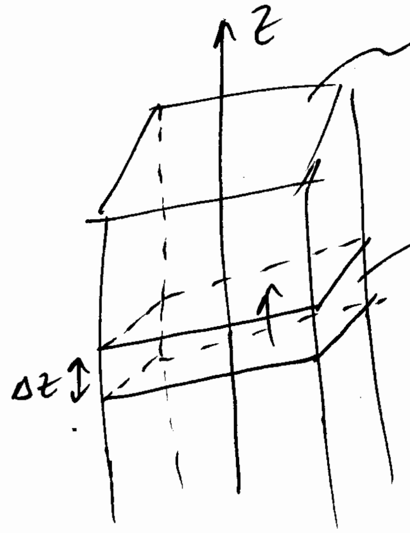
2.4 Current, Conductors, & Conductivity

→ conductors are materials where there are charges (electrons) free to move when an electric field is applied.

2.4.1 Current



volume $\Delta V = \Delta S \Delta z$ contains total electric charge ΔQ_e moving in +z-direction @ velocity v_z

vol. charge density

$$\Delta I = \lim_{\Delta t \to 0} \frac{\Delta Q_e}{\Delta t} = \lim_{\Delta t \to 0} q_v \Delta S \frac{\Delta z}{\Delta t} = q_v v_z \Delta S \quad (A)$$

Define current density magnitude as

$$J_z = \lim_{\Delta S \to 0} \frac{\Delta I}{\Delta S} = q_v v_z \quad (A/m^2)$$

In vector form

$$\bar{J} = q_v \bar{v} \equiv \text{Convection current density}$$

This can be broken into positive & negative charges/parts

$$\bar{J} = q_v^+ \bar{v}^+ + q_v^- \bar{v}^-$$

2.4.2 Conductors

→ Materials where valence electrons are <u>not</u> tightly bound ⇒ 'free' electrons

→ At rest, the free electrons move in random directions ⇒ $I_{net} = 0$ through any surface

→ Excess free charges introduced into a conductor migrate/flow to surface due to repulsive Coulomb forces. The volume charge density inside the conductor is governed by

$$q_v(t) = q_{v0}\, e^{-t/t_r}$$

where $t_r = \epsilon/\sigma$ (s) $\equiv$ relaxation time constant

$\sigma \equiv$ conductivity (S/m) ← Next section

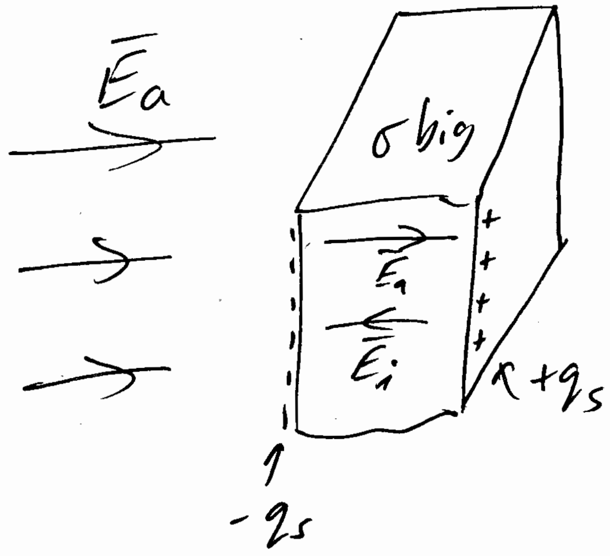
<u>Other properties</u>

→ For perfect electrical conductors (PEC) and very good conductors (σ very large)

$$\Delta V_{inside} = 0, \quad \bar{E}_{t,inside} = \bar{D}_{t,inside} = 0$$

2.4.2 cont.

Why?



① Applied electric field $\bar{E}_a$ causes charges to move $\bar{F} = q\bar{E}_a$ leading to positive & negative surface charge densities

② The newly formed surface charge densities cause/create an induced electric field $\bar{E}_i$. $\bar{E}_i$, in opposition to $\bar{E}_a$ will keep getting bigger + bigger until $\bar{E}_t = \bar{E}_i + \bar{E}_a = 0$ and charges stop moving!

③ By $\bar{D} = \epsilon \bar{E} \Rightarrow \underline{\underline{\bar{D}_t = 0}}$

By $\Delta V = -\int_A^B \bar{E} \cdot d\bar{\ell} \Rightarrow \Delta V = 0$ inside conductor

$\Rightarrow$ all points in conductor @ same potential!

2.4.3 Conductivity

To characterize the flow of charges in a conductor, we'll define a quantity / parameter called conductivity σ

→ Apply an external electric field $\bar{E}_a$ to a conductor. The free electrons which were moving in random directions w/ random velocities now, <u>on average</u>, move at a <u>drift velocity $\bar{v}_e$</u> in a direction <u>opposite</u> to $\bar{E}_a$ ($\bar{F} = q\bar{E}_a$ w/ q negative)

⇒ Conduction current ⇐

Define $\bar{v}_e = -\mu_e \bar{E}$ where $\mu_e \equiv$ electron mobility $\left(\frac{m^2}{V \cdot s}\right)$

use $\bar{v}_e$ in $\bar{J} = q_v \bar{v}$ to get

$$\bar{J} = -q_{ve}\mu_e \bar{E} = \sigma_s \bar{E}$$

where the static conductivity $\equiv \sigma_s = -q_{ve}\mu_e$ (S/m)

Older texts resistivity $= \rho = \frac{1}{\sigma}$ $(\Omega \cdot m)$

⇒ <u>See Table 2-3</u> ⇐

TABLE 2-3 Typical conductivities of insulators, semiconductors, and conductors

Material	Class	Conductivity σ (S/m)
Fused quartz	Insulator	$\sim 10^{-17}$
Ceresin wax	Insulator	$\sim 10^{-17}$
Sulfur	Insulator	$\sim 10^{-15}$
Mica	Insulator	$\sim 10^{-15}$
Paraffin	Insulator	$\sim 10^{-15}$
Hard rubber	Insulator	$\sim 10^{-15}$
Porcelain	Insulator	$\sim 10^{-14}$
Glass	Insulator	$\sim 10^{-12}$
Bakelite	Insulator	$\sim 10^{-9}$
Distilled water	Insulator	$\sim 10^{-4}$
Gallium arsenide (GaAs)*	Semiconductor	$\sim 2.38 \times 10^{-7}$
Fused silica*	Semiconductor	$\sim 2.1 \times 10^{-4}$
Cross-linked polystyrene (unreinforced)*	Semiconductor	$\sim 3.7 \times 10^{-4}$
Beryllium Oxide (BeO)*	Semiconductor	$\sim 3.9 \times 10^{-4}$
Intrinsic silicon	Semiconductor	$\sim 4.39 \times 10^{-4}$
Sapphire*	Semiconductor	$\sim 5.5 \times 10^{-4}$
Glass-reinforced Teflon (microfiber)*	Semiconductor	$\sim 7.8 \times 10^{-4}$
Teflon quartz (woven)*	Semiconductor	$\sim 8.2 \times 10^{-4}$
Dry soil	Semiconductor	$\sim 10^{-4} - 10^{-3}$
Ferrite(Fe_2O_3)*	Semiconductor	$\sim 1.3 \times 10^{-3}$
Glass-reinforced Polystyrene*	Semiconductor	$\sim 1.45 \times 10^{-3}$
Polyphenelene oxide (PPO)*	Semiconductor	$\sim 2.27 \times 10^{-3}$
Glass-reinforced Teflon (woven)*	Semiconductor	$\sim 2.43 \times 10^{-3}$
Plexiglas*	Semiconductor	$\sim 5.1 \times 10^{-3}$
Wet soil	Semiconductor	$\sim 10^{-3} - 10^{-2}$
Fresh water	Semiconductor	$\sim 10^{-2}$
Human and animal tissue	Semiconductor	$\sim 0.2 - 0.7$
Intrinsic germanium	Semiconductor	~ 2.227
Seawater	Semiconductor	~ 4
Tellurium	Conductor	$\sim 5 \times 10^{-2}$
Carbon	Conductor	$\sim 3 \times 10^{-4}$
Graphite	Conductor	$\sim 3 \times 10^{4}$
Cast iron	Conductor	$\sim 10^{6}$
Mercury	Conductor	10^{6}
Nichrome	Conductor	10^{6}
Silicon steel	Conductor	2×10^{6}
German silver	Conductor	2×10^{6}
Lead	Conductor	5×10^{6}
Tin	Conductor	9×10^{6}
Iron	Conductor	1.03×10^{7}
Nickel	Conductor	1.45×10^{7}
Zinc	Conductor	1.7×10^{7}
Tungsten	Conductor	1.83×10^{7}
Brass	Conductor	2.56×10^{7}
Aluminum	Conductor	3.96×10^{7}
Gold	Conductor	4.1×10^{7}
Copper	Conductor	5.76×10^{7}
Silver	Conductor	6.1×10^{7}

*For most semiconductors the conductivities are representative for a frequency of about 10 GHz.

Advanced Engineering Electromagnetics (Second Edition), Balanis, Wiley, 2012, ISBN-10: 0470589485, ISBN-13: 978-0470589489.

Skip sections <u>2.5 Semiconductors</u>,

<u>2.6 Superconductors</u>, & <u>2.7 Metamaterials</u>

<u>2.8 Linear, Homogenous, Isotropic, and Nondispersive Media</u>

<u>Linear</u> - $\epsilon, \mu, \& \sigma$ are <u>NOT</u> functions of applied electric or magnetic field strengths

examples of nonlinearity

→ dielectric breakdown ⇒ $|E|$ where electrons ripped loose from material

→ magnetic saturation ⇒ $|H|$ where all magnetic dipoles are aligned

<u>Homogenous</u> - $\epsilon, \mu, \& \sigma$ are <u>NOT</u> functions of position/location w/in material; otherwise material is inhomogeneous or nonhomogenous

→ this can be a concern for circuit board manufacturers as well as for TL insulation.

<u>Nondispersive</u> - $\epsilon, \mu, \& \sigma$ are <u>NOT</u> functions of frequency (or time)

→ Reality is that all materials are dispersive. However, over limited BW they may be effectively nondispersive.

2.8 cont.

<u>Isotropic</u> - ϵ, μ, & σ are <u>NOT</u> functions of direction w/in the material

→ Many crystalline materials are anisotropic or nonisotropic. In this case, ϵ, μ, & σ are usually written as 3x3 tensors instead of a single value. E.g.,

$$\bar{D} = \bar{\bar{\epsilon}} \cdot \bar{E}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$\Downarrow$$

$$D_x = \epsilon_{xx} E_x + \epsilon_{xy} E_y + \epsilon_{xz} E_z$$

$$D_y = \epsilon_{yx} E_x + \epsilon_{yy} E_y + \epsilon_{yz} E_z$$

$$D_z = \epsilon_{zx} E_x + \epsilon_{zy} E_y + \epsilon_{zz} E_z$$

→ Each of the terms in the $\bar{\bar{\epsilon}}$ tensor can be complex

→ ϵ_{xx}, ϵ_{yy}, & ϵ_{zz} are called <u>principal</u> permittivities