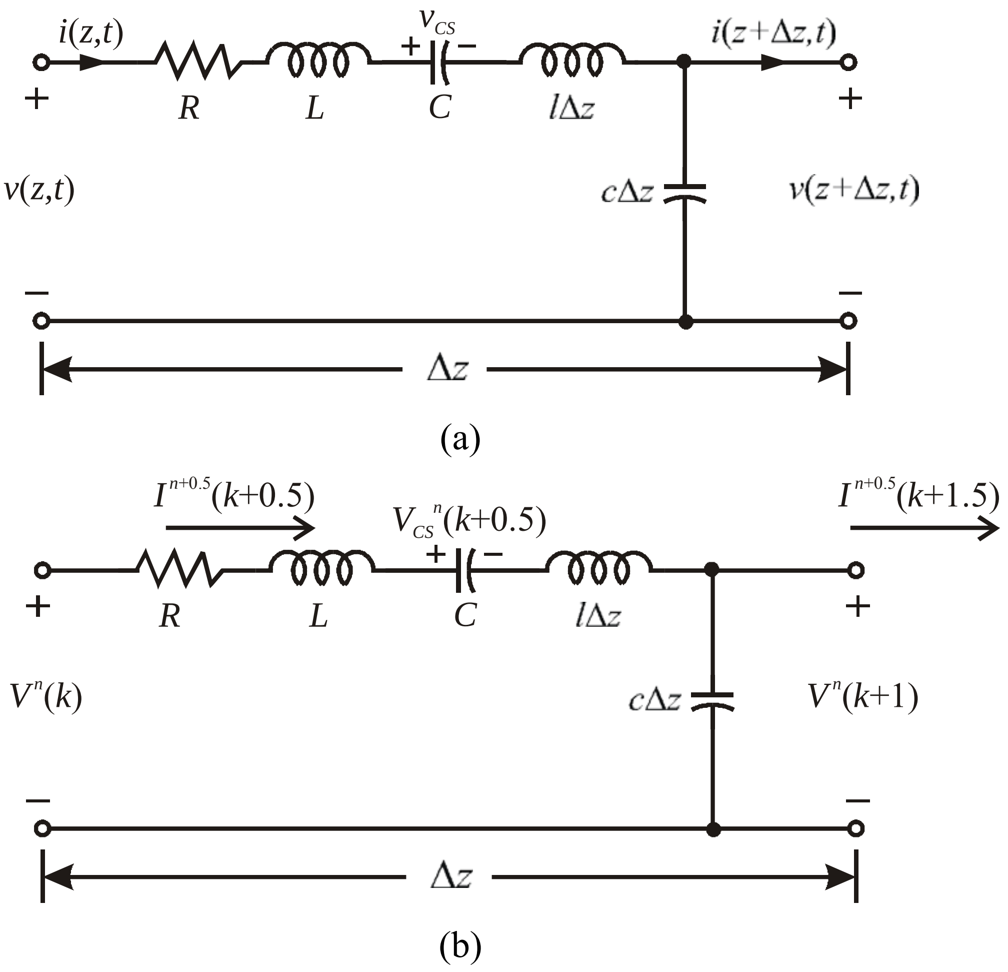


Figure 4 (a) Incremental and (b) discretized incremental section of lossless 1D transmission line with lumped element series RLC load in series.

Applying KCL to the top right node of Figure 4a yields

$$i(z,t)-i(z+\Delta z,t)-c\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t}=0\ .$$

Applying KVL clockwise around the outside loop of Figure 4a yields

$$v(z+\Delta z,t)-v(z,t)+Ri(z,t)+L\frac{\partial i(z,t)}{\partial t}+v_{CS}+l\Delta z\frac{\partial i(z,t)}{\partial t}=0$$

where

$$v_{CS}(z,t) = \frac{1}{C}\int_{t_0}^{t} i(z,t)\partial t + v_{CS}(z,t_0) \ .$$

To save time, the intermediate step of letting Δz→0 and finding the derivatives with respect to *z* will be skipped. Instead, we will directly discretize the KCL equation about position $z = (k+1)\Delta z$ and time $t = (n+0.5)\Delta t$ to get

$$I^{n+0.5}(k+0.5) - I^{n+0.5}(k+1.5) - c\Delta z\left[\frac{V^{n+1}(k+1) - V^{n}(k+1)}{\Delta t}\right] = 0 \ .$$

The KVL and series capacitor equations are discretized about position $z = (k+0.5)\Delta z$ and time $t = n\Delta t$, to get

$$V^{n}(k+1) - V^{n}(k) + R I^{n}(k+0.5) + L\left[\frac{I^{n+0.5}(k+0.5) - I^{n-0.5}(k+0.5)}{\Delta t}\right]$$
$$+ {V_{CS}}^{n}(k+0.5) + l\Delta z\left[\frac{I^{n+0.5}(k+0.5) - I^{n-0.5}(k+0.5)}{\Delta t}\right] = 0$$

and

$$V_{CS}^{n+0.5}(k+0.5) = \frac{1}{C}\left[I^{n-0.5}(k+0.5)\Delta t\right] + V_{CS}^{n-0.5}(k+0.5) .$$

In the KVL equation, a problem arises with the term dealing with the voltage drop across the series resistor *R*. Specifically, the current through *R* is needed at time $t = n\Delta t$, a point not included in the temporal grid. To approximate this voltage, the simple average

$$I^{n}(k+0.5) \approx \frac{I^{n+0.5}(k+0.5) + I^{n-0.5}(k+0.5)}{2}$$

is used yielding

$$V^{n}(k+1) - V^{n}(k) + R\left[\frac{I^{n+0.5}(k+0.5) + I^{n-0.5}(k+0.5)}{2}\right]$$
$$+ \left(L + l\Delta z\right)\left[\frac{I^{n+0.5}(k+0.5) - I^{n-0.5}(k+0.5)}{\Delta t}\right] + {V_{CS}}^{n}(k+0.5) = 0 \ .$$

These equations, when re-arranged and simplified, yield the update equations

$$\boxed{{V_{CS}}^{n}(k+0.5) = {V_{CS}}^{n-1}(k+0.5) + \frac{\Delta t}{C} I^{n-0.5}(k+0.5)},$$

$$\boxed{\begin{gathered} I^{n+0.5}(k+0.5) = B_1 I^{n-0.5}(k+0.5) - B_2 \frac{1}{Z_C}\left(\frac{v_p \Delta t}{\Delta z}\right)\left[V^n(k+1) - V^n(k)\right] \\ -B_2 \frac{1}{Z_C}\left(\frac{v_p \Delta t}{\Delta z}\right){V_{CS}}^{n}(k+0.5) \end{gathered}},$$

and

$$\boxed{V^{n+1}(k+1) = V^n(k+1) - Z_C\left(\frac{v_p \Delta t}{\Delta z}\right)\left[I^{n+0.5}(k+1.5) - I^{n+0.5}(k+0.5)\right]}$$

where

$$\boxed{B_1 = \frac{1 - \frac{1}{Z_C}\left(\frac{v_p \Delta t}{\Delta z}\right)\left(\frac{R}{2} - \frac{L}{\Delta t}\right)}{1 + \frac{1}{Z_C}\left(\frac{v_p \Delta t}{\Delta z}\right)\left(\frac{R}{2} + \frac{L}{\Delta t}\right)}}$$

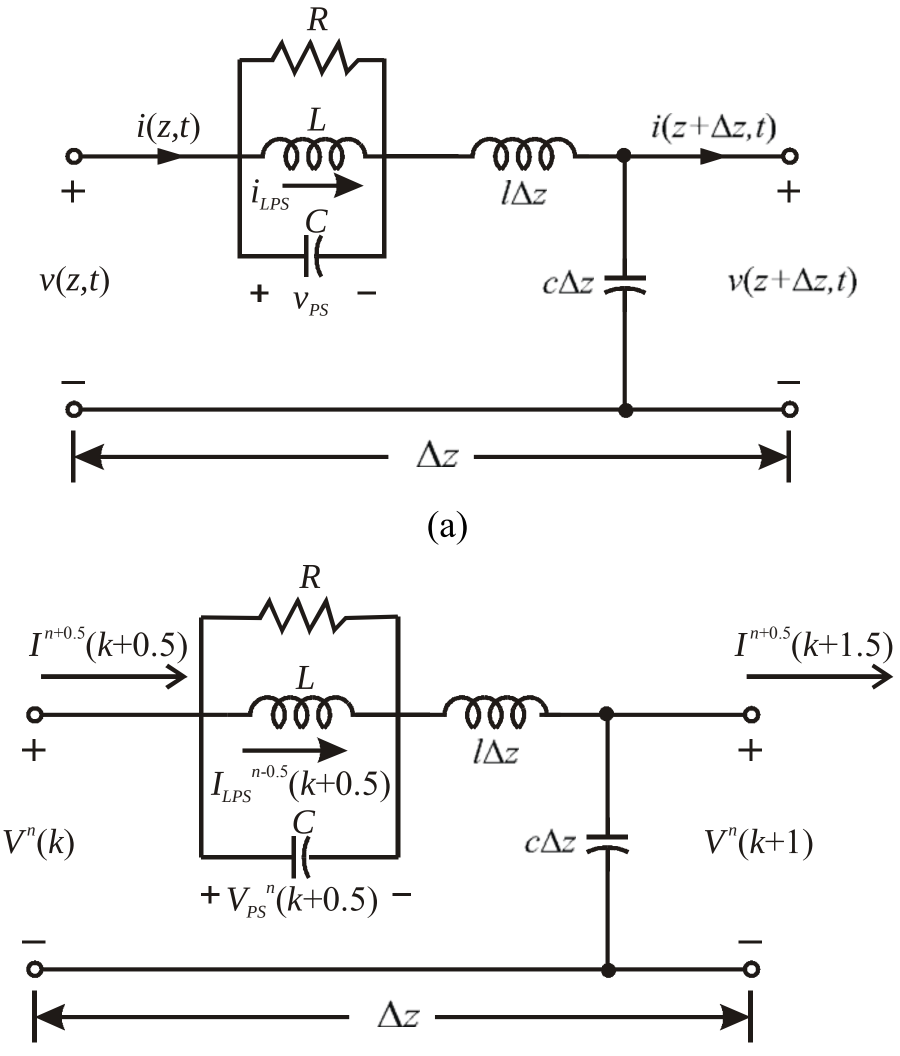
and

$$\boxed{B_2 = \frac{1}{1 + \frac{1}{Z_C}\left(\frac{v_p \Delta t}{\Delta z}\right)\left(\frac{R}{2} + \frac{L}{\Delta t}\right)}}.$$

Note, there is now an auxiliary equation to update the voltage across the series lumped capacitor V_{CS}. Further, the current update equation has an additional term and coefficients (when compared to that for a 1D lossless transmission line) that account for the series RLC load in series. Note that the update equation for the voltage is unchanged from that for a 1D lossless transmission line. Also, the series RLC load in series with the lossless transmission line is located at $z = (k + 0.5)\Delta z$ in the FDTD grid.

Parallel RLC Load in Series

Figure 5a shows a circuit model for an incremental section (Δz) of lossless transmission line with a parallel lumped element RLC load in series. Figure 5b shows this incremental section after making the FDTD discretizations.



(a)

(b)

Figure 5 (a) Incremental and (b) discretized incremental section of a lossless 1D transmission line with a lumped element parallel RLC load in series.

Applying KCL to the top right node of Figure 5a yields

$$i(z,t) - i(z+\Delta z,t) - c\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} = 0 \ .$$

Applying KCL to the top left node of Figure 5a yields

$$i(z,t)-\frac{v_{PS}}{R}-i_{LPS}-C\frac{\partial v_{PS}}{\partial t}=0$$

where

$$i_{LPS}(z,t)=\frac{1}{L}\int_{t_0}^{t}v_{PS}(z,t)\partial t+i_{LPS}(z,t_0)\ .$$

Applying KVL clockwise around the outside loop of Figure 5a yields

$$v(z+\Delta z,t)-v(z,t)+v_{PS}+l\Delta z\frac{\partial i(z,t)}{\partial t}=0\ .$$

To save time, the intermediate step of letting Δz→0 and finding the derivatives with respect to z will be skipped. Instead, we will first directly discretize the KCL equation at the top right node about position $z=(k+1)\Delta z$ and time $t=(n+0.5)\Delta t$ to get

$$I^{n+0.5}(k+0.5)-I^{n+0.5}(k+1.5)-c\Delta z\left[\frac{V^{n+1}(k+1)-V^{n}(k+1)}{\Delta t}\right]=0\ .$$

Next, the KCL equation at the top left node and the equation for the current through the inductor are discretized about position $z=(k+0.5)\Delta z$ and time $t=(n-0.5)\Delta t$ to get

$$I^{n-0.5}(k+0.5)-\frac{V_{PS}^{n-0.5}(k+0.5)}{R}-I_{LPS}^{n-0.5}(k+0.5)-C\left[\frac{V_{PS}^{n}(k+0.5)-V_{PS}^{n-1}(k+0.5)}{\Delta t}\right]=0$$

and

$$I_{LPS}^{n-0.5}(k+0.5)=\frac{1}{L}\left[V_{PS}^{n-1}(k+0.5)\Delta t\right]+I_{LPS}^{n-1.5}(k+0.5)\ .$$

A problem arises with the term dealing with the voltage v_{PS} being needed at integer as well as half-integer time steps in the temporal grid. To resolve this issue, the simple average

$$V_{PS}^{n-0.5}(k+0.5)\approx\frac{V_{PS}^{n}(k+0.5)+V_{PS}^{n-1}(k+0.5)}{2}$$

is used yielding

$$\begin{aligned}I^{n-0.5}(k+0.5)-\left[\frac{V_{PS}^{n}(k+0.5)+V_{PS}^{n-1}(k+0.5)}{2R}\right]-I_{LPS}^{n-0.5}(k+0.5)\\-C\left[\frac{V_{PS}^{n}(k+0.5)-V_{PS}^{n-1}(k+0.5)}{\Delta t}\right]=0\end{aligned}\ .$$

Then, the KVL equation about the outer loop is discretized about position $z = (k + 0.5)\Delta z$ and time $t = n\Delta t$, to get

$$V^n(k+1) - V^n(k) + V_{PS}^n(k+0.5) + l\Delta z\left[\frac{I^{n+0.5}(k+0.5) - I^{n-0.5}(k+0.5)}{\Delta t}\right] = 0.$$

These equations, when re-arranged and simplified, yield the update equations

$$\boxed{I_{LPS}^{n-0.5}(k+0.5) = I_{LPS}^{n-1.5}(k+0.5) + \frac{\Delta t}{L}V_{PS}^{n-1}(k+0.5)},$$

$$\boxed{V_{PS}^n(k+0.5) = \left[\frac{\frac{C}{\Delta t} - \frac{1}{2R}}{\frac{C}{\Delta t} + \frac{1}{2R}}\right]V_{PS}^{n-1}(k+0.5) + \left[\frac{1}{\frac{C}{\Delta t} + \frac{1}{2R}}\right]\left[I^{n-0.5}(k+0.5) - I_{LPS}^{n-0.5}(k+0.5)\right]},$$

$$\boxed{\begin{aligned} I^{n+0.5}(k+0.5) = I^{n-0.5}(k+0.5) - \frac{1}{Z_C}\left(\frac{v_p\Delta t}{\Delta z}\right)\left[V^n(k+1) - V^n(k)\right] \\ -\frac{1}{Z_C}\left(\frac{v_p\Delta t}{\Delta z}\right)V_{PS}^n(k+0.5)\end{aligned}},$$

and

$$\boxed{V^{n+1}(k+1) = V^n(k+1) - Z_C\left(\frac{v_p\Delta t}{\Delta z}\right)\left[I^{n+0.5}(k+1.5) - I^{n+0.5}(k+0.5)\right]}.$$

Note that there are now two auxiliary equations to update the voltage across the lumped element capacitor v_{PS} and current through the lumped element inductor i_{LPS}. Also, the update equation for the current has an additional term (when compared to that for a 1D lossless transmission line) that accounts for the parallel lumped element RLC load in series. However, the update equation for the voltage is unchanged from that for a 1D lossless transmission line. Also, the parallel RLC load in series with the lossless transmission line is located at $z = (k + 0.5)\Delta z$ in the FDTD grid.

Verification of FDTD Update Equations for RLC Loads

To demonstrate the validity of the derived FDTD update equations for these RLC loads, a voltage pulse is launched on a long section of 1D lossless transmission line containing each type of the RLC loads (see Figs. 8ab). The voltage reflected from each type of load is then examined. The FDTD results are compared with analytically derived results. For the analytic results, the 1D lossless transmission line is terminated in its characteristic impedance Z_C after the loads (see Fig. 8cd).

Analytically, the time-domain reflected voltage $V_{\text{ref}}(t)$ was found by taking the inverse Fourier transform of the product of the spectrum of the incident voltage and the reflection coefficient looking in at the RLC loads. The reflection coefficient is defined as

$$\Gamma_{\text{in}} = \frac{Z_{\text{in}} - Z_C}{Z_{\text{in}} + Z_C}$$

where $Z_{\text{in}} = Z_{\text{load}} Z_C / (Z_{\text{load}} + Z_C)$ for a parallel or series RLC load, with impedance Z_{load}, in parallel and $Z_{\text{in}} = Z_{\text{load}} + Z_C$ for the RLC loads in series. A unit-amplitude, Gaussian-pulse, incident voltage pulse was used

$$v_G(t) = e^{-0.5(t-\tau_d)^2/\tau_p^2}$$

where τ_p is the characteristic time and τ_d is a time delay.

The analytic results used for comparison were obtained in the following manner:

1) The input Gaussian pulse $v_G(t)$ is calculated in the time-domain for a period $0 \le t = n\Delta t \le 50\tau_p$ where the step-size Δt is relatively small.
2) The discrete Fourier transform $V_G(\omega_k)$ of $v_G(t)$ is computed.
3) At each discrete frequency ω_k, the impedances of the capacitor $Z_C(\omega_k)$, inductor $Z_L(\omega_k)$, and overall impedance $Z_{\text{in}}(\omega_k)$ of the parallel or series RLC load are calculated. This is used to calculate the input impedance $Z_{\text{in}}(\omega_k)$ and reflection coefficient $\Gamma_{\text{in}}(\omega_k)$.
4) The frequency-domain reflected voltage is then calculated as $V_{\text{ref}}(\omega) = \Gamma_{\text{in}}(\omega) V_G(\omega)$.
5) Finally, an inverse discrete Fourier transform of $V_{\text{ref}}(\omega)$ is performed to obtain $V_{\text{ref}}(t)$.

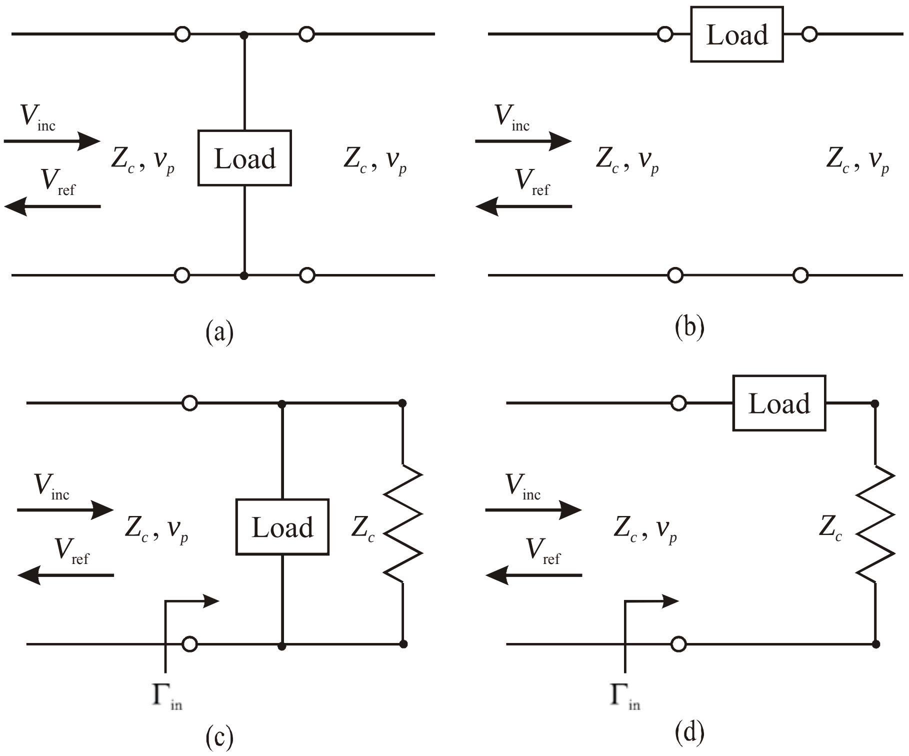


Figure 8 Transmission line circuits modeled using FDTD method for loads in (a) parallel and (b) series, and equivalent transmission line circuits used analytic results for loads in (c) parallel and (d) series.

Figures 9 - 12 show examples of the voltages reflected from each type of load. The RLC element values were arbitrarily selected for each type of load so that the contribution from each element type was evident. In the FDTD models, the transmission line ($Z_C = 50\ \Omega$, $v_p = 2.998 \times 10^8$ m/s) is driven by a one-way voltage source injector and terminated at each end by an absorbing boundary condition (discussed later), $\Delta z = 0.76$ mm, $\tau_p = 16.732$ ps, $\tau_d = 6\tau_p$, and $S = 0.5$. As shown, there is excellent agreement between the analytic and FDTD results.

<u>References</u>

[1] T. P. Montoya and G. S. Smith, “Modeling Transmission Line Circuit Elements in the FDTD Method,” *Microwave and Optical Technology Letters*, vol. 21, no. 7, pp.105-114, April 20, 1999.

[2] T. P. Montoya and W. R. Scott, Jr., “Modeling Parallel and Series RLC Loads in a 1-D FDTD Transmission Line,” *USNC/URSI National Radio Science Meeting*, Salt Lake City, UT, p. 30, July 16-21, 2000.

[3] T. P. Montoya, “Improved 1-D FDTD Modeling of Parallel and Series RLC Loads in a Lossless Transmission Line,” 2006 *IEEE APS International Symp.*, Albuquerque, NM, pp. 1583-1586, July 9-14, 2006.

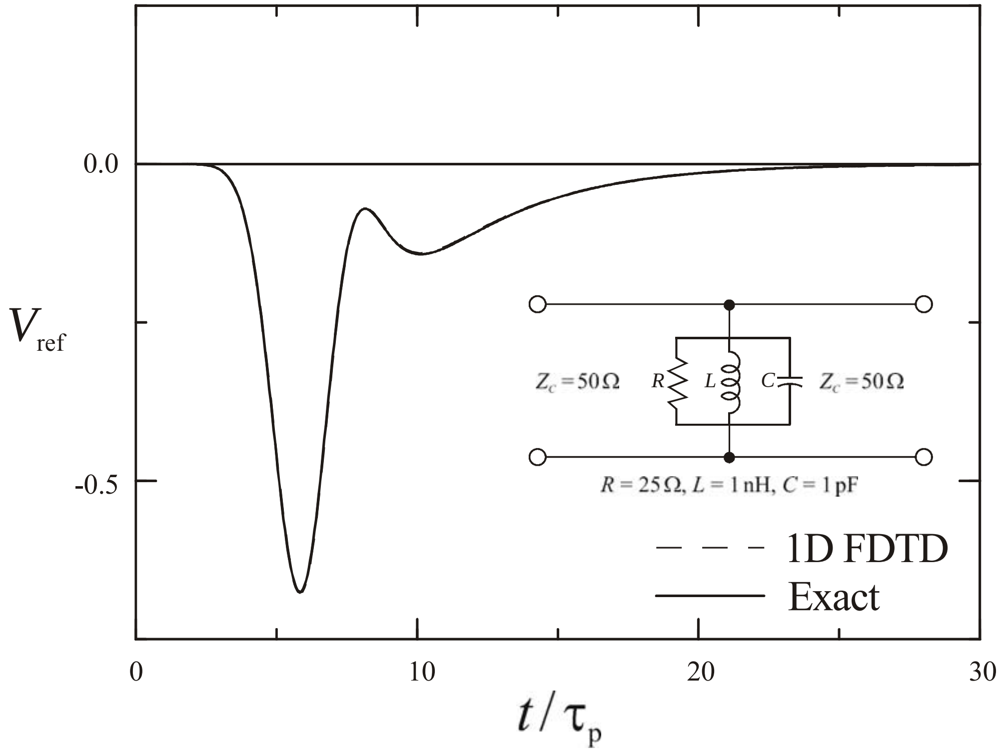


Figure 9 Reflected voltage from a transmission line with a parallel RLC load placed in parallel for **Gaussian** pulse excitation.

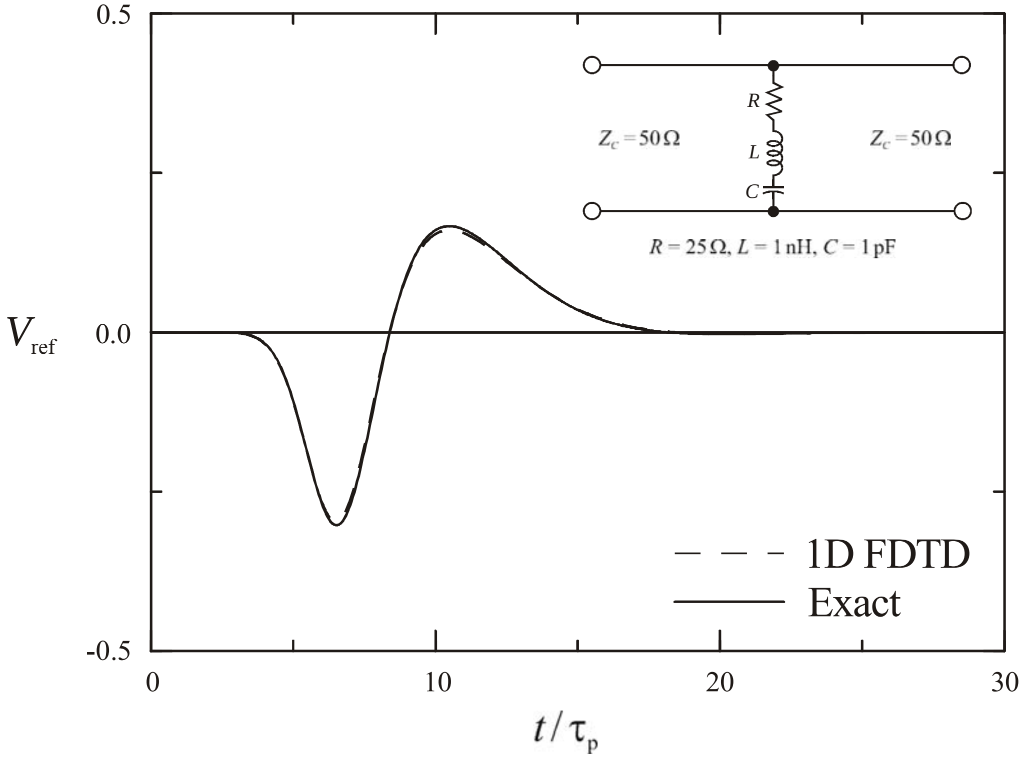


Figure 10 Reflected voltage from a transmission line with a series RLC load placed in parallel for **Gaussian** pulse excitation.

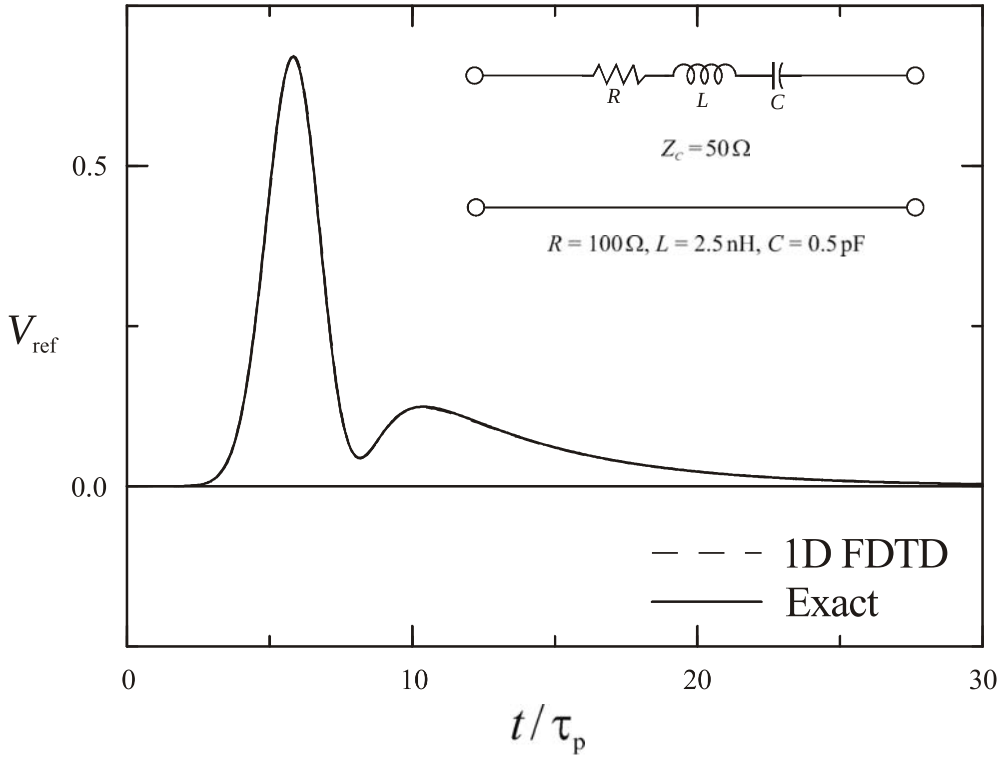


Figure 11 Reflected voltage from a transmission line with a series RLC load placed in series for **Gaussian** pulse excitation.

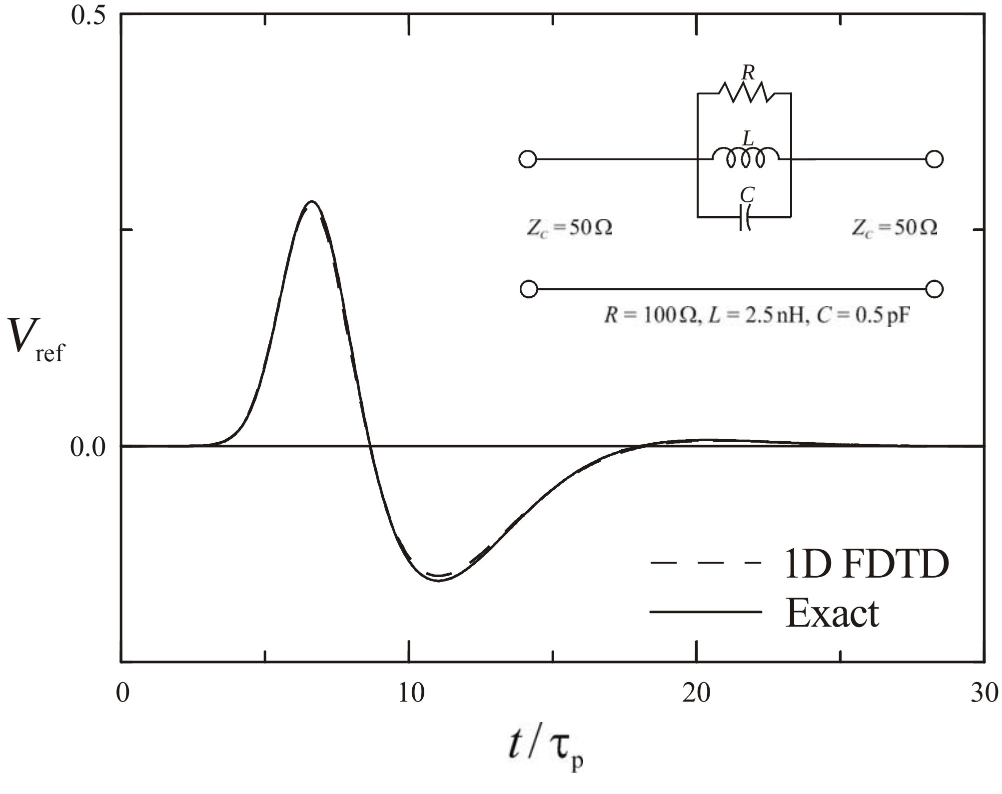


Figure 12 Reflected voltage from a transmission line with a parallel RLC load placed in series for **Gaussian** pulse excitation.