EE 483/583 Antennas for Wireless Communications
Examination #3 (Spring 2026)

Name <u>KEY A</u>

Instructions: Place answers in indicated spaces. Use notation as given in class for coordinates & variables. **Show/label all** work for credit. Insert equation sheets. Where applicable, let $\boldsymbol{c} = \mathbf{2.998 \times 10^8}$ **m/s**.

1) Design a 7-turn axial mode helical antenna to operate in free space at 749.5 MHz with a turn spacing of 0.246λ. Determine the spacing (cm), diameter (cm & x.xλ), pitch angle (deg), height (cm & x.xλ), and overall wire length (cm & x.xλ) of the antenna. Then, find the axial ratio AR and estimated HPBW (deg) and maximum directivity $D_{\max}$ (unitless & dBi) using Kraus' formula (based on dimensions).

$\lambda = c/f = 2.998\times10^8/749.5\times10^6 = 0.4$ m $= 40$ cm.

Spacing S = 0.246(40) $\Rightarrow$ **$S = 9.84$ cm.**

Axial mode $\Rightarrow C = \lambda = \pi D \Rightarrow D = \lambda/\pi = 40/\pi$ $\Rightarrow$ **$D = 0.31831\lambda = 12.7324$ cm.**

Per (10-24), $\alpha = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{0.246\lambda}{\lambda}\right)$ $\Rightarrow$ **$\alpha = 13.82034°$.**

Per Fig. 10.13, the helix height $L = NS = 7(0.246\lambda) = 7(0.246)40 \Rightarrow$ **$L = 1.722\lambda = 68.88$ cm.**

Per Fig. 10.13, the 1-turn length is

$$L_0 = \sqrt{C^2 + S^2} = \sqrt{\lambda^2 + (0.246\lambda)^2} = 1.029813575\lambda = 41.1925 \text{ cm}.$$

The overall wire length is then $L_n = N L_0 = 7(1.0298\lambda) = 7(41.1925)$

$\Rightarrow$ **$L_n = 7.208695\lambda = 288.3478$ cm.**

Per (10-34), $\text{AR} = \frac{2N+1}{2N} = \frac{2(7)+1}{2(7)} = \frac{15}{14}$ $\Rightarrow$ **AR = 1.07143.**

Per (10-31), $\text{HPBW} \simeq \frac{52\lambda^{3/2}}{C\sqrt{NS}} = \frac{52(40)^{3/2}}{40\sqrt{7(9.84)}}$ $\Rightarrow$ **HPBW = 39.6266°.**

Per Dr. Kraus' text, $D_{\max} \simeq 12N\frac{C^2 S}{\lambda^3} = 12(7)\frac{\lambda^2(0.246\lambda)}{\lambda^3}$ $\Rightarrow$ **$D_{\max} = 20.664 = 13.152$ dBi.**

spacing = **$S = 9.84$ cm** diameter = **$D = 0.3183\lambda = 12.73$ cm** pitch angle = **$\alpha = 13.82°$**

height = **$L = 1.722\lambda = 68.88$ cm** overall wire length = **$L_n = 7.2087\lambda = 288.35$ cm**

AR = **1.0714** HPBW = **39.6266°** $D_{\max}$ = **20.664 = 13.152 dBi**

2) A 4-turn circular loop antenna is operated at 96.7 MHz in free space. The loop diameter is 7 cm. The antenna is made of copper wire (1.02 mm diameter, $\sigma = 5 \times 10^7$ S/m) with a loop spacing of 1.479 mm. Compute the loop circumference and overall wire length in terms of wavelengths, i.e., C/λ and NC/λ. Is this loop antenna electrically small, electrically large, or neither? Find the radiation & ohmic loss resistances, and radiation efficiency (%) of the antenna. Show all work on given figures.

$C = \pi D = \pi(7) = 21.9911$ cm. $\lambda = c/f = 2.998\times10^8/96.7\times10^6 = 3.1003$ m $= 310.031$ cm.

$NC/\lambda = 4(21.9911)/310.031 \Rightarrow$ **$NC/\lambda = 0.283728$** $\sim< 0.2 \Rightarrow$ '**smallish' loop**.

$ka = C/\lambda = (21.9911)/310.031 \Rightarrow$ **$C/\lambda = 0.070932$** $< 1/10 \Rightarrow$ **small loop**.

For 1-turn (5-24), $R_{r,1} = \eta(\pi/6)(ka)^4 = 376.7303(\pi/6)(0.070932)^4 = 4.993444\times10^{-3}\,\Omega$

Per (5-24a), $R_r = R_{r,1} N^2 = 4.993444\times10^{-3}(4)^2 \Rightarrow$ **$R_r = 0.079895\,\Omega = 79.895\,\text{m}\Omega$**

Wire diameter $2b = 1.02$ mm & wire spacing $2c = 1.479$ mm $\Rightarrow 2c/2b = 1.479/1.02 = 1.45$.

From Fig. 5.3, **$R_P/R_0 = 0.5$** for $c/b = 1.45$ and $N = 4$.

Per (5-25),
$$R_L = N\frac{\ell}{p}\sqrt{\frac{\omega\mu_0}{2\sigma}}\left(\frac{R_p}{R_0}+1\right) = N\frac{2\pi a}{2\pi b}\sqrt{\frac{\omega\mu_0}{2\sigma}}\left(\frac{R_p}{R_0}+1\right)$$
$$= 4\frac{70}{1.02}\sqrt{\frac{2\pi(96.7\times10^6)4\pi\times10^{-7}}{2(5\times10^7)}}(0.5+1)$$
$\Rightarrow$ **$R_L = 1.1377772\,\Omega$**

Per (2-90) $$e_{cd} = \frac{R_r}{R_L + R_r}100\% = \frac{0.079895\,(100\%)}{1.1377772 + 0.079895}$$ $\Rightarrow$ **$e_{cd} = 6.5613$ %**

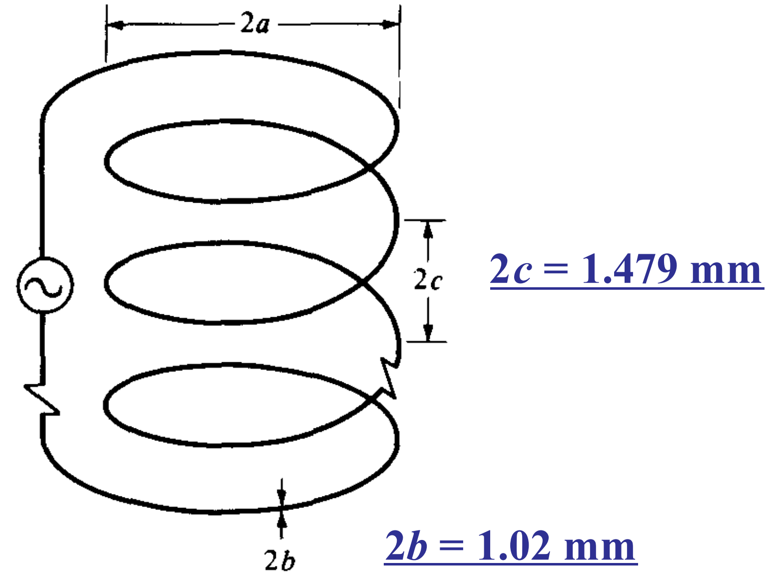
Note: There was a typo, the diameter was meant to be 2.3 cm. Unfortunately, the loop antenna is in the grey zone between being electrically small and large (closer to small). I took either answer as being correct.

Electrically **small**, large, or neither? (circle correct)

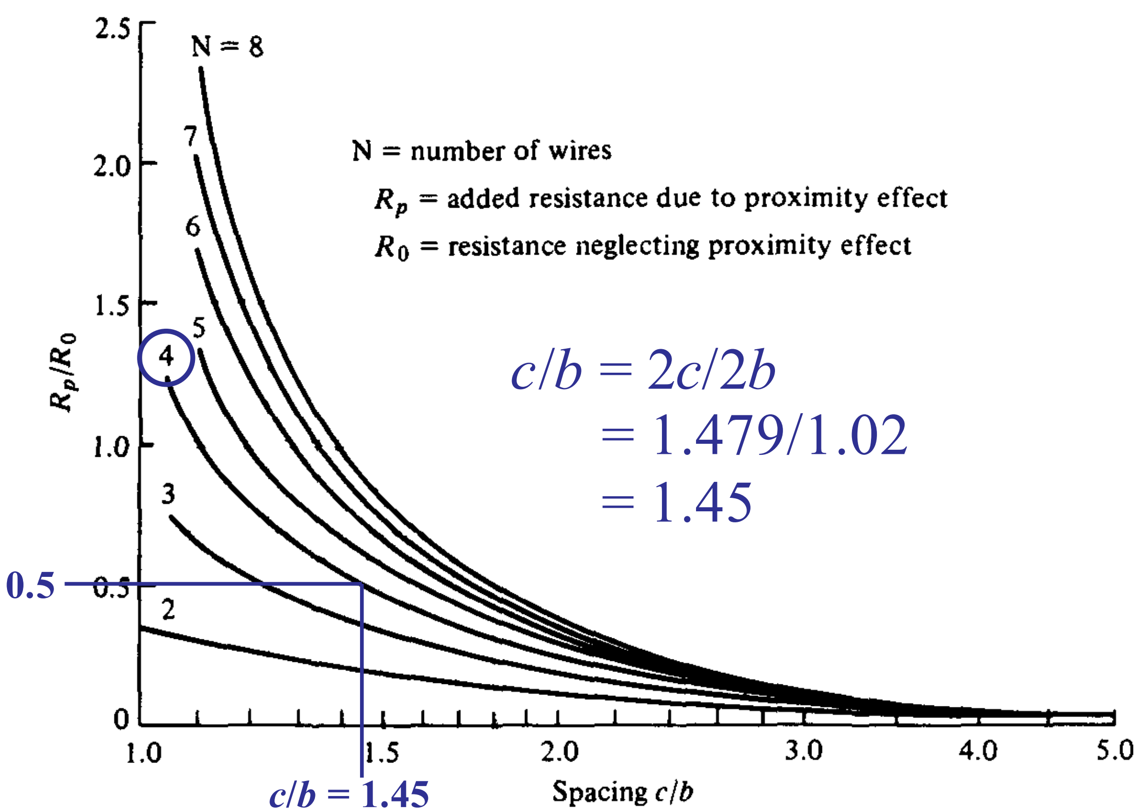
Why? **$C/\lambda = 0.071 < 1/10$ or $NC/\lambda = 0.284 \sim< 0.2$**

Rad. resistance = **$R_r = 79.895\,\text{m}\Omega$** ohmic loss resistance = **$R_L = 1.137777\,\Omega$**

$C/\lambda =$ **0.070932** $NC/\lambda =$ **0.28373** Rad. efficiency = **$e_{cd} = 6.56$ %**



(a) N-turn circular loop



(b) Ohmic resistance due to proximity (after G. N. Smith)

Figure 5.3 *N*-turn circular loop and ohmic resistance due to proximity effect. (SOURCE: G. S. Smith, "Radiation Efficiency of Electrically Small Multiturn Loop Antennas," *IEEE Trans. Antennas Propagat.*, Vol. AP-20, No. 5, pp. 656-657, Sept. 1972© (1972) IEEE).

3) A rectangular microstrip antenna is designed to operate at a frequency of 1.8 GHz on an RF substrate (ε_r = 1.83, h = 3.175 mm) in the TM_{010} mode. The antenna is 7 cm wide and 5.946 cm long with an effective length of 6.2971 cm. The antenna is fed by a 50 Ω microstrip transmission line.

a) The slot conductance G_1 is found using which definite integral? (circle correct answer)

$k_0 = \omega/c = 2\pi(1.8\times10^9)/2.998\times10^8 = 37.724$ rad/m.

Notes $G_1 = \dfrac{1}{\pi\eta_0}\displaystyle\int_{\theta=0}^{\pi}\left[\frac{\sin\left(\frac{k_0 W}{2}\cos\theta\right)}{\cos\theta}\right]^2 \sin^3\theta\, d\theta$. Here, $k_0W/2 = 37.724(0.07)/2 = 1.3203$.

$$\frac{1}{\pi\,376.703}\int_0^{\pi}\left(\frac{\sin(1.1215\cos\theta)}{\cos\theta}\right)^2 \sin^3\theta\, d\theta = 1.306\times10^{-3}\text{ S}$$

$$\frac{1}{\pi\,376.703/\sqrt{1.83}}\int_0^{\pi}\left(\frac{\sin(1.3203\cos\theta)}{\cos\theta}\right)^2 \sin^3\theta\, d\theta = 2.377\times10^{-3}\text{ S}$$

$$\frac{1}{\pi\,376.703}\int_0^{\pi}\left(\frac{\sin(2.243\cos\theta)}{\cos\theta}\right)^2 \sin^3\theta\, d\theta = 4.218\times10^{-3}\text{ S}$$

$$\boxed{\frac{1}{\pi\,376.703}\int_0^{\pi}\left(\frac{\sin(1.3203\cos\theta)}{\cos\theta}\right)^2 \sin^3\theta\, d\theta = 1.757\times10^{-3}\text{ S}}$$

b) The mutual conductance G_{12} was found using which definite integral? (circle correct answer)

$G_{12} = \dfrac{1}{\pi\eta_0}\displaystyle\int_{\theta=0}^{\pi}\left[\frac{\sin\left(\frac{k_0 W}{2}\cos\theta\right)}{\cos\theta}\right]^2 J_0\left(k_0 L\sin\theta\right)\sin^3\theta\, d\theta$ where $k_0L = 37.724(0.05946) = 2.243$.

$$\frac{1}{\pi\,376.703}\int_0^{\pi}\left(\frac{\sin(1.1215\cos\theta)}{\cos\theta}\right)^2 J_0(2.641\sin\theta)\sin^3\theta\, d\theta = 3.629\times10^{-5}\text{ S}$$

$$\boxed{\frac{1}{\pi\,376.703}\int_0^{\pi}\left(\frac{\sin(1.3203\cos\theta)}{\cos\theta}\right)^2 J_0(2.243\sin\theta)\sin^3\theta\, d\theta = 3.745\times10^{-4}\text{ S}}$$

$$\frac{1}{\pi\,376.703/\sqrt{1.83}}\int_0^{\pi}\left(\frac{\sin(1.3203\cos\theta)}{\cos\theta}\right)^2 J_0(2.243\sin\theta)\sin^3\theta\, d\theta = 5.067\times10^{-4}\text{ S}$$

$$\frac{1}{\pi\,376.703}\int_0^{\pi}\left(\frac{\sin(2.243\cos\theta)}{\cos\theta}\right)^2 J_0(1.3203\sin\theta)\sin^3\theta\, d\theta = 2.277\times10^{-3}\text{ S}$$

c) Assuming the imaginary portions of the slot admittances cancel, find the input impedance Z_{in} at the edge of the rectangular microstrip antenna.

$$Z_{in} = R_{in} = \frac{1}{2\left(G_1 + G_{12}\right)} = \frac{1}{2\left(1.757\times10^{-3} + 3.745\times10^{-4}\right)}$$

Z_{in} = **<u>234.58 Ω</u>**

d) Given the following definite integrals (circle one used), estimate the maximum single-slot directivity D_0 (unitless & dBi) as well as the maximum directivity D_{max} (unitless & dBi) of the antenna.

$$\text{Notes- } I_1 = \int_{\theta=0}^{\pi} \left[\frac{\sin\left(\frac{k_0 W}{2}\cos\theta\right)}{\cos\theta} \right]^2 \sin^3\theta \, d\theta \text{ where } k_0W/2 = 1.3203.$$

$$\boxed{\int_0^{\pi} \left(\frac{\sin(1.3203\cos\theta)}{\cos\theta} \right)^2 \sin^3\theta \, d\theta = 2.0794}$$

$$\int_0^{\pi} \left(\frac{\sin(1.215\cos\theta)}{\cos\theta} \right)^2 \sin^3\theta \, d\theta = 1.546$$

$$\int_0^{\pi} \left(\frac{\sin(2.243\cos\theta)}{\cos\theta} \right)^2 J_0(1.3203\sin\theta)\sin^3\theta \, d\theta = 3.2832$$

$$\int_0^{\pi} \left(\frac{\sin(2.243\cos\theta)}{\cos\theta} \right)^2 \sin^3\theta \, d\theta = 4.9924$$

$$\int_0^{\pi} \left(\frac{\sin(1.3203\cos\theta)}{\cos\theta} \right)^2 J_0(2.243\sin\theta)\sin^3\theta \, d\theta = 0.4433$$

$$\lambda_0 = \frac{c}{f} = \frac{2.998\times10^8}{1.8\times10^9} = 0.166555555 \text{ m}$$

$$\text{Notes- one slot } D_0 = \left(\frac{2\pi W}{\lambda_0}\right)^2 \frac{1}{I_1} = \left(\frac{2\pi(0.07)}{0.166555555}\right)^2 \frac{1}{2.0794} \Rightarrow \underline{\boldsymbol{D_0 = 3.3535 = 5.255 \text{ dBi}}}.$$

$$\text{Two slots } D_{max}^{tot} = D_0^{tot} = D_0\left(\frac{2}{1+G_{12}/G_1}\right) = 3.3535\left(\frac{2}{1+0.3745/1.757}\right)$$

$$\Rightarrow \underline{\boldsymbol{D_{max} = 5.5268 = 7.426 \text{ dBi}}}.$$

D_0 = **<u>3.3535 = 5.255 dBi</u>** D_{max} = **<u>5.5268 = 7.426 dBi</u>**

e) Estimate the E-plane and H-plane half-power beamwidths (degrees) for this antenna.

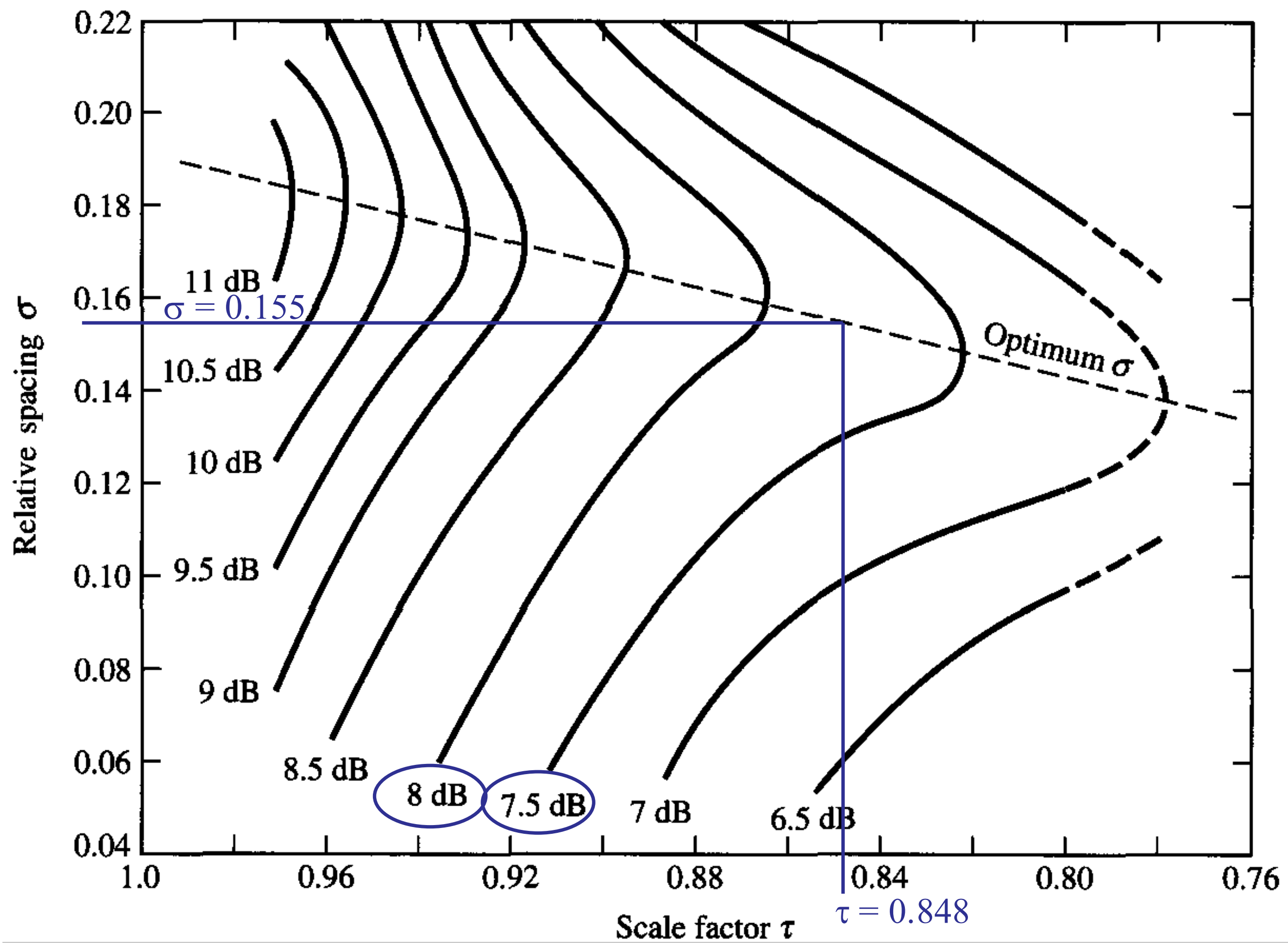
$$\text{Per (14-58), } \Theta_E \simeq 2\sin^{-1}\sqrt{\frac{7.03\lambda_0^2}{4\pi^2\left(3L_{eff}^2 + h^2\right)}} \simeq 2\sin^{-1}\sqrt{\frac{7.03(0.166555555)^2}{4\pi^2\left(3(0.062971)^2 + (0.003175)^2\right)}}$$

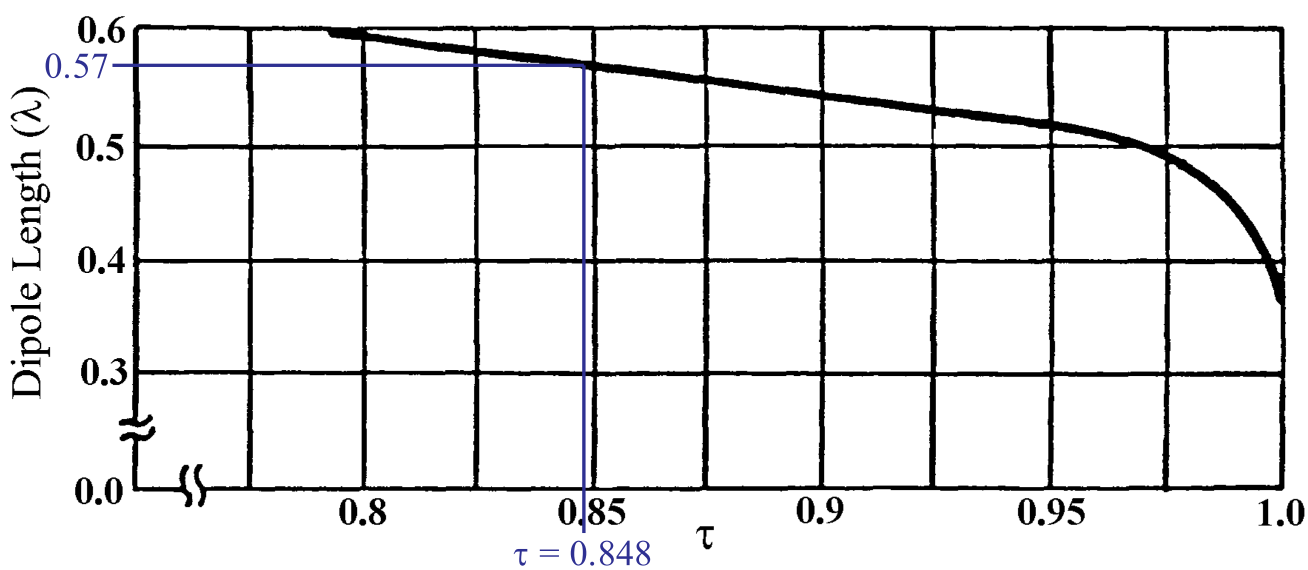
$$\Rightarrow \underline{\textbf{HPBW}_\textbf{E} \textbf{ = 80.2°}}.$$

$$\text{Per (14-59), } \Theta_H \simeq 2\sin^{-1}\sqrt{\frac{1}{2+k_0W}} \simeq 2\sin^{-1}\sqrt{\frac{1}{2+37.724(0.07)}} \Rightarrow \underline{\textbf{HPBW}_\textbf{H} \textbf{ = 55.3°}}.$$

$HPBW_E$ = **<u>80.2°</u>** $HPBW_H$ = **<u>55.3°</u>**

4) Design an optimum LPDA to have a 450 - 750 MHz frequency range and 7.8 dBi directivity. The booms are to be made with 2.25 cm (OD) aluminum pipe. Find the relative spacing, scale factor, apex angle (deg), length l_1 (cm) & location R_1 of the longest element, total bandwidth, and estimated integer number of elements N_{est} & length L_{est} (cm) of the LPDA. Show all work on given figures.





$\alpha = \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right) = \tan^{-1}\left(\frac{1-0.848}{4(0.155)}\right) = 13.775°$ ⇒ apex angle **2α = 27.55°**

$$\lambda_{\max} = \frac{c}{f_{low}} = \frac{2.998\times10^{8}}{450\times10^{6}} = 0.6662222222 \text{ m} = 66.6222222 \text{ cm}$$

Using bottom fig. on prior page & τ, $l_1 = 0.57\lambda_{\max} = 0.57(66.6222)$

⇒ **l_1= 37.975 cm**

$R_1 = 0.5\, l_1 \cot(\alpha) = 0.5\,(37.975)\cot(13.775°)$ ⇒ **R_1= 77.448 cm**

specified bandwidth (BW)- $B = f_{high} / f_{low} = 750/450 = 1.66666667$

active region BW- $B_{ar} = 1.1 + 7.7\,(1 - \tau)^2 \cot(\alpha) = 1.1 + 7.7\,(1 - 0.848)^2 \cot(13.775°)$

Total BW- $B_S = B_{ar}\, B = (1.82565)\, 1.66667$ ⇒ **B_S = 3.04275**

Est. # elements- $N_{est} = 1 + \log(B_S)/\log(1/\tau) = 1 + \log(3.04275)/\log(1/0.848) = 7.74$

⇒ **N_{est} = 8**

Est. length- $L_{est} = 0.5\, l_1\,(1 - 1/B_S)\cot(\alpha) = 0.5(37.975)\,(1 - 1/3.04275)\cot(13.775°)$

⇒ **L_{est} = 51.995 cm**

σ = **0.155** τ = **0.848** apex angle = **2α = 27.55°** l_1 = **37.975 cm**

R_1 = **77.45 cm** total BW = **B_S = 3.04275** N_{est} = _**8**_ L_{est} = **52.00 cm**