

EE 483/583 Antennas for Wireless Communications Quiz #6 (Spring 2025)

Name KeyInstructions: Open notes. Place answers in indicated spaces and **show/indicate all work** for credit.

A Yagi-Uda antenna, operating at $\lambda_c = 50$ cm, is to be driven using a T-match. The driven element has a diameter of 0.8 cm and length of 23 cm. The T-match has a diameter of 0.4 cm, length of 10 cm, and spacing of 2.6 cm. Calculate the characteristic impedance of transmission line mode of the T-match as well as the input impedance of transmission line mode. Next, calculate the current divisor factor and equivalent radius (cm) of the T-match section for the antenna mode. Then, find the overall input impedance (rectangular format) if the impedance of the antenna mode was found to be $13.6 + j3.1 \Omega$.

$$Z_0 = \frac{1}{2\pi} \cosh^{-1} \left[\frac{s^2 - a^2 - a'^2}{2aa'} \right] \quad \begin{array}{l} s = 2.6 \text{ cm} \\ a = 0.4 \text{ cm} \\ a' = 0.2 \text{ cm} \end{array} \quad l' = 10 \text{ cm}$$

$$= \frac{376.7303}{2\pi} \cosh^{-1} \left[\frac{2.6^2 - 0.4^2 - 0.2^2}{2(0.4)(0.2)} \right]$$

$$\underline{Z_0 = 264.2111 \Omega}$$

$$Z_t = jZ_0 \tan\left(\frac{kl'}{2}\right) = j264.2111 \tan\left(\frac{2\pi}{0.5} \frac{0.1}{2}\right)$$

$$\underline{Z_t = j191.9606 \Omega}$$

$$u = \frac{a}{a'} = \frac{0.4}{0.2} = 2 \quad v = \frac{s}{a'} = \frac{2.6}{0.2} = 13$$

$$\alpha = \frac{\cosh^{-1}\left(\frac{v^2 - u^2 + 1}{2v}\right)}{\cosh^{-1}\left(\frac{v^2 + u^2 - 1}{2vu}\right)} = \frac{\cosh^{-1}\left(\frac{13^2 - 2^2 + 1}{2(13)}\right)}{\cosh^{-1}\left(\frac{13^2 + 2^2 - 1}{2(13)2}\right)}$$

$$= \frac{\cosh^{-1}(6.38461)}{\cosh^{-1}(3.30769)} = \underline{1.36186}$$

$$\begin{aligned}
 a_e &= a' e^{\frac{1}{(1+u)^2} (u^2 \ln u + 2u \ln v)} \\
 &= 0.2 e^{\frac{1}{(1+2)^2} (2^2 \ln 2 + 2(2) \ln 13)} \\
 &= 0.2 e^{1.448} = \underline{0.85096 \text{ cm}}
 \end{aligned}$$

$$\begin{aligned}
 Y_{in} &= \frac{Y_t}{2} + \frac{Y_a}{(1+\alpha)^2} = \frac{j191.96}{2} + \frac{13.6 + j3.1}{(1+1.36186)^2} \\
 &= 0.0125301 - j0.0054608 \text{ S}
 \end{aligned}$$

$$\begin{aligned}
 Z_{in} &= \frac{1}{0.01253 - j0.00546} \\
 &= \underline{67.0689 + j29.2297 \Omega}
 \end{aligned}$$

char. imp. of TL mode = 264.21 Ω

input imp. of TL mode = j191.96 Ω

current divisor factor = 1.362

equiv. radius of ant. mode = 0.851 cm

$Z_{in} =$ 67.07 + j29.23 Ω