Chapter 2 Electrical Properties of Matter

2.1 Introduction

→ Much interest of late in metamaterials. These artificial materials can have negative ϵ_r +/or μ_r over a frequency range. Can also create artificial magnetic conductors. This is an active research area in EM.

Basics of Matter

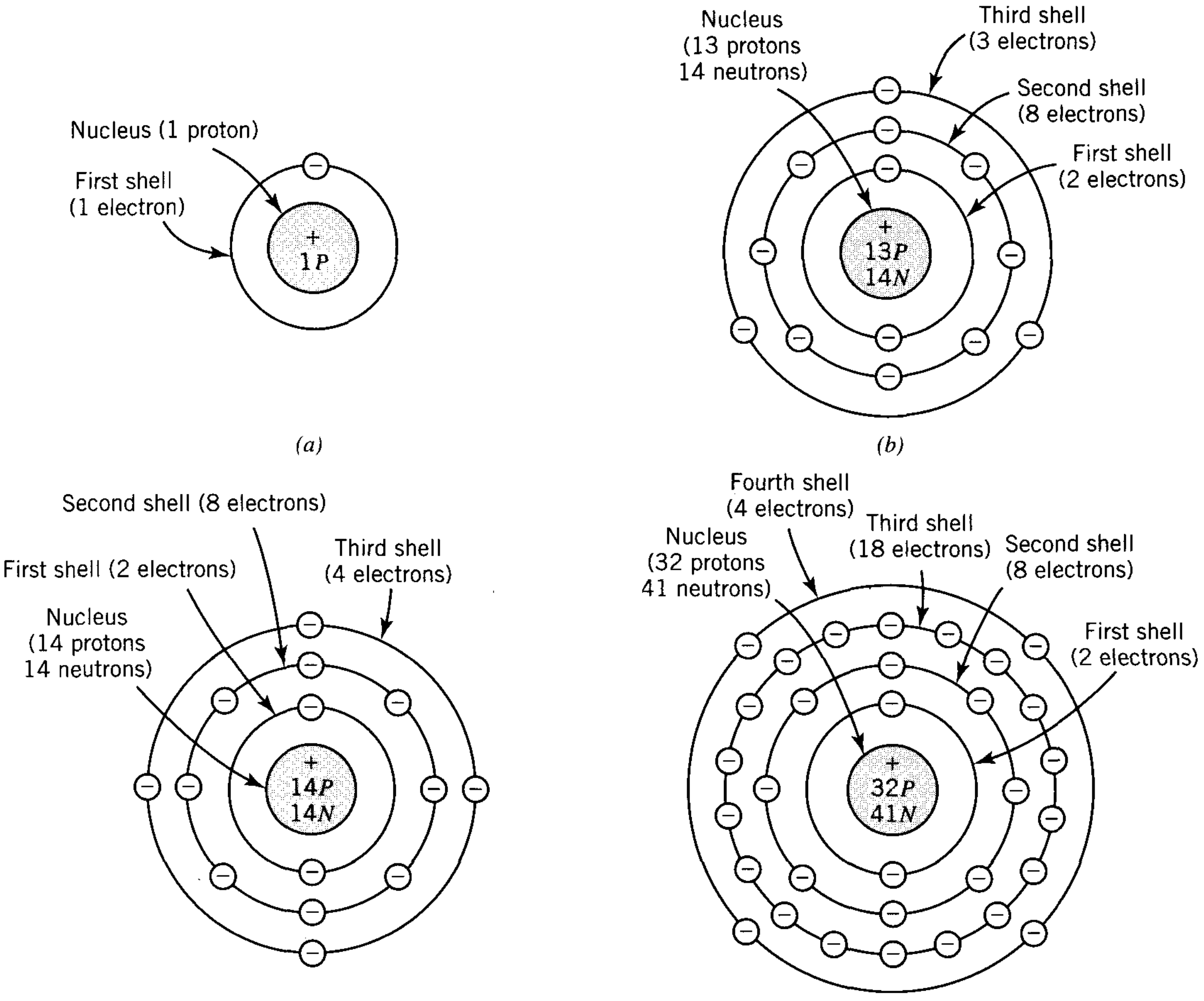
→ atom consists of a nucleus (protons + neutrons) surrounded by electrons.

→ Elements consist of a single type of atom (ignore isotopes). Can be organized in molecules/crystals...

→ Compounds are composed of 2 or more elements and are typically organized as molecules

→ Typically, atoms/molecules are charge neutral, i.e., same # of protons + electrons

→ The electrons for an atom exist in various 'shells' w/ the outermost called the valence shell/band w/ valence electrons. In a given shell, the electrons have the same energy level (orbit/shell/band used interchangeably) ⇒ Bohr Model



(a) *(b)*

(c) *(d)*

Figure 2-1 Atoms of representative elements of most interest in electronics. (*a*) Hydrogen atom. (*b*) Aluminum atom. (*c*) Silicon atom. (*d*) Germanium atom. (Source: R. R. Wright and H. R. Skutt, *Electronics: Circuits and Devices*, 1965; reprinted by permission of John Wiley and Sons, Inc.)

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2.1 cont.

Bohr Model

1) Electrons exist in discrete states w/ discrete energies @ discrete radii for an atom

2) To move from a lower to higher energy level/state/orbit/shell, an electron absorbs a discrete amount of energy (quanta)

3) To move from a higher to lower energy level, an electron radiates/emits a discrete amount of energy (quanta), e.g., a photon.

4) When electron stays in a energy level, it does not absorb or emit energy.

2.2 Dielectrics, Polarization, and Permittivity

→ Dielectrics/insulators are materials w/ bound charges, i.e., not free to travel

→ when unperturbed, the centroids of electrons + protons tend to be aligned, or, if the molecules/ions are polarized, they are randomly oriented.

→ However, when an external electric field is applied ($\bar{E}_a$), Coulomb force $\bar{F} = q\bar{E}_a$, causes the formation of electric dipoles, $d\bar{p}_i = Q\bar{\ell}_i$, or the alignment of existing electric dipoles, i.e., polarized molecules (H_2O) or ionic molecules ($NaCl$) as shown in Figures 2-2 to 2-4.

→ The net effect of this polarization of the dielectric material is accounted for by defining an electric polarization vector $\bar{P}$

$$\text{Total dipole moment} \equiv \bar{p}_t = \sum_{i=1}^{N_e \Delta V} d\bar{p}_i \quad (C\cdot m)$$

ΔV - volume

N_e - electric dipoles per unit volume

$$\bar{P} = \lim_{\Delta V \to 0}\left(\frac{\bar{p}_t}{\Delta V}\right) = \lim_{\Delta V \to 0}\left[\frac{1}{\Delta V}\sum_{i=1}^{N_e \Delta V} d\bar{p}_i\right] \quad \left(C/m^2\right)$$

If we define an average dipole moment $d\bar{p}_i = d\bar{p}_{ave} = Q\bar{\ell}_{ave}$ per molecule/atom/ion $\bar{P} = N_e\, d\bar{p}_{ave} = N_e Q \bar{\ell}_{ave}$

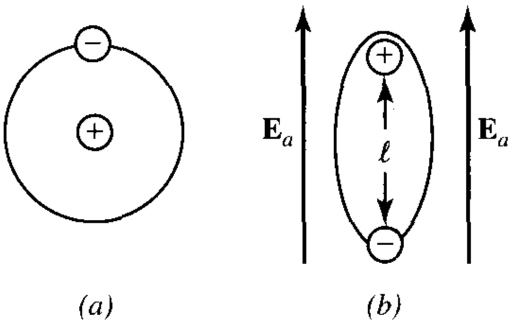


Figure 2-2 A typical atom. (*a*) Absence of applied field. (*b*) Under applied field.

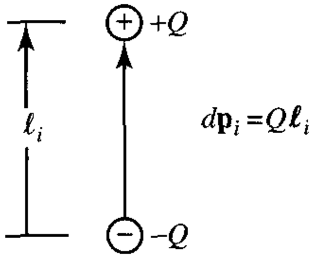


Figure 2-3 Formation of a dipole between two opposite charges of equal magnitude Q.

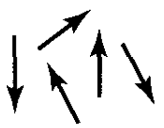
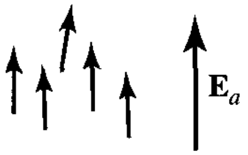
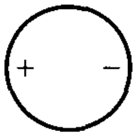
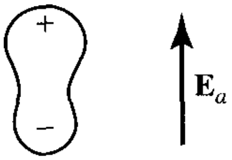
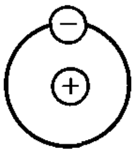
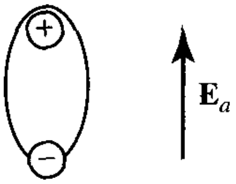
Mechanism	No applied field	Applied field
Dipole *or* orientational polarization		
Ionic *or* molecular polarization		
Electronic polarization		

Figure 2-4 Mechanisms producing electric polarization in dielectrics.

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2.2 cont.

→ Most materials, whether non-polar w/ $d\bar{p}_i = 0$, or polar w/ $d\bar{p}_i \neq 0$ but randomly oriented, have $\bar{P} = 0$ when there is no $\bar{E}_a$.

→ The few materials w/ $\bar{P} \neq 0$ when $\bar{E}_a = 0$ are called electrets (credit Oliver Heaviside), e.g., quartz, plastics, ...

→ Also, there are some materials (e.g. Barium titanate $BaTiO_3$) that are called ferroelectrics which have an electric hysteresis loop of P vs E similar to B vs H magnetic hysteresis loops. They also have ferroelectric Curie temps & residual/remnant polarizations

→ See Figures 2-5 & 2-6

→ As shown due to polarization, we 'see' bound surface charge densities $q_{sp} = \hat{a}_n \cdot \bar{P}$

and there can be a bound volume charge density $q_{vp} = -\bar{\nabla} \cdot \bar{P}$ (Usually zero as net # of negative & positive charges in the interior of a dielectric are equal, even when polarized)

How do $\bar{D}$, $\bar{E}$, + $\bar{P}$ inter-relate?

For free space, $\bar{D}_0 = \epsilon_0 \bar{E}_a$ See Fig 2.6a

For polarized dielectric, $\underline{\bar{D} = \epsilon_0 \bar{E}_a + \bar{P}}$

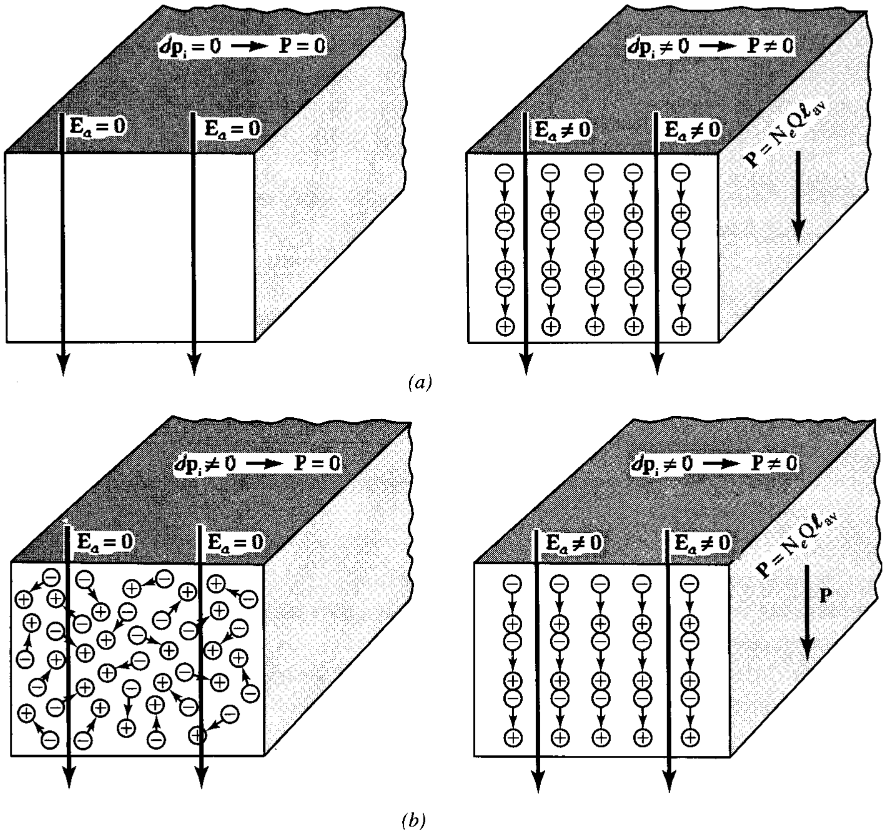

Figure 2-5 Macroscopic scale models of materials. (*a*) Nonpolar. (*b*) Polar.

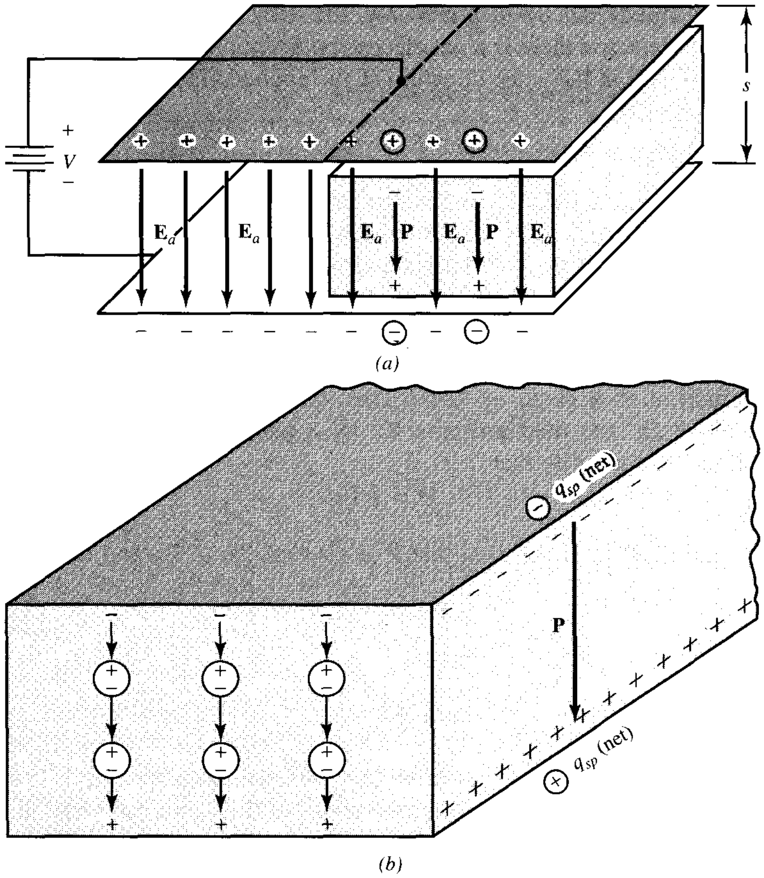

Figure 2-6 Dielectric slab subjected to an applied electric field $\mathbf{E}_a$. (*a*) Total charge. (*b*) Net charge.

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2.2 cont.

For most dielectrics, we can relate $\bar{P}$ to the applied $\bar{E}_a$ as $\underline{\bar{P} = \epsilon_0 \chi_e \bar{E}_a}$ where

$\chi_e \equiv$ electric susceptibility $= \dfrac{|\bar{P}|}{\epsilon_0 |\bar{E}_a|}$ (unitless)

Then, $\bar{D} = \epsilon_0 \bar{E}_a + \epsilon_0 \chi_e \bar{E}_a = \epsilon_0 (1+\chi_e) \bar{E}_a = \underset{\substack{\uparrow \\ \text{static permittivity (F/m)}}}{\epsilon_s} \bar{E}_a$

and we define a static relative permittivity $\equiv \epsilon_{sr} = \dfrac{\epsilon_s}{\epsilon_0} = 1 + \chi_e$

AKA: relative permittivity, $\epsilon_{sr} \geq 1$
dielectric constant

Note: Index of refraction $= n = \sqrt{\epsilon_r}$

→ Reality: relative permittivity or permittivity <u>does</u> vary w/ frequency as we'll discuss in section 2.9.1

→ The value of ϵ_{sr} is a measure of a materials ability/tendency to polarize and hence store electric energy.

→ <u>Table 2-1</u> shows some typical ϵ_{sr} for a variety of materials. Note, the overall static permittivity $\underset{\uparrow \text{F/m}}{\epsilon_s} = \underset{\uparrow \text{F/m}}{\epsilon_0}(1 + \underset{\nwarrow \text{unitless}}{\chi_e}) = \epsilon_0 \underset{\text{unitless} \nearrow}{\epsilon_{sr}}$

TABLE 2-1 Approximate static dielectric constants (relative permittivities) of dielectric materials

Material	Static dielectric constant (ε_r)
Air	1.0006
Styrofoam	1.03
Paraffin	2.1
Teflon	2.1
Plywood	2.1
RT/duroid 5880	2.20
Polyethylene	2.26
RT/duroid 5870	2.35
Glass-reinforced teflon (microfiber)	2.32–2.40
Teflon quartz (woven)	2.47
Glass-reinforced teflon (woven)	2.4–2.62
Cross-linked polystyrene (unreinforced)	2.56
Polyphenelene oxide (PPO)	2.55
Glass-reinforced polystyrene	2.62
Amber	3
Soil (dry)	3
Rubber	3
Plexiglas	3.4
Lucite	3.6
Fused silica	3.78
Nylon (solid)	3.8
Quartz	3.8
Sulfur	4
Bakelite	4.8
Formica	5
Lead glass	6
Mica	6
Beryllium oxide (BeO)	6.8–7.0
Marble	8
Sapphire	$\varepsilon_x = \varepsilon_y = 9.4$ $\varepsilon_z = 11.6$
Flint glass	10
Ferrite (Fe_2O_3)	12–16
Silicon (Si)	12
Gallium arsenide (GaAs)	13
Ammonia (liquid)	22
Glycerin	50
Water	81
Rutile (TiO_2)	$\varepsilon_x = \varepsilon_y = 89$ $\varepsilon_z = 173$

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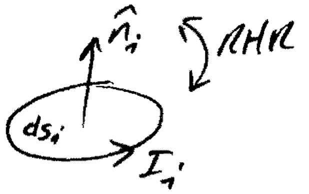
2.3 Magnetics, Magnetization, and Permeability

→ Materials that show magnetic polarization when subjected to an applied magnetic field $\bar{B}_a$ are called magnetic materials.

→ Magnetization occurs when magnetic dipoles in materials align w/ $\bar{B}_a$



magnetic dipole moment $(A\text{-}m^2)$ $\equiv d\bar{m}_i = I_i d\bar{s}_i = \hat{n}_i I_i ds_i$

→ These are caused by electrons orbiting atoms/molecules.

→ To characterize magnetization effects, we define

total magnetic dipole moment $(A\cdot m^2)$ $\equiv \bar{m}_t = \sum_{i=1}^{N_m \Delta V} d\bar{m}_i = \sum_{i=1}^{N_m \Delta V} \hat{n}_i I_i ds_i$

N_m = # of orbiting electrons per unit volume

Magnetization vector (A/m) $\equiv \bar{M} = \lim_{\Delta V \to 0}\left(\frac{\bar{m}_t}{\Delta V}\right) = \lim_{\Delta V \to 0}\left(\frac{\sum d\bar{m}_i}{\Delta V}\right)$

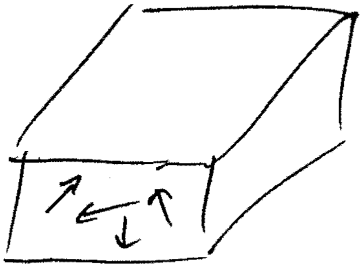
If we define an average magnetic dipole moment

$$d\bar{m}_i = d\bar{m}_{ave} = \hat{n}(I\,ds)_{ave},$$

$$\bar{M} = N_m d\bar{m}_{ave} = \hat{n} N_m (I\,ds)_{ave}$$

2.3 cont.

→ For a non-magnetized material w/ $\bar{B}_a = 0$, $d\bar{m}_i \neq 0$ but $\bar{M} = 0$ since they are randomly oriented



→ when an external magnetic field is applied, the magnetic dipoles experience a torque $\bar{T} = \bar{m} \times \bar{B}_a$ that causes them to tend to align w/ $\bar{B}_a$ so that $\bar{T} = 0$ (See Fig 2-8)

→ An externally magnetized material (See Fig 2-9) will have $\bar{M}$ + $\bar{B}_a$ aligned so that

$$\underline{\bar{B} = \mu_0 \bar{H}_a + \mu_0 \bar{M}}$$

For normal materials, $\bar{M} = \chi_m \bar{H}_a$ where $\chi_m \equiv$ magnetic susceptibility (unitless). Then,

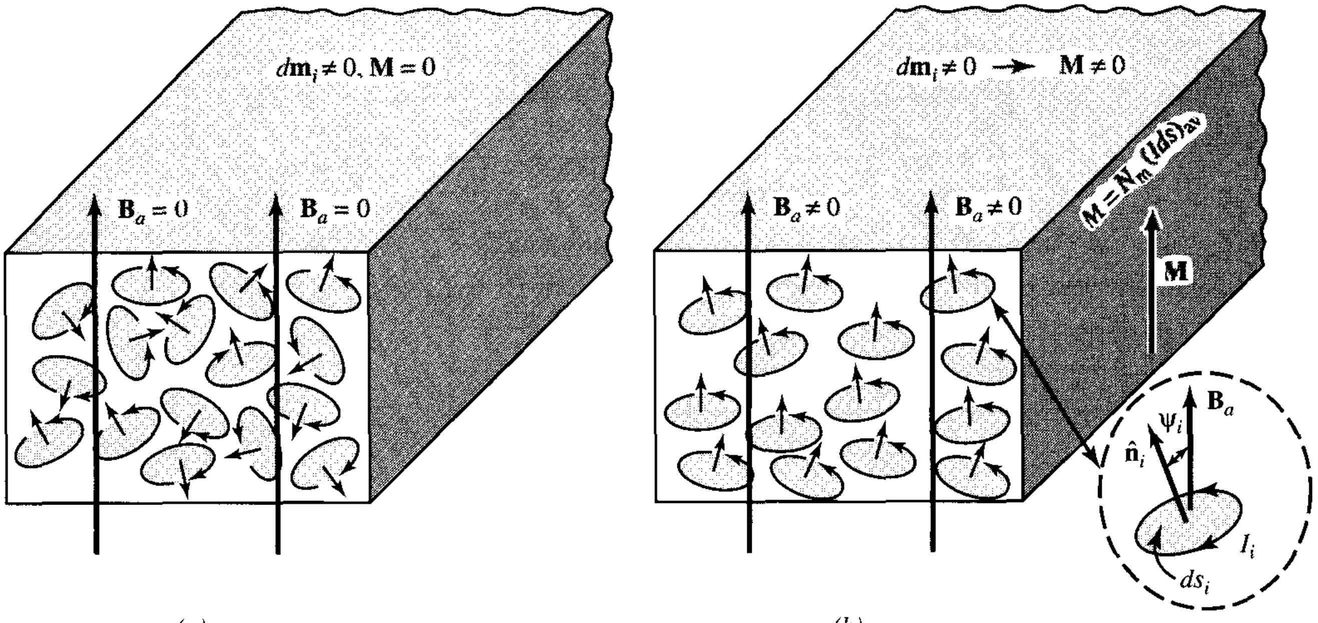
$$\bar{B} = \mu_0 (\bar{H}_a + \chi_m \bar{H}_a) = \mu_0 (1 + \chi_m) \bar{H}_a = \mu_s \bar{H}_a$$

$\mu_s \equiv$ static permeability ($^H/_m$)

$= \mu_0 (1 + \chi_m) = \mu_0 \mu_{sr}$

$1 + \chi_m = \mu_{sr} \equiv$ static relative permeability (unitless)

See Table 2-2 for some examples



(a) (b)

Figure 2-8 Orientation and alignment of magnetic dipoles. (*a*) Random in absence of an applied field. (*b*) Aligned under an applied field.

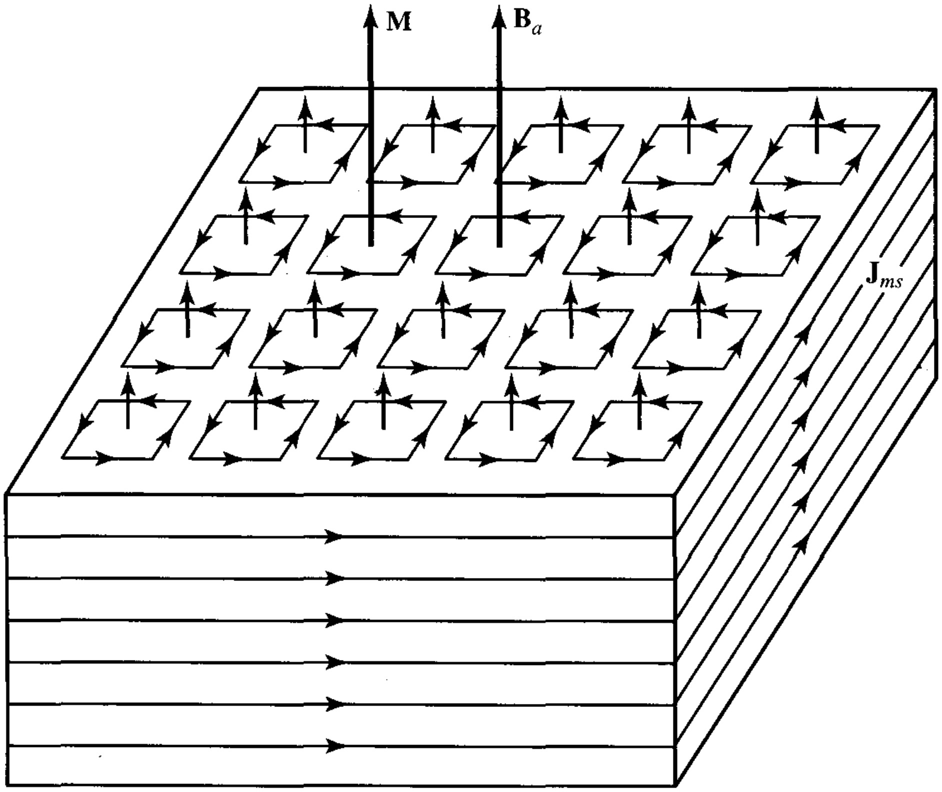


Figure 2-9 Magnetic slab subjected to an applied magnetic field and the formation of the magnetization current density $\mathbf{J}_{ms}$.

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2.3 cont.

The alignment of the magnetic dipoles in a magnetized material gives rise to the appearance of an equivalent magnetic current surface current density (see Fig 2-9) $\bar{J}_{ms}$

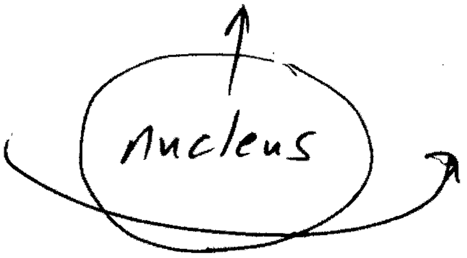
$$\bar{J}_{ms} = \bar{M} \times \hat{n} \leftarrow \text{surface normal} \quad (A/m)$$ bound surface magnetic current density

and $\bar{J}_m = \bar{\nabla} \times \bar{M}$ (A/m^2) ≡ bound volume magnetic current density

which is included in Ampere's Law

$$\bar{\nabla} \times \bar{H} = \bar{J}_i + \underset{\to \sigma\bar{E}}{\bar{J}_c} + \underset{\to \bar{\nabla}\times\bar{M}}{\bar{J}_m} + \overset{\to j\omega\epsilon\bar{E}}{\bar{J}_d}$$

In addition to magnetic dipoles from orbiting electrons



Smaller → effect by ~ 10^{-3} than others

nuclear spin magnetic moment

&

electron spin

'spin' quantum state of electrons

→ only from non-filled shells gives a net contribution

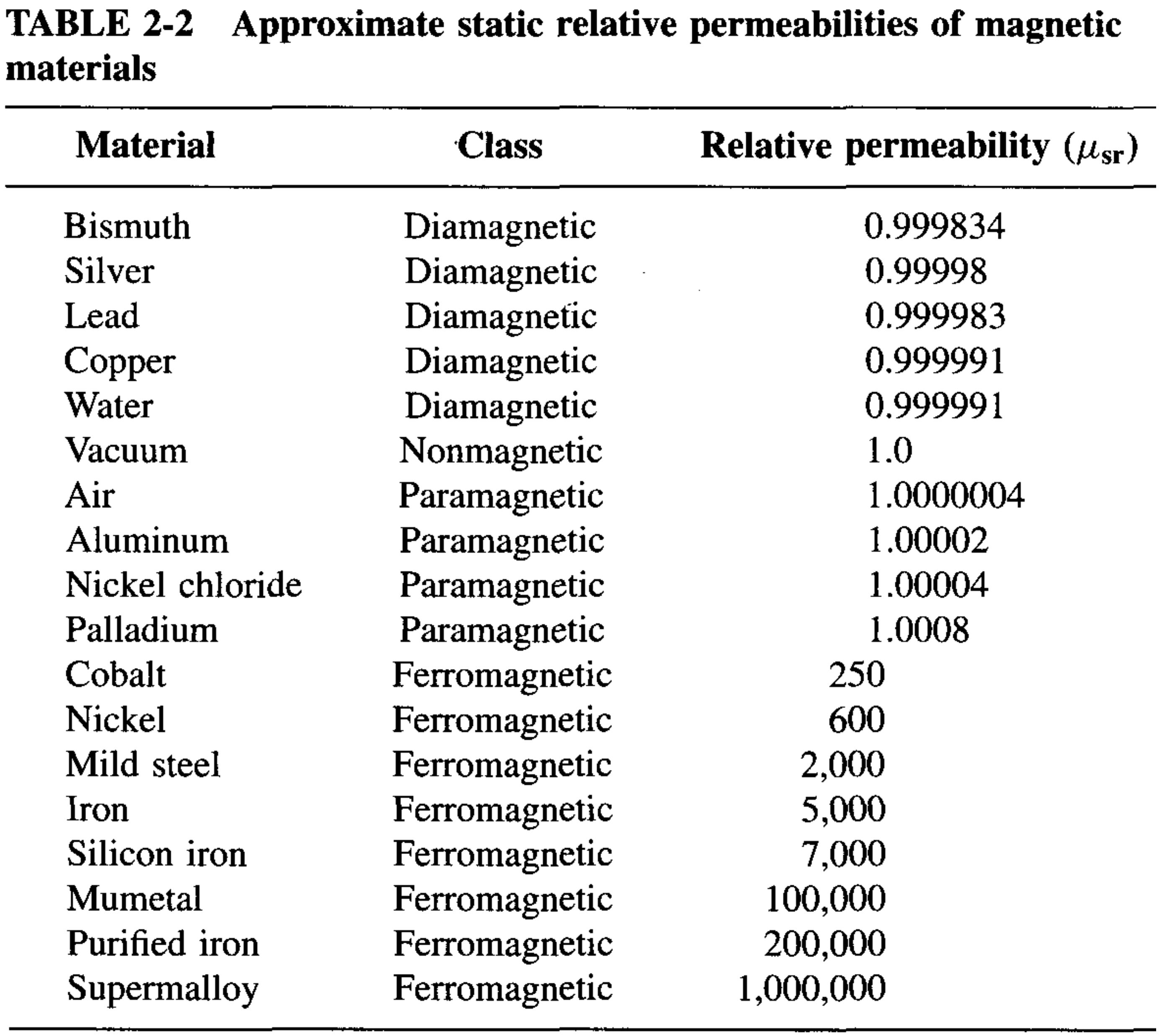


TABLE 2-2 Approximate static relative permeabilities of magnetic materials

Material	Class	Relative permeability (μ_{sr})
Bismuth	Diamagnetic	0.999834
Silver	Diamagnetic	0.99998
Lead	Diamagnetic	0.999983
Copper	Diamagnetic	0.999991
Water	Diamagnetic	0.999991
Vacuum	Nonmagnetic	1.0
Air	Paramagnetic	1.0000004
Aluminum	Paramagnetic	1.00002
Nickel chloride	Paramagnetic	1.00004
Palladium	Paramagnetic	1.0008
Cobalt	Ferromagnetic	250
Nickel	Ferromagnetic	600
Mild steel	Ferromagnetic	2,000
Iron	Ferromagnetic	5,000
Silicon iron	Ferromagnetic	7,000
Mumetal	Ferromagnetic	100,000
Purified iron	Ferromagnetic	200,000
Supermalloy	Ferromagnetic	1,000,000

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2.3 cont.

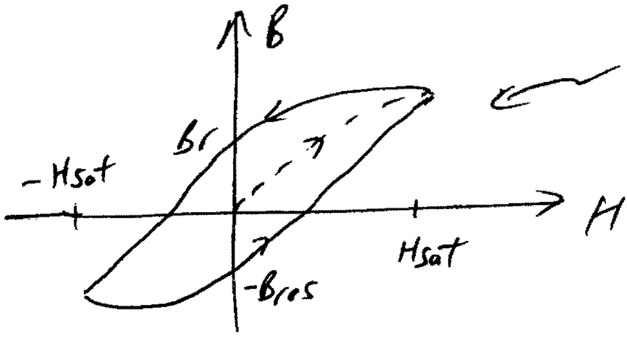
→ Like ϵ, the value of μ is a function of frequency which will be discussed in 2.9.2

→ Some materials have $\overline{M}$ that opposes $\overline{B}_a$. They are called Diamagnetic. Usually, it is a small effect w/ $\mu_r \lessapprox 1$ (see Table 2-2)

→ Some materials have $\overline{M}$ that slightly reinforces $\overline{B}_a$. They are called paramagnetic and antiferromagnetic (e.g., chromium). Here, $\mu_r \gtrapprox 1$.
$\chi_m = 0.00028$

→ Then, we have materials that are strongly magnetic – ferromagnetic (σ large) and ferrimagnetic (σ small usually)

→ These materials have hysteresis curves



μ nonlinear!

→ Magnetic materials tend to be lossy. Two main mechanisms are hysteresis loss (related to area w/in hysteresis curve) and eddy currents (related to induced currents + σ)

→ Mention Curie temp.