

**EE 483/583 Examination #2 (Spring 2026)**Name KEY

Instructions: Place answers in indicated spaces, use notation as given in class for coordinates & vectors, and show all work for credit. Insert equation sheets inside front page and hand-in with exam.  $c = 2.998 \times 10^8$  m/s.

- 1) A lead-tin dipole antenna ( $\sigma = 4 \times 10^6$  S/m, length 50 cm, diameter 1.6 mm) is located and centered on the  $z$ -axis. It operates at 30 MHz in air. Calculate length of this antenna as a fraction of a wavelength  $\ell/\lambda$ . Is this antenna infinitesimal, small, finite, or half-wavelength? Find the radiation resistance, loss resistance, and radiation efficiency (%) for the dipole.

Wavelength is  $\lambda = c/f = 2.998 \times 10^8 / 30 \times 10^6 \Rightarrow \underline{\lambda = 9.993333 \text{ m}}$ .

$\ell/\lambda = 0.5 / 9.993333 \Rightarrow \underline{\ell/\lambda = 0.050033 \sim 1/20}$  (small dipole!)

Per (4-37),  $R_r = \eta \frac{\pi}{6} \left( \frac{\ell}{\lambda} \right)^2 = 376.7303 \frac{\pi}{6} (0.050033)^2 \Rightarrow \underline{R_r = 0.493797 \Omega}$

Per (2-90b),

$$R_{hf} = \frac{\ell}{P} \sqrt{\frac{\omega \mu_0}{2\sigma}} = \frac{\ell}{\pi d} \sqrt{\frac{\omega \mu_0}{2\sigma}} = \frac{0.5}{\pi(0.0016)} \sqrt{\frac{2\pi(30 \times 10^6)4\pi \times 10^{-7}}{2(4 \times 10^6)}} \Rightarrow \underline{R_{hf} = 0.541266 \Omega}$$

From notes, for a triangular current distribution  $R_L = \frac{R_{hf}}{3} = \frac{0.541266}{3} \Rightarrow \underline{R_L = 0.18042 \Omega}$

Per (2-90)  $e_{cd} = R_r / (R_r + R_L) = 0.493797 / (0.493797 + 0.180422) * 100\% \Rightarrow \underline{e_{cd} = 73.24 \%}$

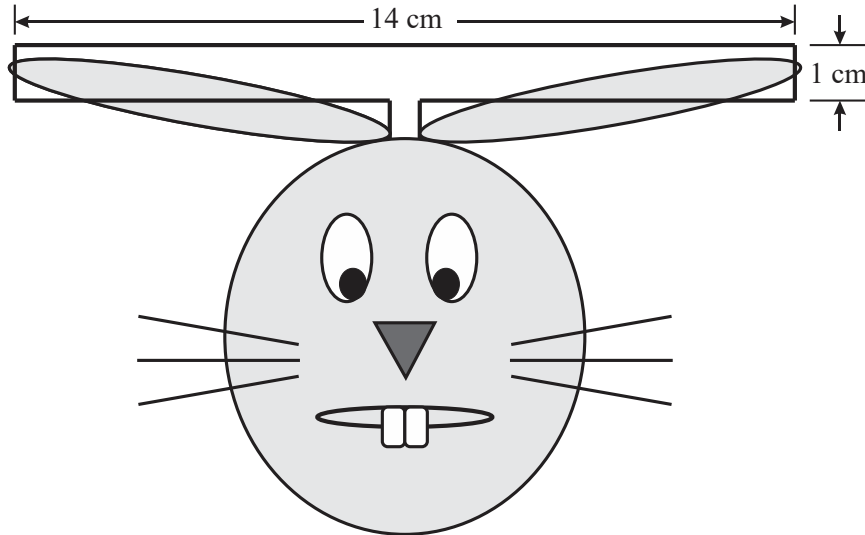
$\ell/\lambda = \underline{0.050033}$  infinitesimal, small, finite, or half-wavelength? (Circle best/correct)

$R_r = \underline{0.4938 \Omega}$

$R_L = \underline{0.1804 \Omega}$

rad. effic. =  $\underline{e_{cd} = 73.24 \%}$

- 2) Peter Rabbit has an ear-mounted folded dipole (see below) made of 4 mm diameter wires cast using a magical chocolate-gold alloy. If Peter uses his ear-mounted folded dipole at 980 MHz to communicate Mr. McGregor's location to his sisters Flopsy, Mopsy and Cotton-tail, find the transmission line (TL) characteristic impedance and TL mode input impedance. Then, calculate the effective radius of the antenna mode dipole (mm) and select the correct antenna mode input impedance  $Z_A$ . Last, find the overall estimated & exact input impedance of the antenna. Assume free space conditions.



$$\lambda = c/f = 2.998 \times 10^8 / 980 \times 10^6 \Rightarrow \lambda = 0.30592 \text{ m. } k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.30592} = 20.53876 \text{ rad/m.}$$

$$Z_0 = \frac{\eta}{\pi} \cosh^{-1} \left( \frac{s}{2a} \right) = \frac{376.7303}{\pi} \cosh^{-1} \left( \frac{0.01}{0.004} \right) \Rightarrow \underline{Z_0 = 187.8858 \Omega.}$$

$$Z_t = jZ_0 \tan \left( \frac{kl}{2} \right) = j187.8858 \tan \left( \frac{20.53876(0.14)}{2} \right) \Rightarrow \underline{Z_t = j1403.4517 \Omega.}$$

$$a_e = \sqrt{as} = \sqrt{\left( \frac{0.004}{2} \right) 0.01} = 0.00447214 \text{ m} \Rightarrow \underline{a_e = 4.472 \text{ mm}} \Rightarrow \text{choose correct } Z_A$$

$$Z_{in} = \frac{4Z_t Z_A}{2Z_A + Z_t} = \frac{4(j1403.45)(78.346 + j8.266)}{2(78.346 + j8.266) + j1403.45} \Rightarrow \underline{Z_{in} = 302.447 + j66.053 \Omega.}$$

$$Z_{in,est.} = 4 Z_A = 4(78.346 + j8.266) \Rightarrow \underline{Z_{in,est.} = 313.384 + j33.064 \Omega.}$$

TL char. imp. =  $Z_0 = 187.886 \Omega$  TL mode input imp. =  $Z_t = j1403.45 \Omega$  eff. rad. =  $a_e = 4.472 \text{ mm}$

**Circle** one:  $Z_A$  (radius = 10 mm) = 65.444 - j26.836  $\Omega$        $Z_A$  (radius = 5.477 mm) = 71.482 - j16.987  $\Omega$

**$Z_A$  (radius = 4.472 mm) = 78.346 + j8.266  $\Omega$**        $Z_A$  (radius = 3.873 mm) = 75.456 + j7.581  $\Omega$

$$Z_{in,est.} = \underline{313.384 + j33.064 \Omega} \qquad Z_{in} = \underline{302.447 + j66.053 \Omega}$$

- 3) A Yagi-Uda antenna has an input impedance of  $Z_A = 14 + j22 \Omega$  at 500 MHz. We need to match it to a lossless transmission line ( $Z_0 = 50 \Omega$ ,  $u = 2.4 \times 10^8$  m/s) using a discrete **inductor** connected in *series* as close to antenna as possible. As part of the solution process, calculate the normalized input impedance for the antenna. Then, find the normalized  $z_M$  & un-normalized  $Z_M$  impedances of the appropriate match point and its distance  $d_M$  (in  $\lambda$  & cm) from the antenna. Calculate the required inductance  $L$  for the series inductor. Show, with labels, all work done on Smith chart. Draw a fully labeled sketch of final match circuit design.

The wavelength is  $\lambda = c/f = 2.4 \times 10^8 / 500 \times 10^6 = 0.48$  m = 48 cm.

### Steps

- 1) Calculate the normalized impedance  $z_A = Z_A/Z_0 = (14 + j22)/50 \Rightarrow \underline{z_A = 0.28 + j0.44 \Omega/\Omega}$  and plot on **Smith chart**.
- 2) Draw circle, centered on Smith chart, through  $z_A$  point. This circle of constant  $|\Gamma|$  includes the locus of all possible  $z_{in}$  (and  $y_{in}$ ) along the transmission line with this load.
- 3) The two match points are  $z_{m,i} = 1 \pm j1.6 \Omega/\Omega$ . To use a discrete series inductor for matching, select  $\underline{z_{m,2} = z_M = 1 - j1.6 \Omega/\Omega}$  as it has a capacitive reactance. Note, the match point impedance is  $Z_M = z_{m,2} Z_0 = (1 - j1.6) 50 \Rightarrow \underline{Z_M = 50 - j80 \Omega}$ .
- 4) Find the distance  $d_2 = d_M$  from  $z_A$  to  $z_{m,2}$  using scales on Smith chart,  $d_M = (0.3216 - 0.07) \lambda \Rightarrow \underline{d_M = 0.2516\lambda}$  or  $d_2 = 0.2516(48$  cm)  $\Rightarrow \underline{d_M = 12.0768$  cm.
- 5) At match point, add a discrete inductor in series with reactance  $Z_{ind} = j\omega L = j80 \Omega$ . Solving for  $L$  yields  $L = 80/(2\pi 500 \times 10^6) = 2.5465 \times 10^{-8}$  H  $\Rightarrow \underline{L = 25.465$  nH.
- 6) Everywhere toward the source from the location of  $L$  will be matched, i.e.,  $Z_{in} = 500 \Omega$ .

$$z_A = \underline{0.28 + j0.44 \Omega/\Omega}$$

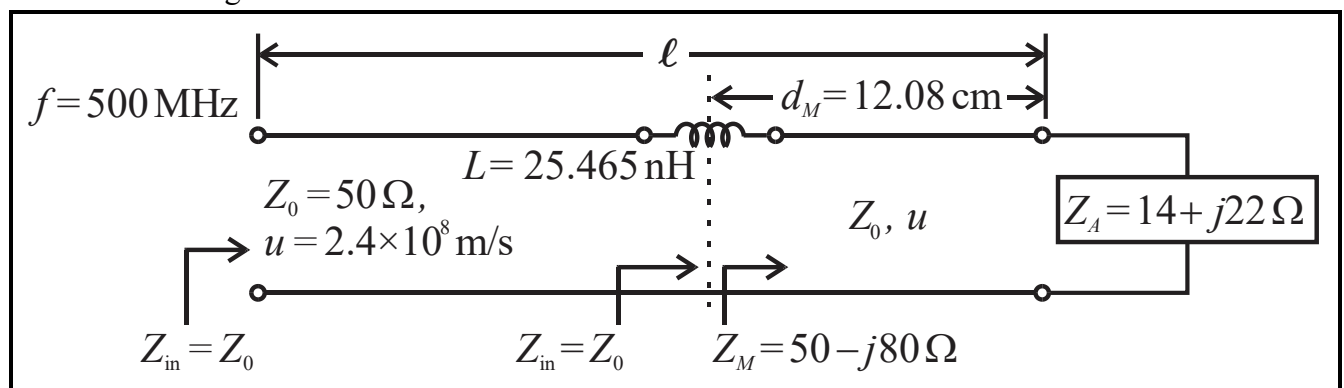
$$z_M = \underline{1 - j1.6 \Omega/\Omega}$$

$$Z_M = \underline{50 - j80 \Omega}$$

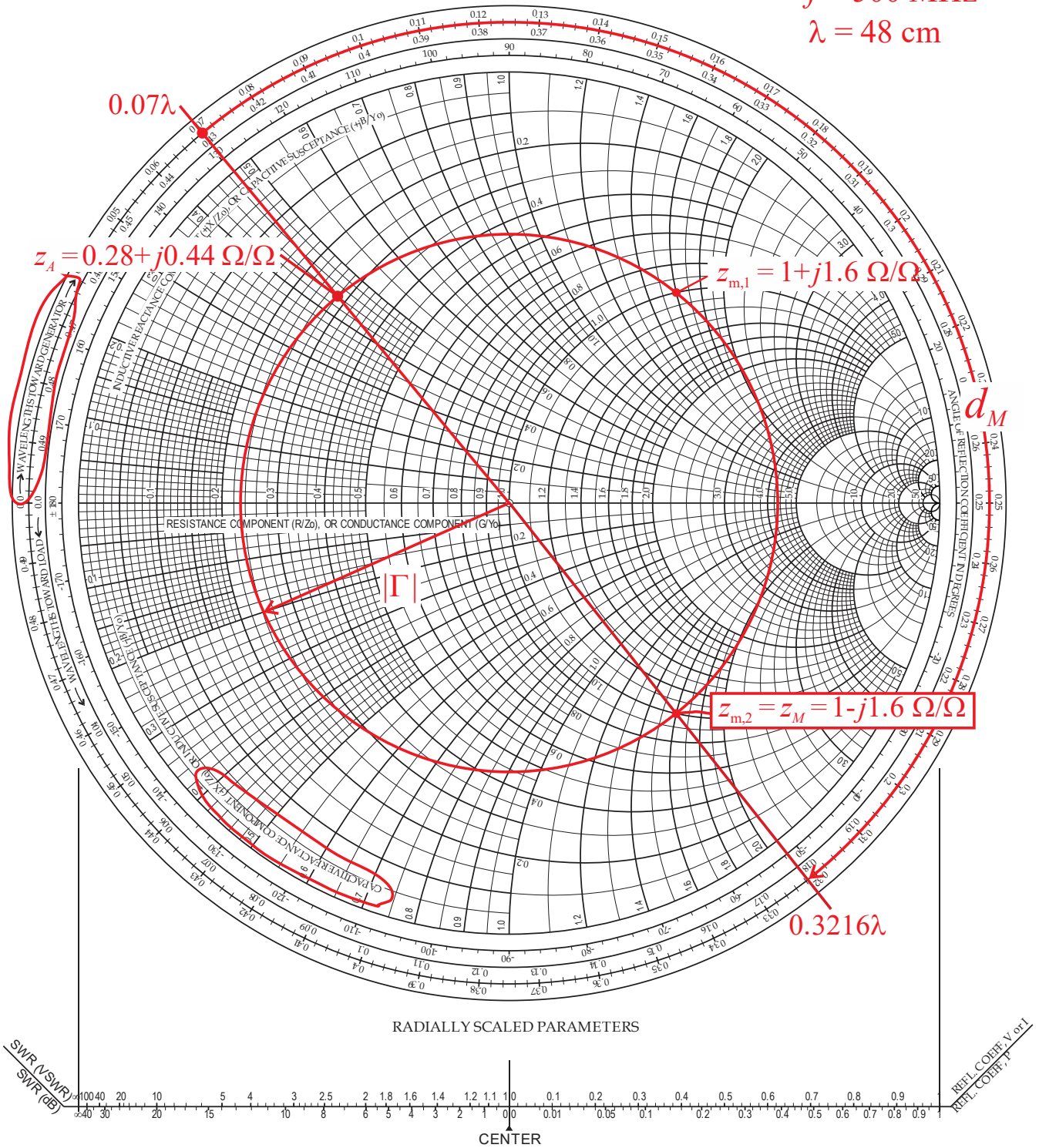
$$d_M = \underline{0.2516\lambda = 12.08$$
 cm

$$L = \underline{25.465$$
 nH

final circuit design



$Z_0 = 50 \Omega$   
 $f = 500 \text{ MHz}$   
 $\lambda = 48 \text{ cm}$



4) A Yagi-Uda antenna is to be designed for a center frequency of 900 MHz, input impedance of  $75 \Omega$ , and a gain greater than 14 dBi. Due to parts availability, 1.266 cm diameter copper pipe is used for the boom and 0.333 cm diameter brass pipe is used for the elements.

a) In terms of the number of elements, what is the shortest standard design that will work? (Also,  on table.) What is the expected gain (in dBi and dBd)?

$$14 \text{ dBi} - 2.15 \text{ dB} = 11.85 \text{ dBd} \Rightarrow \text{From Table 10.6, use 12 element design} \\ (12.25 \text{ dBd} = 14.4 \text{ dBi})$$

$$\underline{\underline{12}} \text{ elements} \quad \text{Gain} = \underline{\underline{12.25 \text{ dBd}}} = \underline{\underline{14.4 \text{ dBi}}}$$

b) What is the center-to-center spacing between the reflector and driven element (in  $\lambda$  & cm)?

$$\text{From Table 10.6, } s_{12} = 0.2\lambda = 0.2 (2.998 \times 10^8 / 900 \times 10^6) = 0.2 (0.33311) = 6.662 \text{ cm}$$

$$\text{spacing} = s_{12} = \underline{\underline{0.2\lambda}} = \underline{\underline{6.662 \text{ cm}}}$$

c) What is the center-to-center spacing between the first director and driven element as well as between the directors (in  $\lambda$  & cm)?

$$\text{From Table 10.6, } s_{ij} = 0.2\lambda = 0.2 (0.33311) = 6.662 \text{ cm}$$

$$\text{spacing} = s_{ij} = \underline{\underline{0.2\lambda}} = \underline{\underline{6.662 \text{ cm}}}$$

d) Excluding the boom, how long will be the antenna measured from the **outside** edges of the reflector and last director (in cm)?

From Table 10.6, the antenna length is  $2.2\lambda$ . Adding the radius of the reflector and radius of the last director then gives

$$\text{outside edge-to-edge length} = 2.2 (33.31111) + 0.333 = 73.617 \text{ cm}$$

$$\text{antenna length} = \underline{\underline{73.617 \text{ cm}}}$$

e) For the boom and elements given, can this antenna design be completed using the given tables &/or graphs? Why or why not? Justify answer.

$$d/\lambda = 0.333/33.3111 \Rightarrow d/\lambda = 0.00999666 \approx \mathbf{0.01}. \text{ Inside } 0.001 \leq d/\lambda \leq 0.04 \text{ range.}$$

$$D/\lambda = 1.266/33.3111 \Rightarrow D/\lambda = \mathbf{0.038}. \text{ Inside } 0.001 \leq D/\lambda \leq 0.04 \text{ range.}$$

/ No (circle correct) Why or why not? **Inside  $0.001 \leq d/\lambda$  &  $D/\lambda \leq 0.04$  ranges**

f) Find the length (in  $\lambda$  & cm) of the first director **after** all corrections are made (show & label all work on design figures).

From Figure 10.25, plot vertical line at  $d/\lambda = 0.01$ . Draw horizontal line where it intersects curve C. Read off  $l_3' = 0.4295 \lambda$ .

From Figure 10.26, plot vertical line at  $D/\lambda = 0.038$ . Draw horizontal line where it intersects curve. Read off  $\Delta l = 0.0282 \lambda$ .

$$\text{We get } l_3'' = (0.4295 + 0.0282) \lambda \Rightarrow l_3'' = \mathbf{0.4577 \lambda} = 0.4577(33.31111) = \mathbf{15.2465 \text{ cm.}}$$

$$\text{first director length} = \underline{\underline{0.4577 \lambda}} = \underline{\underline{15.2465 \text{ cm}}}$$

**Table 10.6** OPTIMIZED UNCOMPENSATED LENGTHS OF PARASITIC ELEMENTS FOR YAGI-UDA ANTENNAS OF SIX DIFFERENT LENGTHS

$d/\lambda = 0.0085$ $s_{12} = 0.2\lambda$		LENGTH OF YAGI-UDA (IN WAVELENGTHS)					
		0.4	0.8	1.20	2.2	3.2	4.2
LENGTH OF REFLECTOR ( $l_1/\lambda$ )		0.482	0.482	0.482	0.482	0.482	0.475
LENGTH OF DIRECTORS, $\lambda$	$l_3$	0.442	0.428	0.428	0.432	0.428	0.424
	$l_4$		0.424	0.420	0.415	0.420	0.424
	$l_5$		0.428	0.420	0.407	0.407	0.420
	$l_6$			0.428	0.398	0.398	0.407
	$l_7$				0.390	0.394	0.403
	$l_8$				0.390	0.390	0.398
	$l_9$				0.390	0.386	0.394
	$l_{10}$				0.390	0.386	0.390
	$l_{11}$				0.398	0.386	0.390
	$l_{12}$				0.407	0.386	0.390
	$l_{13}$					0.386	0.390
	$l_{14}$					0.386	0.390
	$l_{15}$					0.386	0.390
	$l_{16}$					0.386	
$l_{17}$					0.386		
SPACING BETWEEN DIRECTORS ( $s_{ij}/\lambda$ )		0.20	0.20	0.25	0.20	0.20	0.308
DIRECTIVITY RELATIVE TO HALF-WAVE DIPOLE (dBd)		7.1	9.2	10.2	12.25	13.4	14.2
DESIGN CURVE (SEE FIGURE 10.25)		(A)	(B)	(B)	(C)	(B)	(D)

SOURCE: Peter P. Vezbicke, *Yagi Antenna Design*, NBS Technical Note 688, December 1976.

