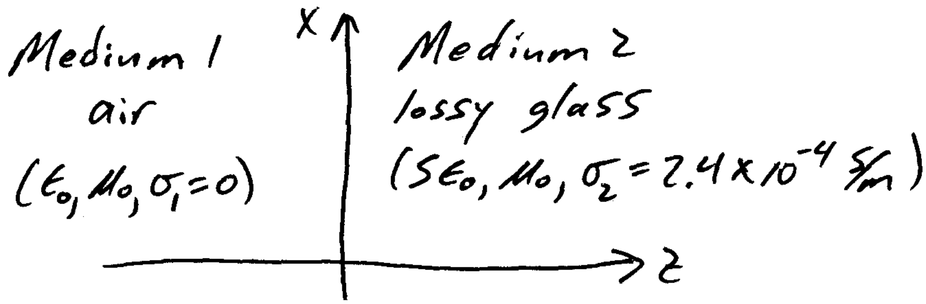
Example- A UPW in air (ε_0, μ_0, $\sigma \approx 0$, $z < 0$) is normally incident on a lossy glass half-space ($5\varepsilon_0$, μ_0, $\sigma = 2.4\times10^{-4}$ S/m, $z > 0$). The 2.4 GHz incident electric field is oriented in the x-direction and has a field strength of 0.6 V/m at $z = 0$. Analyze and determine the various associated fields, related quantities, and power densities at $z = 0$ and at $z = 10$ m.



$$\gamma_1 = \sqrt{j\omega\mu_0(\sigma_1 + j\omega\epsilon_0)} = \sqrt{j2\pi(2.4\times10^9)\, j2\pi(2.4\times10^9)\, 8.854\times10^{-12}}$$

$$= 0 + j50.30028\ (1/m)$$

$$\alpha_1 = 0 \qquad \beta_1 = 50.30028\ \text{rad/m}$$

$$\eta_1 = \sqrt{\frac{j\omega\mu_0}{\sigma_1 + j\omega\epsilon_0}} = \sqrt{\mu_0/\epsilon_0} = 376.7303\ \Omega$$

$$\gamma_2 = \sqrt{j\omega\mu_0(\sigma_2 + j\omega 5\epsilon_0)}$$

$$= \sqrt{j2\pi(2.4\times10^9)(4\pi\times10^{-7})(2.4\times10^{-4} + j2\pi(2.4\times10^9)5(8.854\times10^{-12}))}$$

$$= 0.0202175 + j112.474848\ (1/m)$$

$$\alpha_2 = 0.02022\ (Np/m) \qquad \beta_2 = 112.47485\ \text{rad/m}$$

$$\delta_2 = \frac{1}{\alpha_2} = 49.462\,m$$

$$\eta_2 = \sqrt{\frac{j\omega\mu_0}{\sigma_2 + j\omega 5\epsilon_0}} = 168.47891 + j0.03028\ \Omega$$

$$\eta_2 = 168.4789\angle 0.0103^\circ\ \Omega$$

$$\Gamma^b = \frac{\eta_2 + \eta_1}{\eta_2 + \eta_1} = \frac{168.4789\angle 0.0103^\circ - 376.7303}{168.4789\angle 0.0103^\circ + 376.7303}$$

$$\Gamma^b = -0.38197 + j7.6763\times 10^{-5} = 0.38197\angle 179.9885^\circ$$

$$T^b = \frac{2\eta_2}{\eta_2 + \eta_1} = 0.618034 + j7.6763\times 10^{-5}$$

$$T^b = 0.618034\angle 0.00712^\circ$$

$$\bar{E}^i = \hat{a}_x\, 0.6\, e^{-j50.3z}\ (\mathrm{V/m}) \quad z \le 0 \qquad (5\text{-}47a)$$

$$\bar{H}^i = \hat{a}_y\, 1.59265\, e^{-j50.3z}\ \left(\frac{\mathrm{mA}}{\mathrm{m}}\right) \quad z \le 0 \qquad (5\text{-}47b)$$

$$\bar{E}^r = \hat{a}_x\,(-0.38197 + j7.676\times 10^{-5})\,0.6\, e^{+j50.3z} \qquad (5\text{-}48a)$$

$$\bar{E}^r = -\hat{a}_x\,(0.22918 - j4.606\times 10^{-5})\, e^{+j50.3z}\ (\mathrm{V/m}) \quad z \le 0$$

$$\bar{H}^r = -\hat{a}_y\,\frac{(-0.38197 + j7.676\times 10^{-5})\,0.6}{376.7303}\, e^{+j50.3z} \qquad (5\text{-}48b)$$

$$\bar{H}^r = \hat{a}_y\,(0.60834 - j1.223\times 10^{-4})\, e^{+j50.3z}\ \left(\frac{\mathrm{mA}}{\mathrm{m}}\right) \quad z \le 0$$

$$\bar{E}_{air} = \bar{E}^i + \bar{E}^r = \hat{a}_x\left[0.6\, e^{-j50.3z} - (0.2292 - j4.606\times 10^{-5})\, e^{j50.3z}\right]\left(\frac{\mathrm{V}}{\mathrm{m}}\right)$$

$$\bar{H}_{air} = \hat{a}_y\left[1.59265\, e^{-j50.3z} + (0.60834 - j1.223\times 10^{-4})\, e^{j50.3z}\right]\left(\frac{\mathrm{mA}}{\mathrm{m}}\right)$$

$$z \le 0$$

$$\bar{E}^t = \hat{a}_x (0.618 + j7.676\times10^{-5})\,0.6\; e^{-0.02z} e^{-j112.475z} \quad (5\text{-}49a)$$

$$\bar{E}_{glass} = \bar{E}^t = \hat{a}_x (0.3708 + j4.606\times10^{-5})\, e^{-0.0202z} e^{-j112.475z} \left(\frac{V}{m}\right) \; z \geq 0$$

$$\bar{H}^t = \hat{a}_y \frac{(0.618 + j7.676\times10^{-5})\,0.6}{168.479 + j0.0303}\, e^{-0.0202z} e^{-j112.475z}$$

$$\bar{H}_{glass} = \bar{H}^t = \hat{a}_y (2.201 - j1.223\times10^{-5})\, e^{-0.0202z} e^{-j112.475z} \left(\frac{mA}{m}\right) \; z \geq 0$$

$$\bar{S}^i_{ave} = \tfrac{1}{2}\,Re\left(\bar{E}^i \times \bar{H}^{i*}\right) = \hat{a}_z\, 0.4778 \left(\frac{mW}{m^2}\right) \; z \leq 0$$

$$\bar{S}^r_{ave} = \tfrac{1}{2}\,Re\left(-\hat{a}_x (0.22918 - j4.606\times10^{-5})\, e^{j50.3z} \times \hat{a}_y (0.6083\times10^{-3} + j1.22\times10^{-7})\, e^{-j50.3}\right)$$

$$= \tfrac{1}{2}\,Re\left(-\hat{a}_z\, 1.39419\times10^{-4}\right) = -\hat{a}_z\, 6.971\times10^{-5}$$

$$\bar{S}^r_{ave} = -\hat{a}_z\, 0.0697 \left(\frac{mW}{m^2}\right) \; z \leq 0$$

$$(5\text{-}51c) \quad \bar{S}^t_{ave} = \hat{a}_z\, |T^b|^2 \frac{|E_0|^2}{2}\, e^{-2\alpha_2 z}\, Re\left(\frac{1}{\eta_2^*}\right)$$

$$= \hat{a}_z\, 0.618034^2\, \frac{0.6^2}{2}\, e^{-2(0.0202)z}\, Re\left(\frac{1}{168.479 - j0.0303}\right)$$

$$\bar{S}^t_{ave} = \hat{a}_z\, 0.40809\, e^{-0.04043z} \left(\frac{mW}{m^2}\right) \; z \geq 0$$

$$\bar{S}^t_{ave}(z=0) = \hat{a}_z\, 0.4081 \left(\frac{mW}{m^2}\right)$$

$$\bar{S}^t_{ave}(z=10\,m) = \hat{a}_z\, 0.2724 \left(\frac{mW}{m^2}\right)$$

⇐ 66.74 % of value @ $z=0$