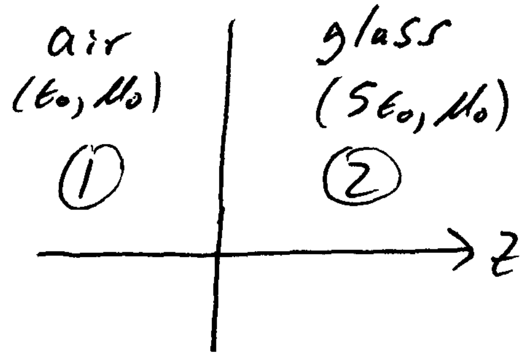
Example- A UPW in air (ε_0, μ_0, $z < 0$) is normally incident on a glass half-space ($5\varepsilon_0$, μ_0, $z > 0$). The 2.4 GHz incident electric field is oriented in the x-direction and has a field strength of 0.6 V/m at $z = 0$. Analyze and determine the various associated fields, power densities, and other related quantities.



$$\beta_1 = \omega\sqrt{\mu_1 \epsilon_1} = 2\pi(2.4\times10^9)\sqrt{4\pi\times10^{-7}(8.8541878\times10^{-12})}$$

$$= \underline{50.3003 \text{ rad/m}}$$

$$\eta_1 = \eta_0 = \underline{376.7303\,\Omega}$$

$$\beta_2 = \omega\sqrt{\mu_2\epsilon_2} = 2\pi(2.4\times10^9)\sqrt{4\pi\times10^{-7}(5)(8.8541878\times10^{-12})}$$

$$= \underline{112.47485 \text{ rad/m}}$$

$$\eta_2 = \sqrt{\mu_2/\epsilon_2} = \eta_0/\sqrt{\epsilon_{r2}} = \frac{376.7303}{\sqrt{5}} = \underline{168.4789\,\Omega}$$

$$\Gamma^b = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{168.48 - 376.73}{168.48 + 376.73} = \underline{-0.381966}$$

$$T^b = 1 + \Gamma^b = 1 - 0.381966 = \underline{0.618034}$$

ex. cont.

$$\bar{E}^i = \hat{a}_x 0.6 e^{-j50.3z} \ (V/m) \quad z \le 0 \quad (air)$$

$$\bar{E}^r = \hat{a}_x (-0.382) 0.6 e^{+j50.3z}$$

$$\bar{E}^r = -\hat{a}_x 0.2292 e^{+j50.3z} \ (V/m) \quad z \le 0 \quad (air)$$

$$\bar{E}_{air} = \bar{E}^i + \bar{E}^r = \hat{a}_x \left[0.6 e^{-j50.3z} - 0.2292 e^{j50.3z} \right] (V/m) \quad z \le 0$$

$$\bar{E}^t = \hat{a}_x (0.618) 0.6 e^{-j112.475z}$$

$$\bar{E}_{glass} = \bar{E}^t = \hat{a}_x 0.3708 e^{-j112.475z} \ (V/m) \quad z \ge 0 \quad (glass)$$

$$\bar{H}^i = \hat{a}_y \frac{0.6}{376.73} e^{-j50.3z} = \hat{a}_y 1.59265 e^{-j50.3z} \left(\frac{mA}{m}\right) \quad z \le 0 \quad (air)$$

$$\bar{H}^r = -\hat{a}_y \frac{(-0.382)0.6}{376.73} e^{+j50.3z}$$

$$\bar{H}^r = \hat{a}_y 0.60834 e^{+j50.3z} \ (mA/m) \quad z \le 0 \quad (air)$$

$$\bar{H}_{air} = \bar{H}^i + \bar{H}^r = \hat{a}_y \left[1.593 e^{-j50.3z} + 0.608 e^{j50.3z} \right] \left(\frac{mA}{m}\right) \quad z \le 0 \quad (air)$$

$$\bar{H}^t = \hat{a}_y \frac{0.618(0.6)}{168.479} e^{-j112.475z}$$

$$\bar{H}_{glass} = \bar{H}^t = \hat{a}_y 2.201 e^{-j112.475z} \left(\frac{mA}{m}\right) \quad z \ge 0 \quad (glass)$$

ex. cont.

$$SWR_{air} = \frac{1+|-0.382|}{1-|-0.382|} = \underline{\underline{2.236}}$$

$$\bar{S}^{i}_{ave} = \tfrac{1}{2}\,Re\left(\hat{a}_x\, 0.6\, e^{-j50.3z} \times \hat{a}_y\, 1.59265\times10^{-3}\, e^{+j50.3z}\right)$$

$$\underline{\bar{S}^{i}_{ave} = \hat{a}_z\, 0.4778\ \frac{mW}{m^2} \quad z \le 0\ (air)}$$

$$\bar{S}^{r}_{ave} = \tfrac{1}{2}\,Re\left(-\hat{a}_x\, 0.2292\, e^{j50.3z} \times \hat{a}_y\, 6.0834\times10^{-4}\, e^{-j50.3z}\right)$$

$$\underline{\bar{S}^{r}_{ave} = -\hat{a}_z\, 0.0697\ \frac{mW}{m^2} \quad z \le 0\ (air)}$$

$$\bar{S}^{t}_{ave} = \tfrac{1}{2}\,Re\left(\hat{a}_x\, 0.3708\, e^{-j112.475z} \times \hat{a}_y\, 2.201\times10^{-3}\, e^{+j112.475z}\right)$$

$$\underline{\bar{S}^{t}_{ave} = \hat{a}_z\, 0.4081\ mW/m^2 \quad z \ge 0\ (glass)}$$

Check $\bar{S}^{i}_{ave} + \bar{S}^{r}_{ave} \stackrel{?}{=} \bar{S}^{t}_{ave}$

$$\hat{a}_z(0.4778 - 0.0697) \stackrel{?}{=} \hat{a}_z\, 0.4081$$

$$\underbrace{\hat{a}_z\, 0.4081\ \frac{mW}{m^2}}_{air} = \underbrace{\hat{a}_z\, 0.4081\ \frac{mW}{m^2}}_{glass}\ \checkmark$$