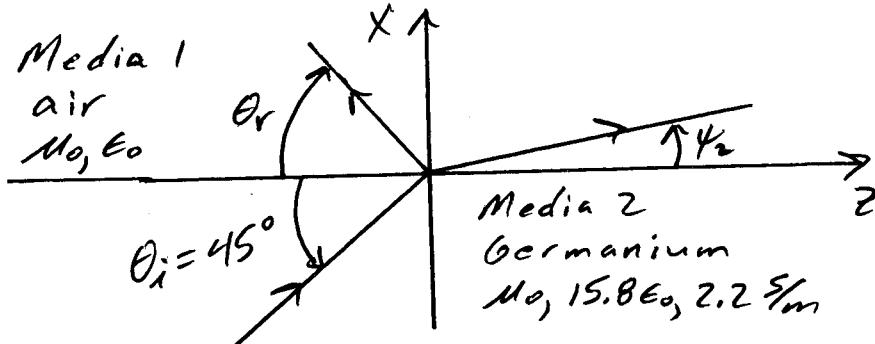


**Example-** A 6 GHz UPW in air ( $\epsilon_0, \mu_0, z < 0$ ) is obliquely incident at  $\theta_i = 45^\circ$  on a germanium half-space ( $15.8\epsilon_0, \mu_0, \sigma = 2.2 \text{ S/m}, z > 0$ ). Find the intrinsic impedance and propagation constants in the two materials, reflected  $\theta_r$  & transmitted  $\theta_t$  angles, reflection & transmission coefficients for both parallel and perpendicular polarizations, effective attenuation  $\alpha_{2e}$  & phase  $\beta_{2e}$  constants, true refracted angle  $\psi_2$ , and phase velocity  $v_{pr}$  in the germanium.



### Propagation constants

$$\text{Per (4-28), } \gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\gamma_1 = \sqrt{j2\pi6 \times 10^9(4\pi \times 10^{-7})(0 + j2\pi6 \times 10^9(8.8542 \times 10^{-12}))}$$

$$\underline{\gamma_1 = j125.750701 \text{ m}^{-1}}$$

$$\gamma_2 = \sqrt{j2\pi6 \times 10^9(4\pi \times 10^{-7})(2.2 + j2\pi6 \times 10^9(15.8)8.8542 \times 10^{-12})}$$

$$\underline{\gamma_2 = 102.143607 + j510.178881 \text{ m}^{-1}}$$

### Intrinsic impedances

$$\text{Per (4-30), } Z_w = \eta_c = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\eta_1 = \sqrt{\frac{j\omega4\pi \times 10^{-7}}{0 + j\omega8.8542 \times 10^{-12}}} \Rightarrow \underline{\eta_1 = \eta_0 = 376.7303 \Omega}$$

$$\eta_2 = \sqrt{\frac{j2\pi6 \times 10^9(4\pi \times 10^{-7})}{2.2 + j2\pi6 \times 10^9(15.8)8.8542 \times 10^{-12}}}$$

$$\underline{\eta_2 = 89.27911 + j17.87469 = 91.051 \angle 11.3216^\circ \Omega}$$

Using Snell's law of refraction for lossy media (S-54)  $\gamma_1 \sin \theta_i = \gamma_2 \sin \theta_t$

$$(S-55a) \quad \sin \theta_t = \frac{\gamma_1}{\gamma_2} \sin \theta_i = \frac{j125.75}{102.14+j510.18} \sin 45^\circ \\ = 0.16757 + j0.03355$$

$$(S-55b) \quad \cos \theta_t = \sqrt{1 - \sin^2 \theta_t} = \sqrt{1 - (0.168 + j0.034)^2} \\ = 0.986447 - j0.005699 = 5 e^{j\phi}$$

$$S = |\cos \theta_t| = |0.98645 - j0.0057| = 0.986463$$

$$\phi = \arg \cos \theta_t = \arg (0.98645 - j0.0057) = -0.005778 \text{ rad} \\ = -0.33103^\circ$$

$$\text{Per (S-55a)} \quad \theta_t = \sin^{-1}(\sin \theta_t) = \sin^{-1}(0.1676 + j0.03355)$$

$$\text{or} \quad \begin{aligned} \theta_t &= 0.16827 + j0.03402 \text{ rad} \\ \theta_t &= 9.641 + j1.949^\circ \end{aligned}$$

Snell's law of reflection

$$\text{Per (S-15a), } \theta_r = \theta_i = \pi/4 = 45^\circ$$

Perpendicular Polarization  $\Gamma + T$

$$(S-17a) \quad \Gamma_L^b = \frac{\gamma_2 \cos \theta_i - \gamma_1 \cos \theta_t}{\gamma_2 \cos \theta_i + \gamma_1 \cos \theta_t} \\ = \frac{(89.28 + j17.87) \cos 45^\circ - 376.73 (0.986 - j0.0057)}{(89.28 + j17.87) \cos 45^\circ + 376.73 (0.986 - j0.0057)}$$

$$\Gamma_L^b = -0.70835 + j0.051106 = 0.71019 \underline{|175.8734^\circ|}$$

$$(S-17b) T_{\perp}^b = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$$

$$= \frac{2(89.28 + j17.87) \cos 45^\circ}{(89.28 + j17.87) \cos 45^\circ + 376.73(0.986 - j0.0057)}$$

$$\underline{T_{\perp}^b = 0.29165 + j0.051106 = 0.296093 / 9.93909^\circ}$$

### Parallel Polarization $\Gamma + T$

$$(S-24c) \Gamma_{11}^b = \frac{-\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}$$

$$= \frac{-376.73 \cos 45^\circ + (89.28 + j17.87)(0.986 - j0.0057)}{376.73 \cos 45^\circ + (89.28 + j17.87)(0.986 - j0.0057)}$$

$$\underline{\Gamma_{11}^b = -0.499148 + j0.072402 = 0.29609 / 9.93909^\circ}$$

$$(S-24d) T_{11}^b = \frac{2\eta_2 \cos \theta_i}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}$$

$$= \frac{2(89.28 + j17.87) \cos 45^\circ}{376.73 \cos 45^\circ + (89.28 + j17.87)(0.986 - j0.0057)}$$

$$\underline{T_{11}^b = 0.35871 + j0.053972 = 0.36275 / 8.5566^\circ}$$

### Effective attenuation in media 2

$$(S-57a) \alpha_{ze} = \rho = s(\alpha_2 \cos \delta - \beta_2 \sin \delta)$$

$$= 0.98645(102.144 \cos(-0.331^\circ) - 510.1789 \sin(-0.331^\circ))$$

$$\underline{\alpha_{ze} = 103.6669 \text{ dB/m}}$$

## Effective phase constant in media 2

$$(S-60a) \quad u = \beta_1 \sin \theta_i = 125.75 \sin 45^\circ$$

$$u = 88.919174 \text{ m}^{-1}$$

$$(S-57b) \quad q = s(\alpha_2 \sin \beta + \beta_2 \cos \beta)$$

$$= 0.98645(102.144 \sin(-0.331^\circ) + 510.1789 \cos(-0.331^\circ))$$

$$= 502.682189 \text{ m}^{-1}$$

$$(S-61b) \quad \beta_{2e} = \sqrt{u^2 + q^2} = \sqrt{88.919^2 + 502.682^2}$$

$$\underline{\beta_{2e} = 510.48605 \text{ rad/m}}$$

## True refracted angle in media 2

$$(S-60d) \quad \underline{\psi_2 = \tan^{-1}(u/q)} = \tan^{-1}\left(\frac{88.919}{502.682}\right)$$

$$\underline{\psi_2 = 10.03125^\circ} \quad \leftarrow \text{Not too far from } \\ \text{Re}(\theta_t) = 9.641^\circ \text{ or } |\theta_t| = 9.836^\circ$$

## Phase velocity in media 2

$$(S-62a) \quad \underline{v_{pr} = \frac{\omega}{\beta_{2e}}} = \frac{2\pi(6 \times 10^9)}{510.486}$$

$$\underline{v_{pr} = 0.7384945 \times 10^8 \text{ m/s}}$$

$$\left( \text{Note: } \frac{c}{\sqrt{\epsilon_r}} = 0.7542 \times 10^8 \text{ m/s} \right) \\ \Rightarrow \text{about } 2.1\% \text{ difference}$$