

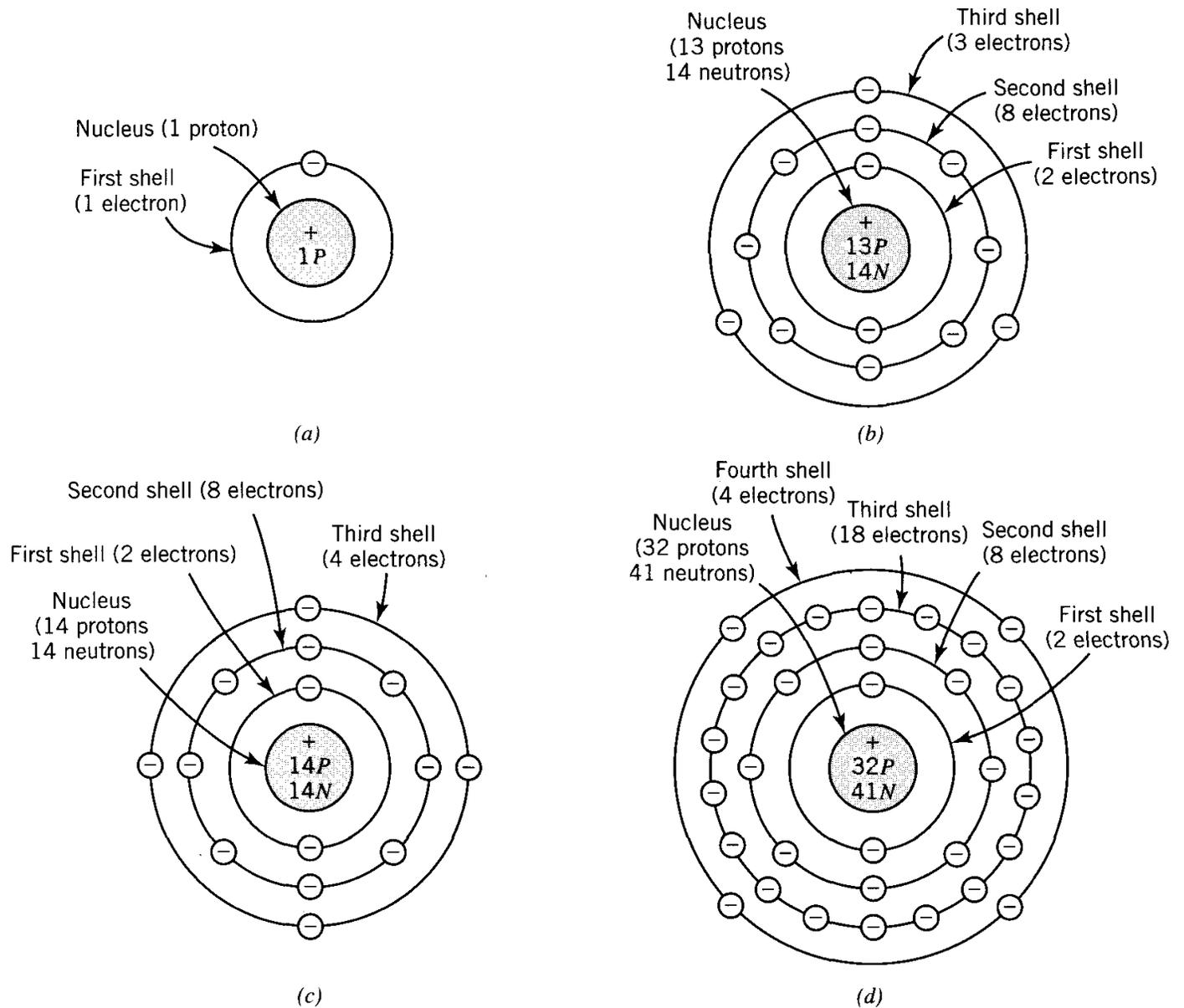
# Chapter 2 Electrical Properties of Matter

## 2.1 Introduction

→ Much interest of late in metamaterials. These artificial materials can have negative  $\epsilon_r$  &/or  $\mu_r$  over a frequency range. Can also create artificial magnetic conductors. This is an active research area in EM.

## Basics of Matter

- atom consists of a nucleus (protons + neutrons) surrounded by electrons.
- Elements consist of a single type of atom (ignore isotopes). Can be organized in molecules/crystals...
- Compounds are composed of 2 or more elements and are typically organized as molecules
- Typically, atoms/molecules are charge neutral, i.e., same # of protons + electrons
- The electrons for an atom exist in various 'shells' w/ the outermost called the valence shell/band w/ valence electrons. In a given shell, the electrons have the same energy level (orbit / shell / band used interchangeably)  $\Rightarrow$  Bohr Model



**Figure 2-1** Atoms of representative elements of most interest in electronics. (a) Hydrogen atom. (b) Aluminum atom. (c) Silicon atom. (d) Germanium atom. (Source: R. R. Wright and H. R. Skutt, *Electronics: Circuits and Devices*, 1965; reprinted by permission of John Wiley and Sons, Inc.)

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## 2.1 cont.

### Bohr Model

- 1) Electrons exist in discrete states w/ discrete energies @ discrete radii for an atom
- 2) To move from a lower to higher energy level/state/orbit/shell, an electron absorbs a discrete amount of energy (quanta)
- 3) To move from a higher to lower energy level, an electron radiates/emits a discrete amount of energy (quanta), e.g., a photon.
- 4) When electron stays in a energy level, it does not absorb or emit energy.

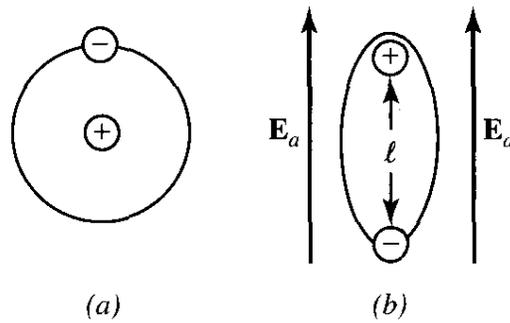
## 2.2 Dielectrics, Polarization, and Permittivity

- Dielectrics/insulators are materials w/ bound charges, i.e., not free to travel
- When unperturbed, the centroids of electrons & protons tend to be aligned, or, if the molecules/ions are polarized, they are randomly oriented.
- However, when an external electric field is applied ( $\vec{E}_a$ ), Coulomb force  $\vec{F} = q\vec{E}_a$ , causes the formation of electric dipoles,  $d\vec{p}_i = Q\vec{l}_i$ , or the alignment of existing electric dipoles, i.e., polarized molecules ( $H_2O$ ) or ionic molecules ( $NaCl$ ) as shown in Figures 2-2 to 2-4.
- The net effect of this polarization of the dielectric material is accounted for by defining an electric polarization vector  $\vec{P}$

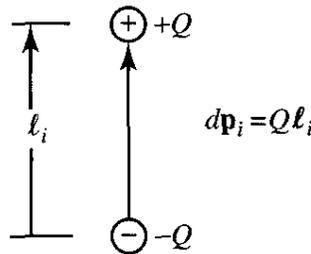
$$\text{Total dipole moment} \equiv \vec{p}_t = \sum_{i=1}^{N_0 \Delta V} d\vec{p}_i \text{ (C}\cdot\text{m)} \quad \begin{array}{l} \Delta V - \text{volume} \\ N_0 - \text{electric dipoles per} \\ \text{unit volume} \end{array}$$

$$\vec{P} = \lim_{\Delta V \rightarrow 0} \left( \frac{\vec{p}_t}{\Delta V} \right) = \lim_{\Delta V \rightarrow 0} \left[ \frac{1}{\Delta V} \sum_{i=1}^{N_0 \Delta V} d\vec{p}_i \right] \text{ (C/m}^2\text{)}$$

If we define an average dipole moment  $d\vec{p}_i = d\vec{p}_{ave} = Q\vec{l}_{ave}$  per molecule/atom/ion  $\vec{P} = N_0 d\vec{p}_{ave} = N_0 Q\vec{l}_{ave}$



**Figure 2-2** A typical atom. (a) Absence of applied field. (b) Under applied field.



**Figure 2-3** Formation of a dipole between two opposite charges of equal magnitude  $Q$ .

Mechanism	No applied field	Applied field
Dipole or orientational polarization		
Ionic or molecular polarization		
Electronic polarization		

**Figure 2-4** Mechanisms producing electric polarization in dielectrics.

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2.2 cont.

- Most materials, whether non-polar w/  $d\bar{p}_i = 0$ , or polar w/  $d\bar{p}_i \neq 0$  but randomly oriented, have  $\bar{P} = 0$  when there is no  $\bar{E}_a$ .
- The few materials w/  $\bar{P} \neq 0$  when  $\bar{E}_a = 0$  are called electrets (credit Oliver Heaviside), e.g., quartz, plastics, ...
- Also, there are some materials (e.g., Barium titanate  $BaTiO_3$ ) that are called ferroelectrics which have an electric hysteresis loop of  $P$  vs  $E$  similar to  $B$  vs  $H$  magnetic hysteresis loops. They also have ferroelectric Curie temps & residual/remnant polarizations
- See Figures 2-5 & 2-6

- As shown due to polarization, we 'see' bound surface charge densities  $q_{sp} = \hat{a}_n \cdot \bar{P}$  and there can be a bound volume charge density  $q_{vp} = -\bar{\nabla} \cdot \bar{P}$  (Usually zero as net # of negative & positive charges in the interior of a dielectric are equal, even when polarized.)

How do  $\bar{D}$ ,  $\bar{E}$ , &  $\bar{P}$  inter-relate?

For free space,  $\bar{D}_0 = \epsilon_0 \bar{E}_a$  See Fig 2.6a

For polarized dielectric,  $\bar{D} = \epsilon_0 \bar{E}_a + \bar{P}$

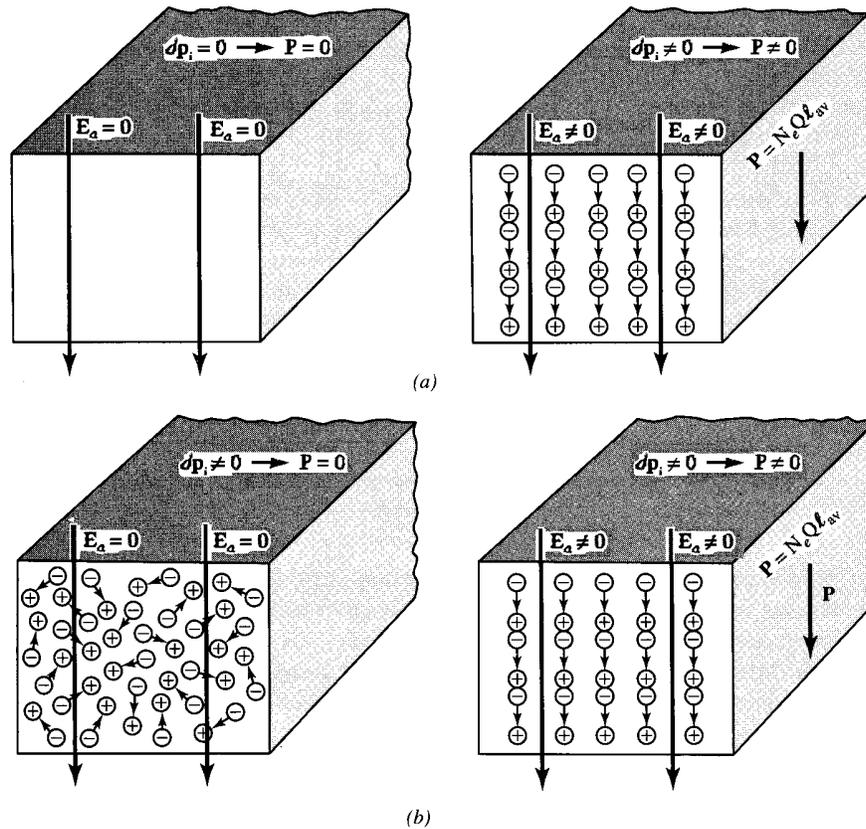


Figure 2-5 Macroscopic scale models of materials. (a) Nonpolar. (b) Polar.

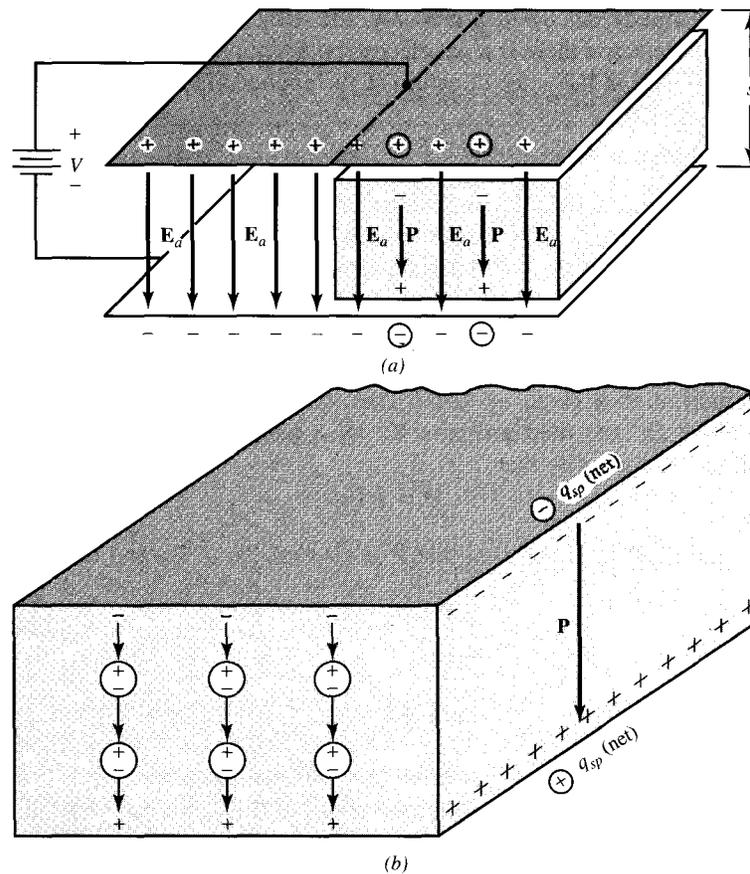


Figure 2-6 Dielectric slab subjected to an applied electric field  $E_a$ . (a) Total charge. (b) Net charge.

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2.2 cont.

For most dielectrics, we can relate  $\bar{P}$  to the applied  $\bar{E}_a$  as  $\bar{P} = \epsilon_0 \chi_e \bar{E}_a$  where

$$\chi_e \equiv \text{electric susceptibility} = \frac{|\bar{P}|}{\epsilon_0 |\bar{E}_a|} \quad (\text{unitless})$$

$$\text{Then, } \bar{D} = \epsilon_0 \bar{E}_a + \epsilon_0 \chi_e \bar{E}_a = \epsilon_0 (1 + \chi_e) \bar{E}_a = \epsilon_s \bar{E}_a$$

$\uparrow$   
 static permittivity (F/m)

and we define a static relative permittivity  $\equiv \epsilon_{sr} = \frac{\epsilon_s}{\epsilon_0} = 1 + \chi_e$

AKA: relative permittivity,  $\epsilon_{sr} \geq 1$   
dielectric constant

Note: Index of refraction =  $n = \sqrt{\epsilon_{sr}}$

→ Reality: relative permittivity or permittivity does vary w/ frequency as well discuss in section 2.9.1

→ The value of  $\epsilon_{sr}$  is a measure of a materials ability/tendency to polarize and hence store electric energy.

→ Table 2-1 shows some typical  $\epsilon_{sr}$  for a variety of materials. Note, the overall static permittivity  $\epsilon_s = \epsilon_0 (1 + \chi_e) = \epsilon_0 \epsilon_{sr}$

$\uparrow$  F/m     $\uparrow$  F/m     $\uparrow$  unitless

**TABLE 2-1 Approximate static dielectric constants (relative permittivities) of dielectric materials**

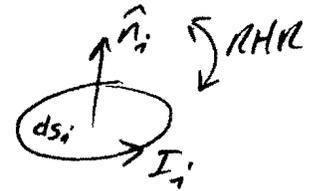
Material	Static dielectric constant ( $\epsilon_r$ )
Air	1.0006
Styrofoam	1.03
Paraffin	2.1
Teflon	2.1
Plywood	2.1
RT/duroid 5880	2.20
Polyethylene	2.26
RT/duroid 5870	2.35
Glass-reinforced teflon (microfiber)	2.32–2.40
Teflon quartz (woven)	2.47
Glass-reinforced teflon (woven)	2.4–2.62
Cross-linked polystyrene (unreinforced)	2.56
Polyphenylene oxide (PPO)	2.55
Glass-reinforced polystyrene	2.62
Amber	3
Soil (dry)	3
Rubber	3
Plexiglas	3.4
Lucite	3.6
Fused silica	3.78
Nylon (solid)	3.8
Quartz	3.8
Sulfur	4
Bakelite	4.8
Formica	5
Lead glass	6
Mica	6
Beryllium oxide (BeO)	6.8–7.0
Marble	8
Sapphire	$\epsilon_x = \epsilon_y = 9.4$ $\epsilon_z = 11.6$
Flint glass	10
Ferrite (Fe <sub>2</sub> O <sub>3</sub> )	12–16
Silicon (Si)	12
Gallium arsenide (GaAs)	13
Ammonia (liquid)	22
Glycerin	50
Water	81
Rutile (TiO <sub>2</sub> )	$\epsilon_x = \epsilon_y = 89$ $\epsilon_z = 173$

## 2.3 Magnetics, Magnetization, and Permeability

→ Materials that show magnetic polarization when subjected to an applied magnetic field  $\vec{B}_a$  are called magnetic materials.

→ Magnetization occurs when magnetic dipoles in materials align w/  $\vec{B}_a$

$$\text{magnetic dipole moment (A}\cdot\text{m}^2) \equiv d\vec{m}_i = I_i d\vec{s}_i = \hat{n}_i I_i ds_i$$



→ These are caused by electrons orbiting atoms/molecules.

→ To characterize magnetization effects, we define

$$\text{total magnetic dipole moment (A}\cdot\text{m}^2) \equiv \vec{M}_t = \sum_{i=1}^{N_m \Delta V} d\vec{m}_i = \sum_{i=1}^{N_m \Delta V} \hat{n}_i I_i ds_i$$

$N_m = \# \text{ of orbiting electrons per unit volume}$

$$\text{Magnetization vector (A/m)} \equiv \vec{M} = \lim_{\Delta V \rightarrow 0} \left( \frac{\vec{M}_t}{\Delta V} \right) = \lim_{\Delta V \rightarrow 0} \left( \frac{\sum d\vec{m}_i}{\Delta V} \right)$$

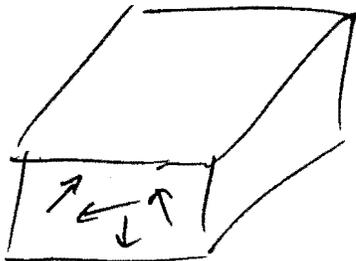
If we define an average magnetic dipole moment

$$d\vec{m}_i = d\vec{m}_{ave} = \hat{n} (I ds)_{ave},$$

$$\vec{M} = N_m d\vec{m}_{ave} = \hat{n} N_m (I ds)_{ave}$$

2.3 cont.

→ For a non-magnetized material w/  $\bar{B}_a = 0$ ,  
 $d\bar{m}_i \neq 0$  but  $\bar{M} = 0$  since they are randomly  
 oriented



→ When an external magnetic field is applied,  
 the magnetic dipoles experience a torque

$\bar{T} = \bar{m} \times \bar{B}_a$  that causes them to tend to  
 align w/  $\bar{B}_a$  so that  $\bar{T} = 0$  (see Fig 2-8)

→ An externally magnetized material (see Fig 2-9)  
 will have  $\bar{M} + \bar{B}_a$  aligned so that

$$\underline{\bar{B}} = \mu_0 \bar{H}_a + \mu_0 \bar{M}$$

For normal materials,  $\bar{M} = \chi_m \bar{H}_a$  where  
 $\chi_m \equiv$  magnetic susceptibility (unitless). Then,

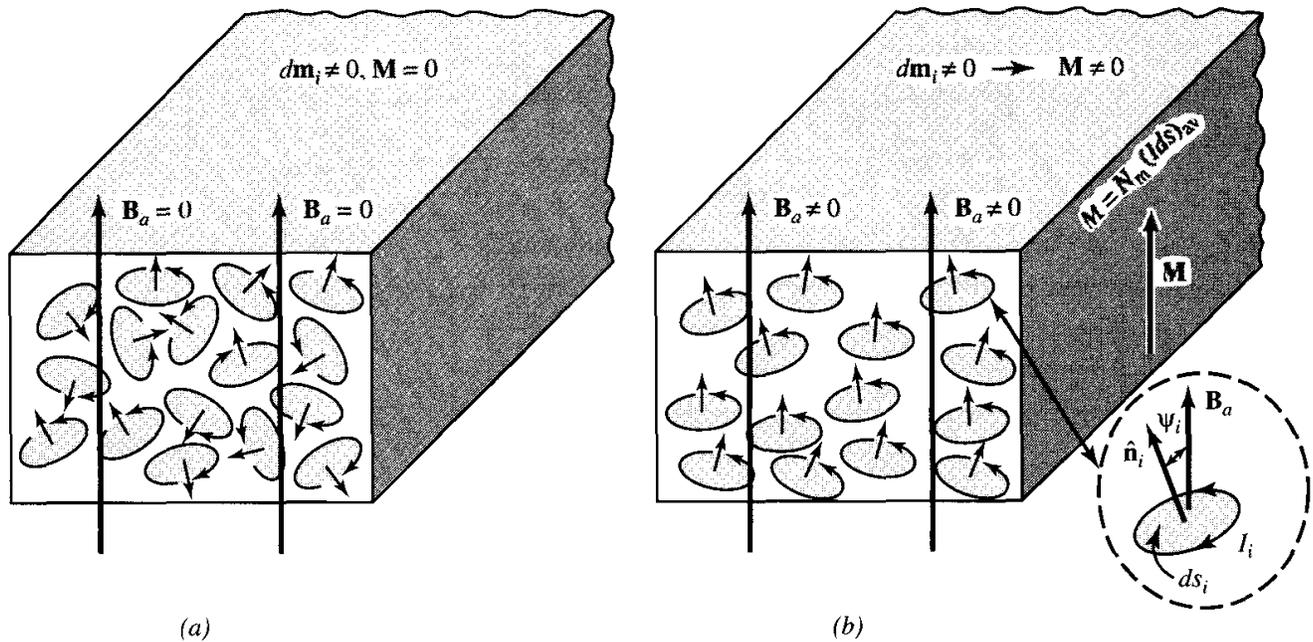
$$\bar{B} = \mu_0 (\bar{H}_a + \chi_m \bar{H}_a) = \mu_0 (1 + \chi_m) \bar{H}_a = \mu_s \bar{H}_a$$

$\mu_s \equiv$  static permeability ( $H/m$ )

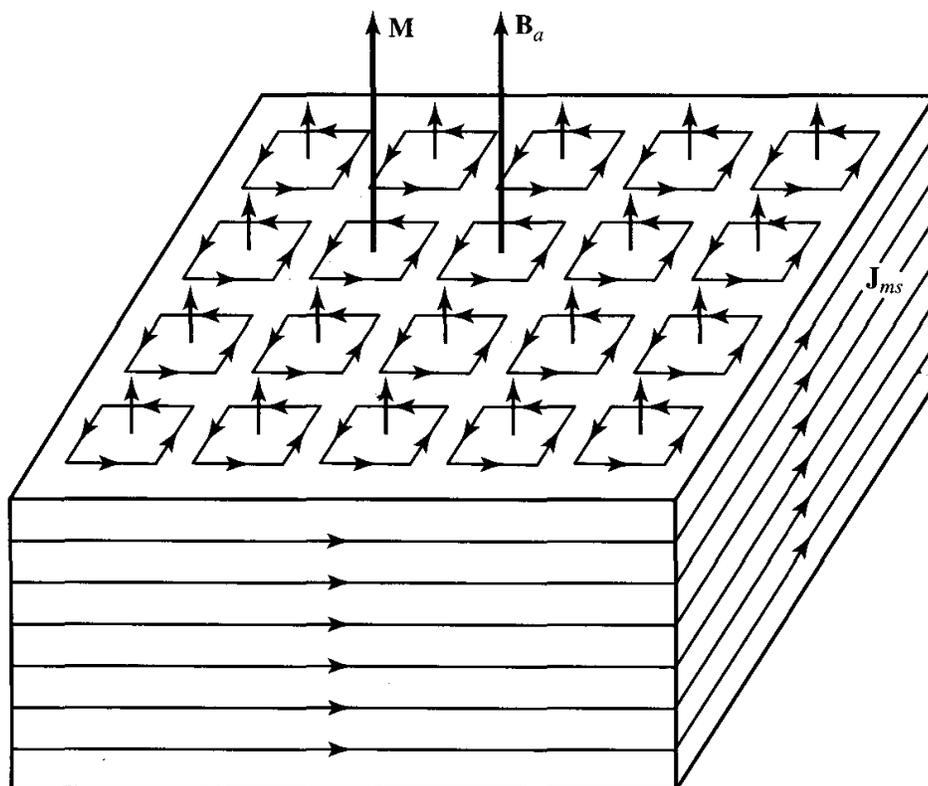
$$= \mu_0 (1 + \chi_m) = \mu_0 \mu_{sr}$$

$1 + \chi_m = \mu_{sr} \equiv$  static relative permeability (unitless)

See Table 2-2 for some examples



**Figure 2-8** Orientation and alignment of magnetic dipoles. (a) Random in absence of an applied field. (b) Aligned under an applied field.



**Figure 2-9** Magnetic slab subjected to an applied magnetic field and the formation of the magnetization current density  $J_{ms}$ .

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2.3 cont.

The alignment of the magnetic dipoles in a magnetized material gives rise to the appearance of an equivalent magnetic current surface current density (see Fig 2-9)  $\vec{J}_{ms}$

$$\vec{J}_{ms} = \vec{M} \times \hat{n} \leftarrow \begin{matrix} \text{surface} \\ \text{normal} \end{matrix} \quad (A/m) \quad \begin{matrix} \text{bound surface} \\ \text{magnetic current} \\ \text{density} \end{matrix}$$

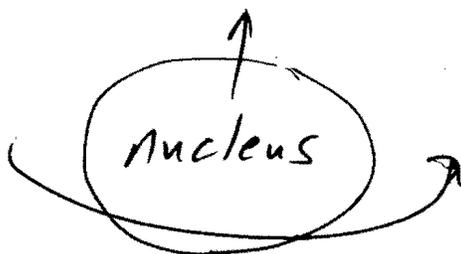
and  $\vec{J}_m = \nabla \times \vec{M} \quad (A/m^2) \equiv \text{bound volume magnetic current density}$

which is included in Ampere's Law

$$\nabla \times \vec{H} = \vec{J}_i + \underbrace{\vec{J}_c}_{\leftarrow \sigma \vec{E}} + \underbrace{\vec{J}_m}_{\leftarrow \nabla \times \vec{M}} + \vec{J}_d \rightarrow j_w \epsilon \vec{E}$$

In addition to magnetic dipoles from orbiting electrons

Smaller  $\rightarrow$   
effect  
by  $\sim 10^{-3}$  than  
others



nuclear spin  
magnetic moment

electron  
spin



'spin' quantum state  
of electrons  
 $\rightarrow$  only from non-filled shells  
gives a net contribution

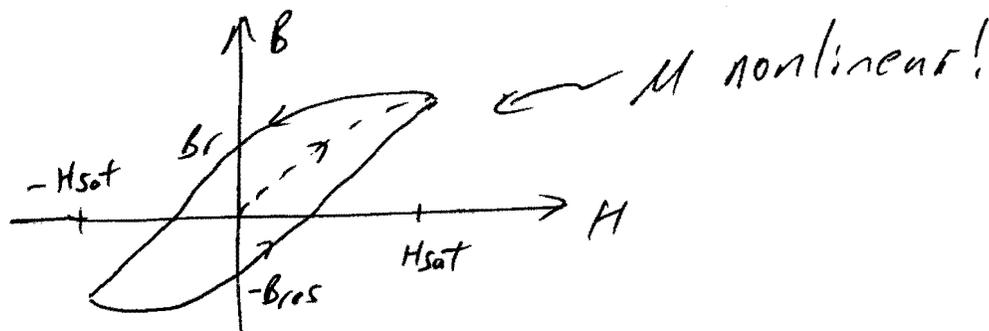
**TABLE 2-2 Approximate static relative permeabilities of magnetic materials**

<b>Material</b>	<b>Class</b>	<b>Relative permeability (<math>\mu_{sr}</math>)</b>
Bismuth	Diamagnetic	0.999834
Silver	Diamagnetic	0.99998
Lead	Diamagnetic	0.999983
Copper	Diamagnetic	0.999991
Water	Diamagnetic	0.999991
Vacuum	Nonmagnetic	1.0
Air	Paramagnetic	1.0000004
Aluminum	Paramagnetic	1.00002
Nickel chloride	Paramagnetic	1.00004
Palladium	Paramagnetic	1.0008
Cobalt	Ferromagnetic	250
Nickel	Ferromagnetic	600
Mild steel	Ferromagnetic	2,000
Iron	Ferromagnetic	5,000
Silicon iron	Ferromagnetic	7,000
Mumetal	Ferromagnetic	100,000
Purified iron	Ferromagnetic	200,000
Supermalloy	Ferromagnetic	1,000,000

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## 2.3 cont.

- Like  $\epsilon$ , the value of  $\mu$  is a function of frequency which will be discussed in 2.9.2
- Some materials have  $\bar{M}$  that opposes  $\bar{B}_0$ . They are called Diamagnetic. Usually, it is a small effect w/  $\mu_r \lesssim 1$  (see Table 2-2)
- Some materials have  $\bar{M}$  that slightly reinforces  $\bar{B}_0$ . They are called paramagnetic and antiferromagnetic (e.g., chromium). Here,  $\mu_r \gtrsim 1$ ,  $X_m = 0.00028$
- Then, we have materials that are strongly magnetic - ferromagnetic ( $\sigma$  large) and ferrimagnetic ( $\sigma$  small usually)
- These materials have hysteresis curves

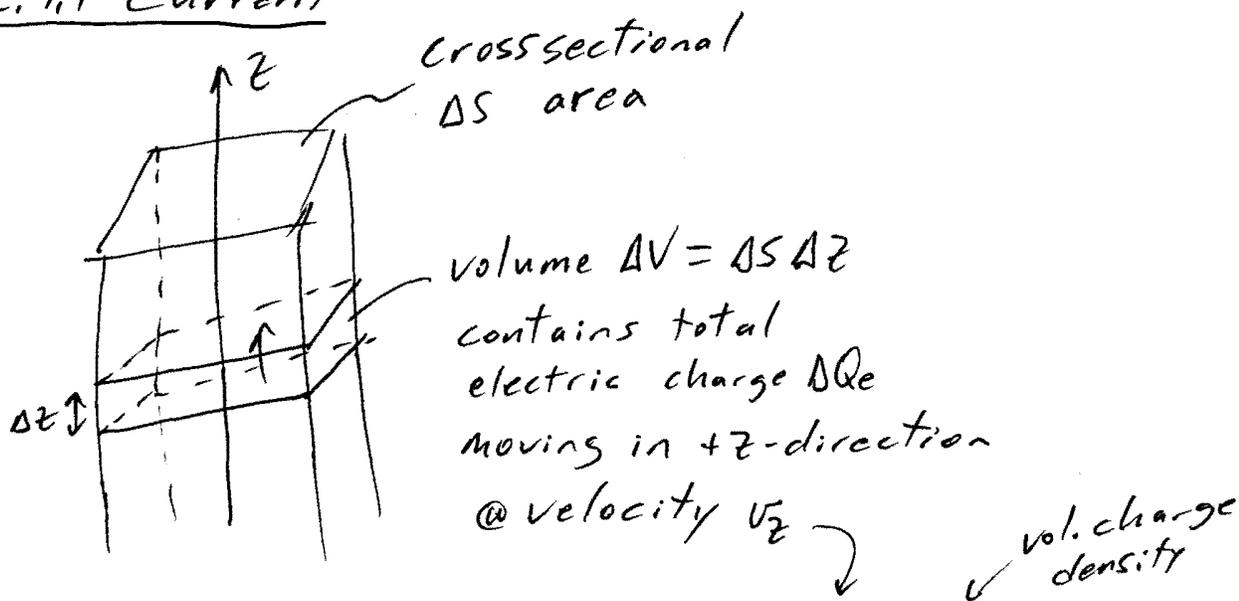


- Magnetic materials tend to be lossy. Two main mechanisms are hysteresis loss (related to area w/in hysteresis curve) and eddy currents (related to induced currents &  $\sigma$ )
- Mention Curie temp.

## 2.4 Current, Conductors, & Conductivity

→ conductors are materials where there are charges (electrons) free to move when an electric field is applied.

### 2.4.1 Current



$$\Delta I = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q_e}{\Delta t} = \lim_{\Delta t \rightarrow 0} q_v \Delta S \frac{\Delta z}{\Delta t} = q_v v_z \Delta S \quad (\text{A})$$

Define current density magnitude as

$$J_z = \lim_{\Delta S \rightarrow 0} \frac{\Delta I}{\Delta S} = q_v v_z \quad (\text{A/m}^2)$$

In vector form

$$\vec{J} = q_v \vec{v} \equiv \text{Convection current density}$$

This can be broken into positive & negative charges/parts

$$\vec{J} = q_v^+ \vec{v}^+ + q_v^- \vec{v}^-$$

## 2.4.2 Conductors

- Materials where valence electrons are not tightly bound  $\Rightarrow$  'free' electrons
- At rest, the free electrons move in random directions  $\Rightarrow I_{net} = 0$  through any surface
- Excess free charges introduced into a conductor migrate/flow to surface due to repulsive Coulomb forces. The volume charge density inside the conductor is governed by

$$q_v(t) = q_{v0} e^{-t/\tau_r}$$

where  $\tau_r = \frac{\epsilon}{\sigma}$  (s)  $\equiv$  relaxation time constant

$\sigma \equiv$  conductivity (S/m)  $\leftarrow$  Next section

### Other properties

- For perfect electrical conductors (PEC) and very good conductors ( $\sigma$  very large)

$$\Delta V_{inside} = 0, \quad \vec{E}_{t,inside} = \vec{D}_{t,inside} = 0$$

2.4.2 cont.

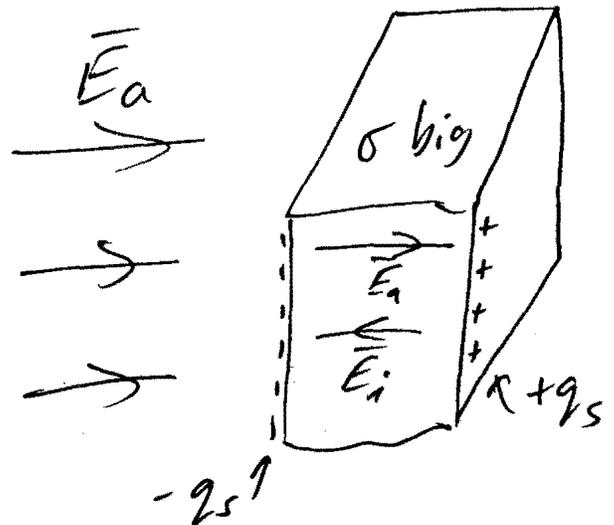
why?

① Applied electric field

 $\vec{E}_a$  causes charges tomove ( $\vec{F} = q\vec{E}_a$ ) leading

to positive &amp; negative

surface charge densities



② The newly formed surface charge densities cause/create an induced electric field  $\vec{E}_i$ .  $\vec{E}_i$ , in opposition to  $\vec{E}_a$  will keep getting bigger & bigger until  $\vec{E}_t = \vec{E}_i + \vec{E}_a = 0$  and charges stop moving!

③ By  $\vec{D} = \epsilon \vec{E} \Rightarrow \underline{\underline{\vec{D}_t = 0}}$

By  $\Delta V = -\int_A^B \vec{E} \cdot d\vec{x} \Rightarrow \Delta V = 0$  inside conductor

$\Rightarrow$  all points in conductor @ same potential!

### 2.4.3 Conductivity

To characterize the flow of charges in a conductor, we'll define a quantity / parameter called conductivity  $\sigma$

→ Apply an external electric field  $\vec{E}_a$  to a conductor. The free electrons which were moving in random directions w/ random velocities now, on average, move at a drift velocity  $\vec{v}_e$  in a direction opposite to  $\vec{E}_a$  ( $\vec{F} = q\vec{E}_a$  w/  $q$  negative)

⇒ Conduction current ←

Define  $\vec{v}_e = -\mu_e \vec{E}$  where  $\mu_e \equiv$  electron mobility ( $\frac{m^2}{V \cdot s}$ )

use  $\vec{v}_e$  in  $\vec{J} = q_v \vec{v}$  to get

$$\vec{J} = -q_v \mu_e \vec{E} = \sigma_s \vec{E}$$

where static conductivity  $\equiv \sigma_s = -q_v \mu_e$  ( $\frac{S}{m}$ )

Older texts resistivity =  $\rho = \frac{1}{\sigma}$  ( $\Omega \cdot m$ )

⇒ See Table 2-3 ←

**TABLE 2-3 Typical conductivities of insulators, semiconductors, and conductors**

Material	Class	Conductivity $\sigma$ (S/m)
Fused quartz	Insulator	$\sim 10^{-17}$
Ceresin wax	Insulator	$\sim 10^{-17}$
Sulfur	Insulator	$\sim 10^{-15}$
Mica	Insulator	$\sim 10^{-15}$
Paraffin	Insulator	$\sim 10^{-15}$
Hard rubber	Insulator	$\sim 10^{-15}$
Porcelain	Insulator	$\sim 10^{-14}$
Glass	Insulator	$\sim 10^{-12}$
Bakelite	Insulator	$\sim 10^{-9}$
Distilled water	Insulator	$\sim 10^{-4}$
Gallium arsenide (GaAs)*	Semiconductor	$\sim 2.38 \times 10^{-7}$
Fused silica*	Semiconductor	$\sim 2.1 \times 10^{-4}$
Cross-linked polystyrene (unreinforced)*	Semiconductor	$\sim 3.7 \times 10^{-4}$
Beryllium Oxide (BeO)*	Semiconductor	$\sim 3.9 \times 10^{-4}$
Intrinsic silicon	Semiconductor	$\sim 4.39 \times 10^{-4}$
Sapphire*	Semiconductor	$\sim 5.5 \times 10^{-4}$
Glass-reinforced Teflon (microfiber)*	Semiconductor	$\sim 7.8 \times 10^{-4}$
Teflon quartz (woven)*	Semiconductor	$\sim 8.2 \times 10^{-4}$
Dry soil	Semiconductor	$\sim 10^{-4} - 10^{-3}$
Ferrite(Fe <sub>2</sub> O <sub>3</sub> )*	Semiconductor	$\sim 1.3 \times 10^{-3}$
Glass-reinforced Polystyrene*	Semiconductor	$\sim 1.45 \times 10^{-3}$
Polyphenelene oxide (PPO)*	Semiconductor	$\sim 2.27 \times 10^{-3}$
Glass-reinforced Teflon (woven)*	Semiconductor	$\sim 2.43 \times 10^{-3}$
Plexiglas*	Semiconductor	$\sim 5.1 \times 10^{-3}$
Wet soil	Semiconductor	$\sim 10^{-3} - 10^{-2}$
Fresh water	Semiconductor	$\sim 10^{-2}$
Human and animal tissue	Semiconductor	$\sim 0.2 - 0.7$
Intrinsic germanium	Semiconductor	$\sim 2.227$
Seawater	Semiconductor	$\sim 4$
Tellurium	Conductor	$\sim 5 \times 10^{-2}$
Carbon	Conductor	$\sim 3 \times 10^{-4}$
Graphite	Conductor	$\sim 3 \times 10^4$
Cast iron	Conductor	$\sim 10^6$
Mercury	Conductor	$10^6$
Nichrome	Conductor	$10^6$
Silicon steel	Conductor	$2 \times 10^6$
German silver	Conductor	$2 \times 10^6$
Lead	Conductor	$5 \times 10^6$
Tin	Conductor	$9 \times 10^6$
Iron	Conductor	$1.03 \times 10^7$
Nickel	Conductor	$1.45 \times 10^7$
Zinc	Conductor	$1.7 \times 10^7$
Tungsten	Conductor	$1.83 \times 10^7$
Brass	Conductor	$2.56 \times 10^7$
Aluminum	Conductor	$3.96 \times 10^7$
Gold	Conductor	$4.1 \times 10^7$
Copper	Conductor	$5.76 \times 10^7$
Silver	Conductor	$6.1 \times 10^7$

\*For most semiconductors the conductivities are representative for a frequency of about 10 GHz.

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Skip sections 2.5 Semiconductors,  
2.6 Superconductors, & 2.7 Metamaterials  
2.8 Linear, Homogenous, Isotropic, and  
Nondispersive Media

Linear -  $\epsilon$ ,  $\mu$ , &  $\sigma$  are NOT functions of applied electric or magnetic field strengths

examples of nonlinearity

- dielectric breakdown  $\Rightarrow |E|$  where electrons ripped loose from material
- magnetic saturation  $\Rightarrow |H|$  where all magnetic dipoles are aligned

Homogenous -  $\epsilon$ ,  $\mu$ , &  $\sigma$  are NOT functions of position/location w/in material; otherwise material is inhomogeneous or nonhomogenous

- this can be a concern for circuit board manufacturers as well as for TL insulation.

Nondispersive -  $\epsilon$ ,  $\mu$ , &  $\sigma$  are NOT functions of frequency (or time)

- Reality is that all materials are dispersive. However, over a limited BW they may be effectively nondispersive.

2.8 cont.

Isotropic -  $\epsilon, \mu, \text{ \& } \sigma$  are NOT functions of direction w/in the material

→ Many crystalline materials are anisotropic or nonisotropic. In this case,  $\epsilon, \mu, \text{ \& } \sigma$  are usually written as  $3 \times 3$  tensors instead of a single value. E.g.,

$$\vec{D} = \bar{\epsilon} \cdot \vec{E}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

⇓

$$D_x = \epsilon_{xx} E_x + \epsilon_{xy} E_y + \epsilon_{xz} E_z$$

$$D_y = \epsilon_{yx} E_x + \epsilon_{yy} E_y + \epsilon_{yz} E_z$$

$$D_z = \epsilon_{zx} E_x + \epsilon_{zy} E_y + \epsilon_{zz} E_z$$

→ Each of the terms in the  $\bar{\epsilon}$  tensor can be complex

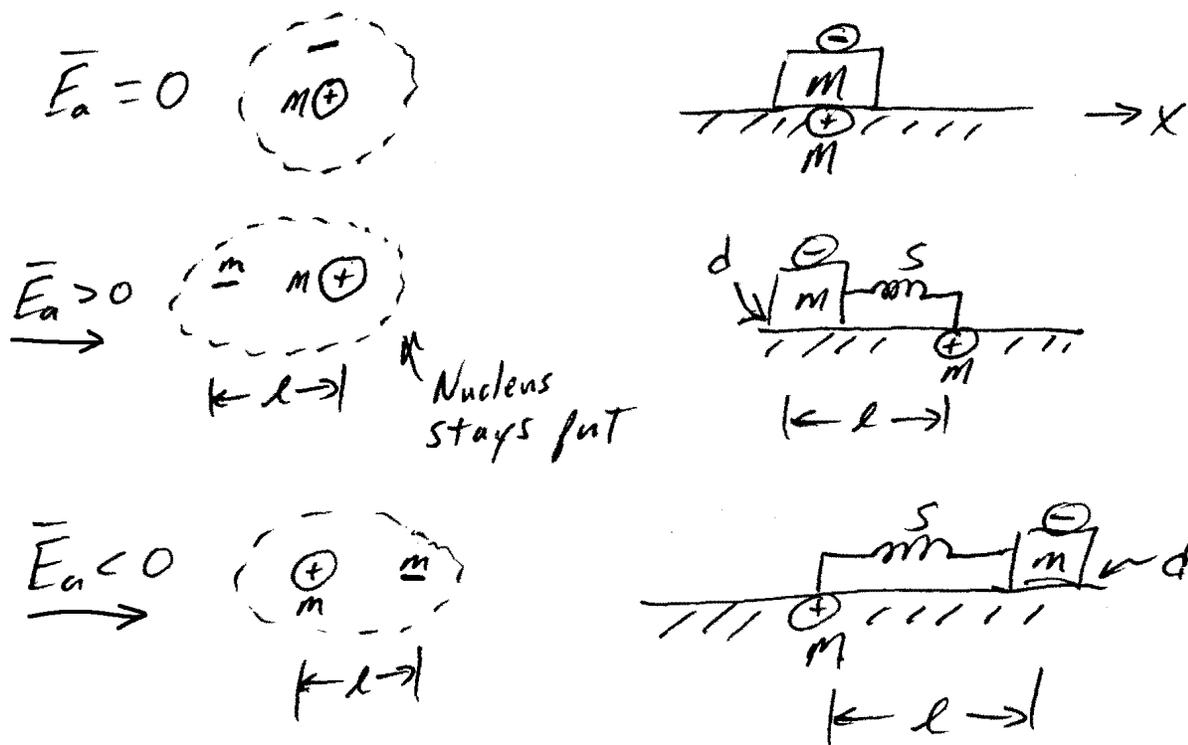
→  $\epsilon_{xx}, \epsilon_{yy}, \text{ \& } \epsilon_{zz}$  are called principal permittivities

## 2.9 AC Variations in Materials

→  $\epsilon, \mu, \sigma$  all vary w/ frequency.

### 2.9.1 Complex Permittivity

→ Make analogy between polarizing an atom w/ a mechanical model of a mass, attached to a spring, being moved back-n-forth



$M \equiv$  mass of nucleus  $+Q$

$m \equiv$  " " electrons  $-Q$

$d \equiv$  damping/friction coefficient  $\rightarrow$  takes power/energy to move charges

$S \equiv$  spring/tension coefficient  $\rightarrow$  stored power/energy

$l \equiv$  displacement distance for centroids of nucleus & electrons

2.9.1 cont.

For a time-harmonic applied electric field,  
leads to a 2<sup>nd</sup> order ODE

$$M \frac{d^2 l}{dt^2} + d \frac{dl}{dt} + S l = Q E(t) = Q E_0 e^{j\omega t}$$

↙ "ma"      "dv"      "sl"      "driving force"

$$\frac{d^2 l}{dt^2} + 2\alpha \frac{dl}{dt} + \omega_0^2 l = \frac{Q}{m} E_0 e^{j\omega t}$$

Where  $\alpha = \frac{d}{2m} + \omega_0 = \sqrt{S/m}$  ↑ very similar to RLC circuits

If  $\alpha > \omega_0 \rightarrow$  overdamped

$\alpha = \omega_0 \rightarrow$  critically damped

$\alpha < \omega_0 \rightarrow$  underdamped

complementary  
(Source-free)  
Transient

Two  
solns

particular  
soln

+

$$l_p(t) = l_0 e^{j\omega t}$$

$$l_c(t) = \begin{cases} \text{underdamped} \\ \text{critically damped} \\ \text{overdamped} \end{cases}$$

where  $l_0 = \frac{Q/m E_0}{(\omega_0^2 - \omega^2) + j\omega(d/m)}$

$$l(t) = l_c(t) + l_p(t)$$

↑

goes to zero as  $t \rightarrow \infty$  as long as  $d \neq 0$

Assuming an underdamped system ( $\alpha < \omega_0$ ),  
we can determine the damped resonant  
frequency (natural frequency) to be

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{S/m - (d/2m)^2}$$

### 2.9.1 cont.

If  $d=0$  (no damping),  $\omega_d = \omega_0 = \sqrt{S/m}$ .

At steady-state (i.e.,  $h_c(t) \rightarrow 0$ ), the time-varying polarization is

$$P = P(t) = N_e Q L(t) = \frac{N_e \left(\frac{Q^2}{m}\right) \overbrace{E_0}^{E(t)} e^{j\omega t}}{(\omega_0^2 - \omega^2) + j\omega(d/m)}$$

↑  
# dipoles  
per unit  
volume

Back in section 2.2,  $\bar{D} = \epsilon_0 \bar{E}_a + \bar{P} = \epsilon_0 \bar{E}_a + \epsilon_0 \chi_e \bar{E}_a$

$$\text{Now } \frac{P}{E} = \frac{N_e \left(\frac{Q^2}{m}\right)}{(\omega_0^2 - \omega^2) + j\omega(d/m)} \quad \text{d } E = E_0 + \frac{|\bar{P}|}{|\bar{E}_a|}$$

← complex #!

and

$$\begin{aligned} \text{complex permittivity} \equiv \dot{\epsilon} &= \epsilon_0 + \frac{P}{E} = \epsilon_0 + \frac{N_e \left(\frac{Q^2}{m}\right)}{(\omega_0^2 - \omega^2) + j\omega(d/m)} \\ &= \epsilon' - j\epsilon'' \end{aligned}$$

$$\text{relative complex permittivity} \equiv \dot{\epsilon}_r = \frac{\dot{\epsilon}}{\epsilon_0} = \epsilon_r' - j\epsilon_r'' = 1 + \frac{\frac{N_e Q^2}{\epsilon_0 m}}{(\omega_0^2 - \omega^2) + j\omega(d/m)}$$

$$\omega / \epsilon_r' = 1 + \frac{\frac{N_e Q^2}{\epsilon_0 m} (\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (\omega d/m)^2}$$

$$\epsilon_r'' = \frac{N_e Q^2}{\epsilon_0 m} \left[ \frac{\omega d/m}{(\omega_0^2 - \omega^2)^2 + (\omega d/m)^2} \right]$$

↑  
multiply top  
& bottom by  
complex conj.  
of denom.

2.9.1 cont.

Define complex index of refraction  $\equiv \dot{n} = \sqrt{\dot{\epsilon}_r}$

Note: If  $d=0$ ,  $\epsilon_r' = 1 + \frac{N_e Q^2}{\epsilon_0 m (\omega_0^2 - \omega^2)}$  &  $\epsilon_r'' = 0$

How does having  $\dot{\epsilon} = \epsilon' - j\epsilon''$  impact Maxwell's equations? Ampere's Law now becomes (phasor form)

$$\begin{aligned} \nabla \times \bar{H} &= \bar{J}_i + \bar{J}_c + j\omega \dot{\epsilon} \bar{E} = \bar{J}_t \\ &= \bar{J}_i + \sigma_s \bar{E} + j\omega (\epsilon' - j\epsilon'') \bar{E} \\ &= \bar{J}_i + (\sigma_s + \omega \epsilon'') \bar{E} + j\omega \epsilon' \bar{E} \\ &= \bar{J}_i + \underbrace{\sigma_e}_{\text{eff. cond. current dens.}} \bar{E} + j\omega \underbrace{\epsilon'}_{\text{effective displacement current density}} \bar{E} \end{aligned}$$

Equivalent conductivity  $\equiv \sigma_e = \sigma_s + \omega \epsilon'' = \sigma_s + \sigma_a$

alternating field conductivity  $\equiv \sigma_a = \omega \epsilon''$

static conductivity  $\equiv \sigma_s$

Some materials have modest  $\sigma_s$  and larger  $\sigma_a$  (think microwave oven +  $H_2O$ )

2.9.1 cont.

We can express the total current density

$$\begin{aligned}\bar{J}_t &= \bar{J}_i + \sigma_e \bar{E} + j\omega \epsilon' \bar{E} = \bar{J}_i + j\omega \epsilon' \left(1 - j \frac{\sigma_e}{\omega \epsilon'}\right) \bar{E} \\ &= \bar{J}_i + j\omega \epsilon' (1 - j \tan \delta_e) \bar{E}\end{aligned}$$

Define

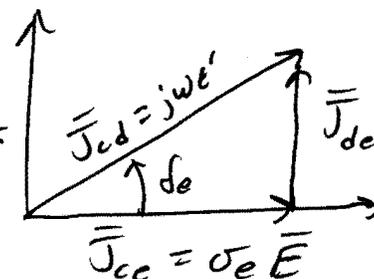
$$\begin{aligned}\text{effective loss tangent} &\equiv \tan \delta_e = \frac{\sigma_e}{\omega \epsilon'} = \frac{\sigma_s}{\omega \epsilon'} + \frac{\overset{\omega \epsilon''}{\uparrow} \sigma_a}{\omega \epsilon'} \\ &= \frac{\sigma_s}{\omega \epsilon'} + \frac{\epsilon''}{\epsilon'} = \tan \delta_s + \tan \delta_a\end{aligned}$$

$$\text{static loss tangent} \equiv \tan \delta_s = \frac{\sigma_s}{\omega \epsilon'}$$

$$\text{alternating loss tangent} \equiv \tan \delta_a = \frac{\epsilon''}{\epsilon'}$$

⇒ Manufacturers of PCBs and TLs when giving conductivity  $\sigma$ , loss tangent  $\tan \delta$ , and relative permittivity  $\epsilon_r$  at selected frequencies typically mean  $\sigma_e$ ,  $\tan \delta_e$ , and  $\epsilon_r'$ . See Table 2-5.

⇒ Look at the effective conduction + displacement current densities on complex plane

$$\begin{aligned}\bar{J}_{cd} &= \bar{J}_{cc} + \bar{J}_{de} \\ &= j\omega \epsilon' \left(1 - j \frac{\sigma_e}{\omega \epsilon'}\right) \bar{E}\end{aligned}$$


$$\bar{J}_{de} = j\omega \epsilon' \bar{E} \Rightarrow \tan \delta_e = \frac{\sigma_e}{\omega \epsilon'}$$

**TABLE 2-5 Dielectric constants and loss tangents of typical dielectric materials**

Material	$\epsilon'_r$	$\tan \delta$
Air	1.0006	
Alcohol (ethyl)	25	0.1
Aluminum oxide	8.8	$6 \times 10^{-4}$
Bakelite	4.74	$22 \times 10^{-3}$
Carbon dioxide	1.001	
Germanium	16	
Glass	4–7	$1 \times 10^{-3}$
Ice	4.2	0.1
Mica	5.4	$6 \times 10^{-4}$
Nylon	3.5	$2 \times 10^{-2}$
Paper	3	$8 \times 10^{-3}$
Plexiglas	3.45	$4 \times 10^{-2}$
Polystyrene	2.56	$5 \times 10^{-5}$
Porcelain	6	$14 \times 10^{-3}$
Pyrex glass	4	$6 \times 10^{-4}$
Quartz (fused)	3.8	$7.5 \times 10^{-4}$
Rubber	2.5–3	$2 \times 10^{-3}$
Silica (fused)	3.8	$7.5 \times 10^{-4}$
Silicon	11.8	
Snow	3.3	0.5
Sodium chloride	5.9	$1 \times 10^{-4}$
Soil (dry)	2.8	$7 \times 10^{-2}$
Styrofoam	1.03	$1 \times 10^{-4}$
Teflon	2.1	$3 \times 10^{-4}$
Titanium dioxide	100	$15 \times 10^{-4}$
Water (distilled)	80	$4 \times 10^{-2}$
Water (sea)	81	4.64
Wood (dry)	1.5–4	$1 \times 10^{-2}$

2.9.1 cont.

Ex. Rogers Corporation RO4003C hydrocarbon/ceramic laminate substrate. From the datasheet

$$\rho_s = 1.7 \times 10^{10} \Omega \cdot m \Rightarrow \underline{\sigma_s = 5.88 \times 10^{-15} \text{ S/m}}$$

$$\begin{aligned} \tan \delta = \tan \delta_e &= 0.0021 @ 2.5 \text{ GHz} \\ &= 0.0027 @ 10 \text{ GHz} \end{aligned}$$

$$\begin{aligned} \epsilon_{re} = \epsilon_r' &= 3.67 @ 2.5 \text{ GHz} \\ &= 3.651 @ 10 \text{ GHz} \end{aligned}$$

Find the effective conductivity, imaginary part of  $\epsilon$ , and  $\epsilon_r$ , the complex permittivity and relative permittivity.

@ 2.5 GHz  $\sigma_e = \omega \epsilon' \tan \delta_e = 2\pi(2.5 \times 10^9) 3.67 \epsilon_0 (0.0021)$

$$\underline{\sigma_e = 0.001072 \text{ S/m} = 1.072 \text{ mS/m}}$$

$$\epsilon'' = \frac{\sigma_e - \sigma_s}{\omega} = \frac{0.001072 - 5.88 \times 10^{-15}}{2\pi(2.5 \times 10^9)} = \underline{6.824 \times 10^{-14} \text{ F/m}}$$

$$\underline{\epsilon_r'' = \frac{\epsilon''}{\epsilon_0} = 0.007707} \text{ (pretty small compared to } \epsilon_r' = 3.67)$$

@ 10 GHz  $\sigma_e = 2\pi(10 \times 10^9) 3.651 (8.9542 \times 10^{-12}) 0.0027$

$$\underline{\sigma_e = 0.005484 \text{ S/m} = 5.484 \text{ mS/m}}$$

$$\epsilon'' = \frac{\sigma_e - \sigma_s}{\omega} = \frac{0.005484 - 5.88 \times 10^{-15}}{2\pi(10 \times 10^9)} = \underline{8.7282 \times 10^{-14} \text{ F/m}}$$

$$\epsilon_r'' = \frac{\epsilon''}{\epsilon_0} = 0.009858 \text{ (small compared to } \epsilon_r' = 3.651)$$

2.9.1 cont.

Consider the extremes

1. Good dielectrics ( $\tan \delta_e = \frac{\sigma_e}{\omega \epsilon'} \ll 1$ )

$$\bar{J}_{cd} \approx j\omega \epsilon' \bar{E} \quad (\text{nearly all displacement current density})$$

2. Good conductors ( $\tan \delta_e = \frac{\sigma_e}{\omega \epsilon'} \gg 1$ )

$$\bar{J}_{cd} \approx \sigma_e \bar{E} \quad (\text{nearly all conduction current density})$$

Going back to section 2.2, it was mentioned that electric polarization could occur due to different mechanisms: 1) dipole/orientational, 2) ionic/molecular, & 3) electronic. Depending on the material, 1, 2, or 3 of these mechanisms may apply and typically would occur @ different frequencies/frequency ranges.

Writing the phasor polarization vector as a function of frequency

$$\bar{P}(\omega) = \epsilon_0 \chi_e(\omega) \bar{E}_a(\omega)$$

we'll note that the electric susceptibility is a function of frequency. Further,  $\chi_e(\omega)$  must be complex since  $\dot{\epsilon}$  is complex.

2.9.1 cont.

$$\chi_e(\omega) = \chi_e'(\omega) - j\chi_e''(\omega)$$

$$= \left[ \underset{\substack{\uparrow \\ \text{dipole}}}{\chi_{ed}'(\omega)} + \underset{\substack{\uparrow \\ \text{ionic}}}{\chi_{ei}'(\omega)} + \underset{\substack{\uparrow \\ \text{electronic}}}{\chi_{ee}'(\omega)} \right] - j \left[ \underset{\substack{\uparrow \\ \text{dipole}}}{\chi_{ed}''(\omega)} + \underset{\substack{\uparrow \\ \text{ionic}}}{\chi_{ei}''(\omega)} + \underset{\substack{\uparrow \\ \text{electronic}}}{\chi_{ee}''(\omega)} \right]$$

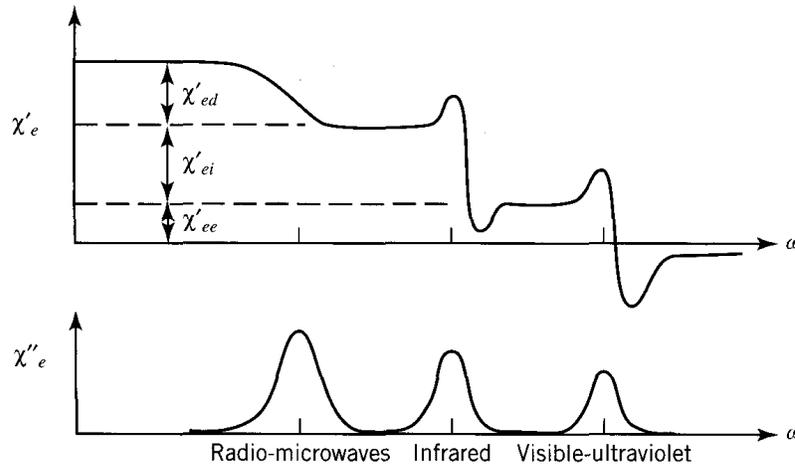
In general,  $\chi_e'(\omega) = \chi_e'(-\omega)$  real part is even

$$\chi_e''(-\omega) = -\chi_e''(\omega) \text{ imag. part is odd}$$

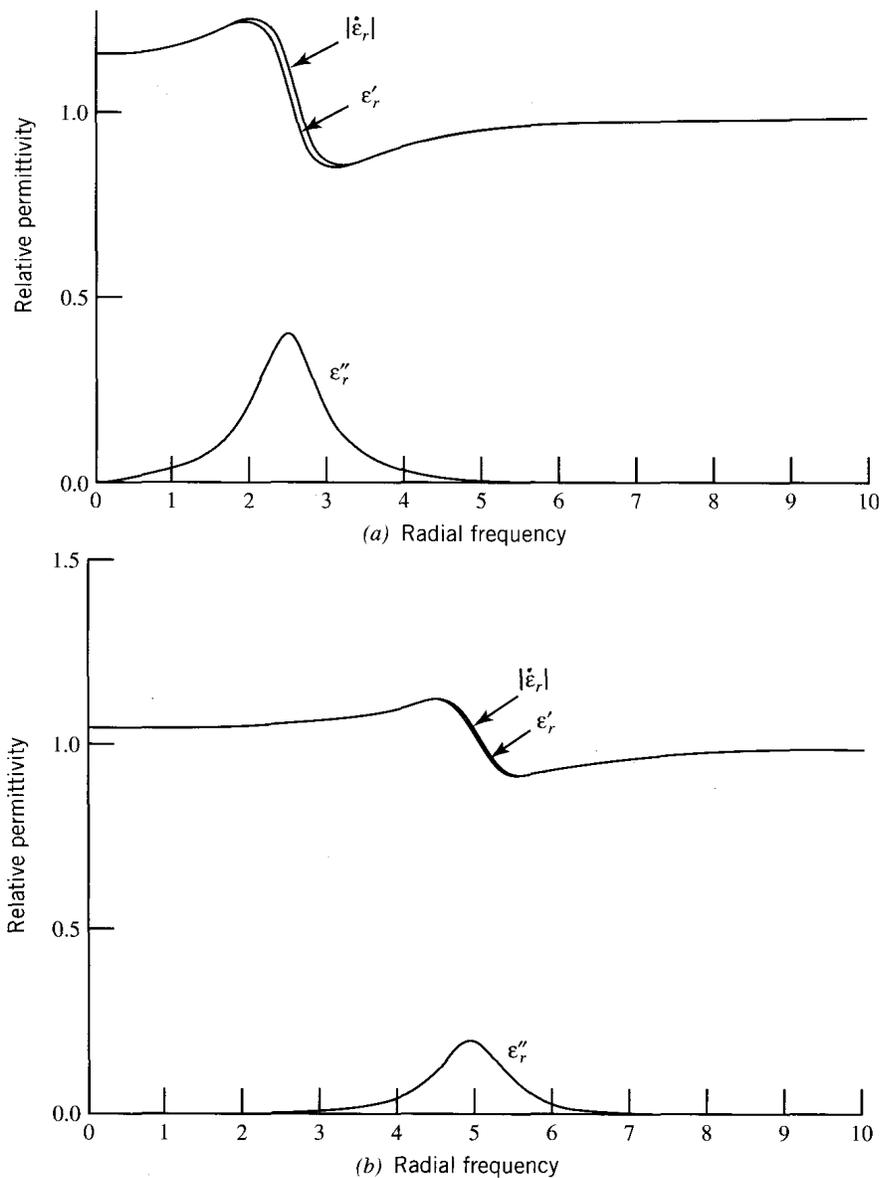
Figures 2-18 & 2-19 show general samples as to how  $\chi_e$  &  $\epsilon_r$  can vary with frequency

→ Note, both  $\chi_e''$  &  $\epsilon_r''$  peak @/near resonant frequencies while  $\chi_e'$  &  $\epsilon_r'$  dip (negative slope), called anomalous dispersion.

→ Other than near resonant frequencies  $|\epsilon_r|$  &  $\epsilon_r'$  exhibit positive slopes, i.e., normal dispersion.



**Figure 2-18** Electric susceptibility (real and imaginary) variations as a function of frequency for a typical dielectric.



**Figure 2-19** Typical frequency variations of real and imaginary parts of relative permittivity of dielectrics. (a)  $N_e Q^2 / \epsilon_0 m = 1, d/m = 1, \alpha / \omega_0 = 1/5, \omega_0 = 2.5$ . (b)  $N_e Q^2 / \epsilon_0 m = 1, d/m = 1, \alpha / \omega_0 = 1/10, \omega_0 = 5$ .

*Advanced Engineering Electromagnetics* (Second Edition), Balanis, Wiley, 2012, ISBN-10: 0470589485, ISBN-13: 978-0470589489.

2.9.1 cont.

For our mechanical analogy, we assumed a single resonant frequency. However, actual material can have multiple resonances due to different mechanisms. To allow for this possibility, or  $P$  resonances, let-

$$\frac{\rho}{\epsilon} = \sum_{s=1}^P \frac{N_e \frac{Q^2}{m}}{(\omega_s^2 - \omega^2) + j \frac{\omega d}{m}}$$

$$\dot{\epsilon} = \epsilon' - j\epsilon'' = \epsilon_0 + \sum_{s=1}^P \frac{N_e \frac{Q^2}{m}}{(\omega_s^2 - \omega^2) + j \frac{\omega d}{m}}$$

$$\dot{\epsilon}_r = \epsilon_r' - j\epsilon_r'' = 1 + \sum_{s=1}^P \frac{N_e \frac{Q^2}{\epsilon_0 m}}{(\omega_s^2 - \omega^2) + j \frac{\omega d}{m}}$$

$$\epsilon_r' = 1 + \sum_{s=1}^P \frac{N_e Q^2 (\omega_s^2 - \omega^2)}{\epsilon_0 m ((\omega_s^2 - \omega^2)^2 + (\frac{\omega d}{m})^2)}$$

$$\epsilon_r'' = \sum_{s=1}^P \frac{N_e Q^2}{\epsilon_0 m} \frac{\omega \frac{d}{m}}{(\omega_s^2 - \omega^2)^2 + (\frac{\omega d}{m})^2}$$

$\Rightarrow \epsilon_r'$  &  $\epsilon_r''$  are related by the Kramers-Kronig equations

$$\epsilon_r'(\omega) = 1 + \frac{2}{\pi} \int_0^{\infty} \frac{\omega' \epsilon_r''(\omega')}{\omega'^2 - \omega^2} d\omega'$$

$$\epsilon_r''(\omega) = \frac{2\omega}{\pi} \int_0^{\infty} \frac{1 - \epsilon_r'(\omega')}{\omega'^2 - \omega^2} d\omega'$$

2.9.1 cont.

For many materials, the Debye equation can be used to find the complex relative permittivity (as well as  $\sigma_e(\omega)$ )

$$\dot{\epsilon}_r(\omega) = \epsilon'_{r\infty} + \frac{\epsilon'_{rs} - \epsilon'_{r\infty}}{1 + j\omega\tau_e}$$

where

$$\begin{array}{l} \text{new relaxation} \\ \text{time constant} \end{array} \equiv \tau_e = \tau \frac{\epsilon'_{rs} + 2}{\epsilon'_{r\infty} + 2}$$

$$\epsilon'_r(\omega) = \epsilon'_{r\infty} + \frac{\epsilon'_{rs} - \epsilon'_{r\infty}}{1 + (\omega\tau_e)^2}$$

$$\Rightarrow \omega_e = \frac{1}{\tau_e}$$

$$f_e = \frac{1}{2\pi\tau_e}$$

relaxation frequencies

$$\epsilon''_r(\omega) = \frac{(\epsilon'_{rs} - \epsilon'_{r\infty})\omega\tau_e}{1 + (\omega\tau_e)^2}$$

provided that the following are known

$\epsilon'_{rs} \equiv$  static relative permittivity ( $\omega=0$ )

$\epsilon'_{r\infty} \equiv$  relative permittivity as  $\omega \rightarrow \infty$

$\tau_e$  or  $\tau$

$\sigma_0 \equiv$  static conductivity (S/m)

$\Rightarrow$  This is good for a single resonance.

More terms can be added to account for multiple resonances.

## 2.9.2 Complex permeability

- For most materials,  $\mu \approx \mu_0$ . So, complex is not of much interest. E.g., diamagnetic, paramagnetic, & antiferromagnetic.
- This leaves ferromagnetic & ferrimagnetic materials which are magnetically lossy. This is accounted for by setting  $\dot{\mu} = \mu' - j\mu''$ .
- In Maxwell's equations, this changes Faraday's Law

$$\begin{aligned}\bar{\nabla} \times \bar{E} &= -\bar{M}_i - j\omega \dot{\mu} \bar{H} = -\bar{M}_i - j\omega(\mu' - j\mu'') \bar{H} \\ &= -\bar{M}_i - j\omega\mu' \bar{H} - \omega\mu'' \bar{H} = -\bar{M}_t\end{aligned}$$

where  $\bar{M}_t \equiv$  total magnetic current density ( $V/m^2$ )

$\bar{M}_i \equiv$  impressed " " "

$\bar{M}_d \equiv$  displacement " " "  $= j\omega\mu' \bar{H}$

$\bar{M}_c \equiv$  conduction " " "  $= \omega\mu'' \bar{H}$

$$\begin{aligned}\bar{M}_t &= \bar{M}_i + j\omega\mu' \left(1 - j \frac{\mu''}{\mu'}\right) \bar{H} \\ &= \bar{M}_i + j\omega\mu' (1 - j \tan \delta_m) \bar{H}\end{aligned}$$

alternating magnetic loss tangent  $\equiv \tan \delta_m = \frac{\mu''}{\mu'}$

## 2.9.2 cont.

Like  $\epsilon$ ,  $\mu$  is dependent on frequency, i.e.,  

$$\tilde{\mu}(\omega) = \mu'(\omega) - j\mu''(\omega)$$

and we can define a complex relative magnetic permeability  $\tilde{\mu}_r(\omega) = \frac{\tilde{\mu}(\omega)}{\mu_0} = \mu_r'(\omega) - j\mu_r''(\omega)$

Practicalities:

→ Ferromagnetic materials ( $\sigma$  large) have such small skin depths that EM waves/fields don't penetrate enough for there to be significant interactions at higher frequencies. At lower frequencies, other losses (hysteresis + eddy currents) are more significant.

→ Ferrimagnetic materials ( $\sigma$  low) do allow for significant interactions w/ EM fields @ higher frequencies. Some applications include isolators, gyrators, phase shifters, etcetera.

⇒ Niche topic, skip to save time.