EE 483/583 Antennas for Wireless Communications
Examination #3 (Spring 2025)

Name KEY

Instructions: Place answers in indicated spaces. Use notation as given in class for coordinates & variables. **Show/label all** work for credit. Insert equation sheets. Where applicable, let $c = 2.998 \times 10^8$ **m/s**.

1) An 8-turn circular loop antenna is operated at 105 MHz in free space. The loop diameter is 1.1 cm. The loop is made with 18 AWG silver wire (1.02 mm diameter, $\sigma_{Ag} = 6.1 \times 10^7$ S/m) and the loop spacing is 1.428 mm. Is this loop antenna electrically small, electrically large, or neither? Find the radiation & ohmic loss resistances, radiation efficiency (%), and gain (dimensionless & dBi) of the antenna.

$C = \pi D = \pi(1.1) = 3.45575$ cm. $\lambda = c/f = 2.998\times10^8/105\times10^6 = 2.855$ m $= 285.5238$ cm.

$NC/\lambda = 8(3.45575)/285.5238 = 0.0968 < 1/10 \Rightarrow$ **small loop**.

Wavenumber $k = 2\pi/\lambda = 2\pi/2.8552386 = 2.200582$ rad/m

$ka = (2.2006)(0.011/2) = 0.0121032$

For 1-turn (5-24), $R_{r,1} = \eta(\pi/6)(ka)^4 = 376.7303(\pi/6)(0.0121032)^4 = 4.23282\times10^{-6}\,\Omega$

Per (5-24a), $R_r = R_{r,1} N^2 = 4.23282\times10^{-6}(8)^2 \Rightarrow$ $\mathbf{R_r = 2.70901\times10^{-4}\,\Omega = 0.2709\,m\Omega}$

Wire diameter $2b = 1.02$ mm & wire spacing $2c = 1.428$ mm $\Rightarrow 2c/2b = 1.428/1.02 = 1.4$.

From Fig. 5.3, $\mathbf{R_P/R_0 = 0.976}$ for $c/b = 1.4$ and $N = 8$

Per (5-25),
$$R_L = N\frac{\ell}{p}\sqrt{\frac{\omega\mu_0}{2\sigma}}\left(\frac{R_p}{R_0}+1\right) = N\frac{2\pi a}{2\pi b}\sqrt{\frac{\omega\mu_0}{2\sigma}}\left(\frac{R_p}{R_0}+1\right)$$
$$= 8\frac{11}{1.02}\sqrt{\frac{2\pi(105\bullet10^6)4\pi\bullet10^{-7}}{2(6.1\bullet10^7)}}(0.976+1)$$
$\Rightarrow$ $\mathbf{R_L = 0.444405\,\Omega}$

Per (2-90) $$e_{cd} = \frac{R_r}{R_L+R_r}100\% = \frac{2.70901\bullet10^{-3}(100\%)}{0.444405+2.70901\bullet10^{-3}}$$ $\Rightarrow$ $\mathbf{e_{cd} = 0.060921\ \%}$

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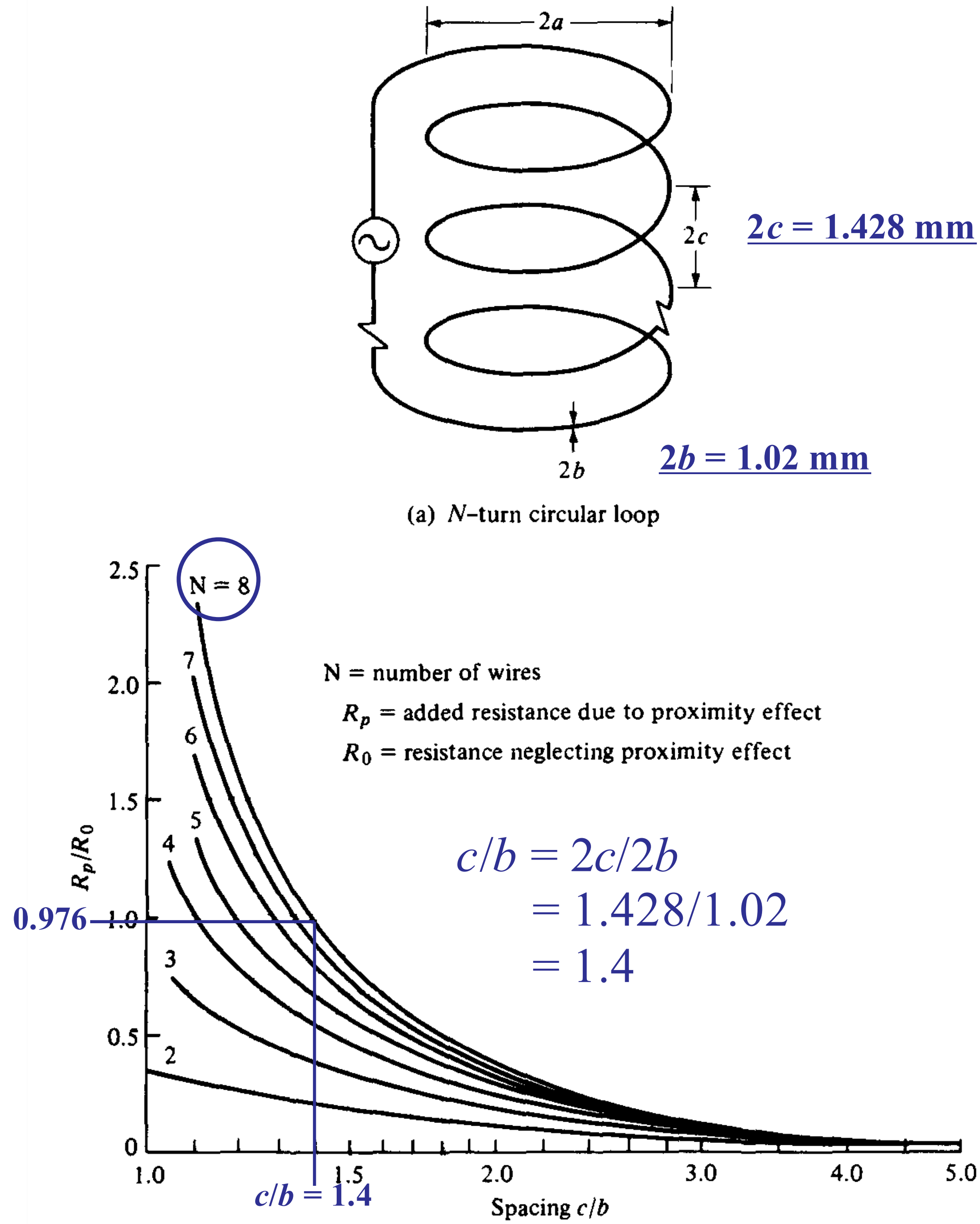
Per (5.31), $D_{max} = 3/2$. $G = e_{cd} D_{max} = 0.00060921(1.5) \Rightarrow$ $\mathbf{G = 0.000913815 = -30.39\ dBi}$

Electrically **small**, large, or neither? (Circle correct) Why? $\mathbf{NC/\lambda = 0.0968 < 1/10}$

Radiation resistance = $\mathbf{R_r = 0.2709\ m\Omega}$ ohmic loss resistance = $\mathbf{R_L = 0.444405\,\Omega}$

Rad. efficiency = $\mathbf{e_{cd} = 0.060921\ \%}$ gain = $\mathbf{G = 0.0009138 = -30.39\ dBi}$



(b) Ohmic resistance due to proximity (after G. N. Smith)

Figure 5.3 *N*-turn circular loop and ohmic resistance due to proximity effect. (SOURCE: G. S. Smith, "Radiation Efficiency of Electrically Small Multiturn Loop Antennas," *IEEE Trans. Antennas Propagat.*, Vol. AP-20, No. 5, pp. 656-657, Sept. 1972© (1972) IEEE).

2) A rectangular microstrip antenna is designed to operate at a frequency of 3 GHz on a high frequency PCB (ε_r = 2.94, h = 0.762 mm). The antenna has L_{eff} = 2.9675 cm, L = 2.8919 cm, W = 3.56 cm, G_1 = 1.3012 mS, and G_{12} = 0.5664 mS. The antenna is fed by a 50 Ω microstrip transmission line.

a) What is the guided wavelength in the patch? **λ = 5.9349 cm**

$L_{eff} = \lambda/2 \Rightarrow \lambda = 2L_{eff} = 2(2.9675) \Rightarrow$ **λ = 5.9349 cm**

b) What is the fringing length at each end of the patch? **ΔL = 0.0378 cm = 0.378 mm**

$L = L_{eff} - 2\Delta L \Rightarrow \Delta L = (L_{eff} - L)/2 = (2.9675-2.8919)/2 = 0.0378 \Rightarrow$ **ΔL = 0.378 mm**

For parts *c*) & *d*) below, $k_0 W/2 = (\omega/c)W/2 = (2\pi 3\times10^9/2.998\times10^8)0.0356/2 = 0.1192$ and $k_0 L = (\omega/c)L = (2\pi 3\times10^9/2.998\times10^8)0.028919 = 1.8182$

c) The slot conductance G_1 was found using which definite integral? (circle correct answer)

$$\frac{1}{\pi 376.703}\int_0^{\pi}\left(\frac{\sin(1.8845\cos\theta)}{\cos\theta}\right)^2 \sin^3\theta\, d\theta \qquad \frac{1}{\pi 376.703/\sqrt{2.94}}\int_0^{\pi}\left(\frac{\sin(1.1192\cos\theta)}{\cos\theta}\right)^2 \sin^3\theta\, d\theta$$

$$\boxed{\frac{1}{\pi 376.703}\int_0^{\pi}\left(\frac{\sin(1.1192\cos\theta)}{\cos\theta}\right)^2 \sin^3\theta\, d\theta} \qquad \frac{1}{\pi 376.703/\sqrt{2.94}}\int_0^{\pi}\left(\frac{\sin(1.8845\cos\theta)}{\cos\theta}\right)^2 \sin^3\theta\, d\theta$$

d) The mutual conductance G_{12} was found using which definite integral? (circle correct answer)

$$\frac{1}{\pi 376.703/\sqrt{2.94}}\int_0^{\pi}\left(\frac{\sin(1.1192\cos\theta)}{\cos\theta}\right)^2 J_0(1.818\sin\theta)\sin^3\theta\, d\theta$$

$$\boxed{\frac{1}{\pi 376.703}\int_0^{\pi}\left(\frac{\sin(1.1192\cos\theta)}{\cos\theta}\right)^2 J_0(1.818\sin\theta)\sin^3\theta\, d\theta}$$

$$\frac{1}{\pi 376.703}\int_0^{\pi}\left(\frac{\sin(1.8845\cos\theta)}{\cos\theta}\right)^2 J_0(3.062\sin\theta)\sin^3\theta\, d\theta$$

$$\frac{1}{\pi 376.703}\int_0^{\pi}\left(\frac{\sin(1.1192\cos\theta)}{\cos\theta}\right)^2 J_0(1.866\sin\theta)\sin^3\theta\, d\theta$$

e) Assuming the imaginary portions of the slot admittances cancel, find the input impedance Z_{in} at the edge of the rectangular microstrip antenna.

$$Z_{in} = R_{in} = \frac{1}{2(G_1 + G_{12})} = \frac{1}{2(0.0013012 + 0.0005664)} \quad \Rightarrow \quad \underline{\mathbf{Z_{in} = 267.723\,\Omega}}$$

Z_{in} = **267.723 Ω**

f) Given the following definite integrals (circle one used), estimate the maximum single-slot directivity D_0 (unitless & dBi) as well as the maximum directivity D_{max} (unitless & dBi) of the antenna.

$$\int_0^{\pi}\left(\frac{\sin(1.8845\cos\theta)}{\cos\theta}\right)^2 J_0(3.062\sin\theta)\sin^3\theta\, d\theta = -0.6517$$

$$\boxed{\int_0^{\pi}\left(\frac{\sin(1.1192\cos\theta)}{\cos\theta}\right)^2 \sin^3\theta\, d\theta = 1.5401}$$

$$\int_0^{\pi}\left(\frac{\sin(1.1192\cos\theta)}{\cos\theta}\right)^2 J_0(1.818\sin\theta)\sin^3\theta\, d\theta = 0.6706$$

$$\int_0^{\pi}\left(\frac{\sin(1.8845\cos\theta)}{\cos\theta}\right)^2 \sin^3\theta\, d\theta = 3.8122$$

$$\int_0^{\pi}\left(\frac{\sin(1.1192\cos\theta)}{\cos\theta}\right)^2 J_0(1.866\sin\theta)\sin^3\theta\, d\theta = 0.6330$$

$I_1 = 1.5401$. So, for a single slot, $D_0 = \left(\frac{2\pi W}{\lambda_0}\right)^2 \frac{1}{I_1} = \left(\frac{2\pi 0.0356}{2.998\bullet 10^8 / 3\bullet 10^9}\right)^2 \frac{1}{1.5401}$

$\Rightarrow$ **D_0 = 3.253 = 5.123 dBi**

For antenna, $D_{max} = D_0\left(\frac{2}{1 + G_{12}/G_1}\right) = 3.253\left(\frac{2}{1 + 0.5664/1.3012}\right)$

$\Rightarrow$ **D_{max} = 4.533 = 6.564 dBi**

D_0 = **3.253 = 5.123 dBi** D_{max} = **4.533 = 6.564 dBi**

g) Estimate the E-plane and H-plane half-power beamwidths (degrees) for this antenna.

$$HPBW_E = 2\sin^{-1}\sqrt{\frac{7.03\lambda_0^2}{4\pi^2(3L_{eff}^2 + h^2)}} = 2\sin^{-1}\sqrt{\frac{7.03(2.998\bullet 10^8 / 3\bullet 10^9)^2}{4\pi^2[3(0.029675)^2 + 0.000762^2)}}$$

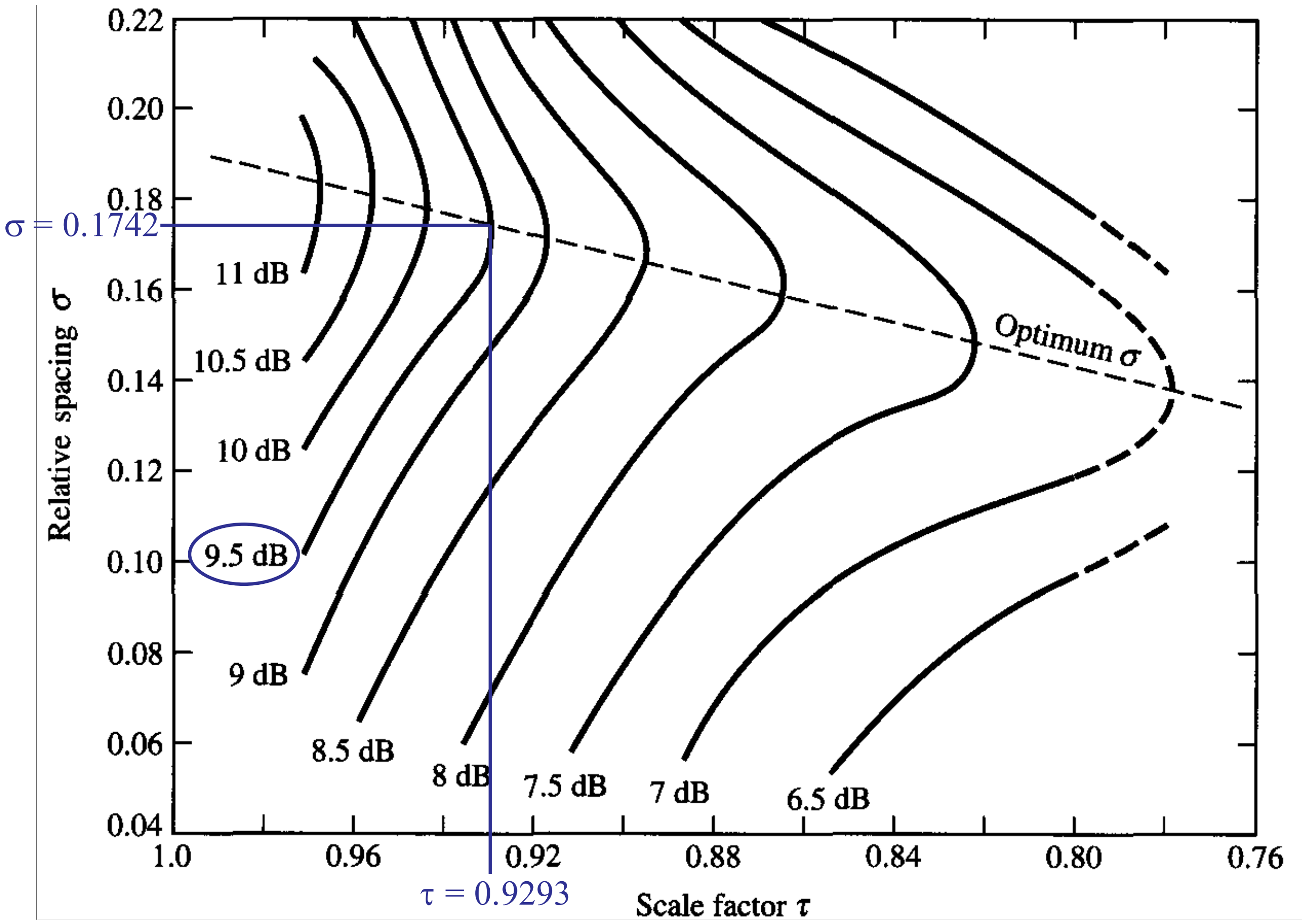
$\Rightarrow$ **$HPBW_E$ = 110.245°**

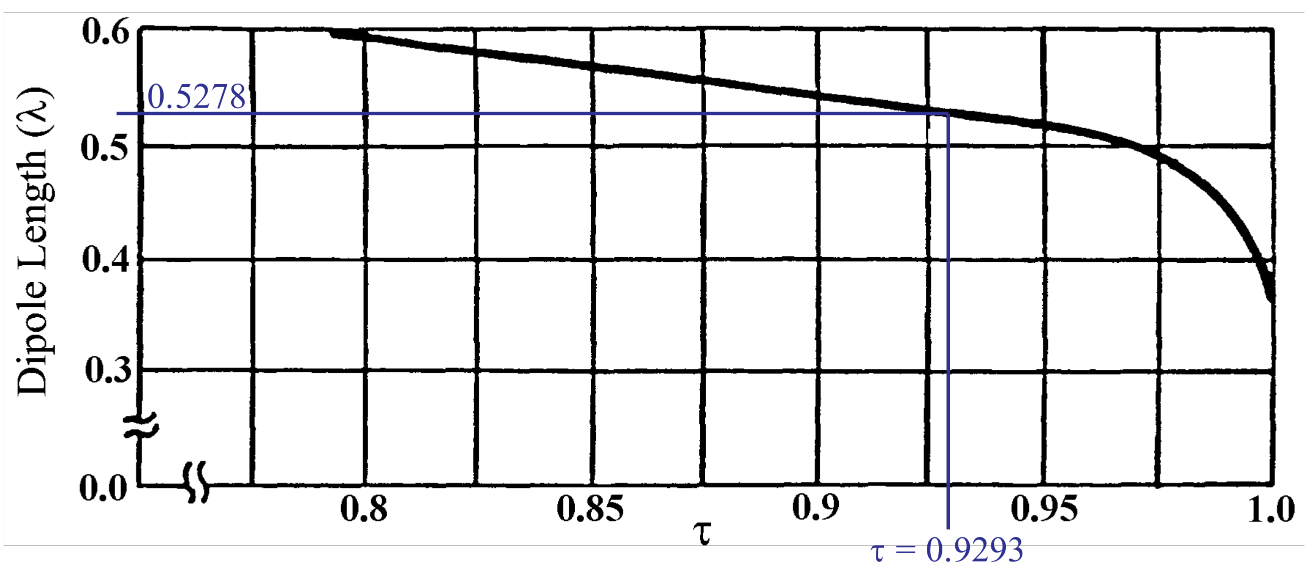
$$HPBW_H = 2\sin^{-1}\sqrt{\frac{1}{2 + k_0 W}} = 2\sin^{-1}\sqrt{\frac{1}{2 + (2\pi 3\bullet 10^9 / 2.998\bullet 10^8)0.0356}}$$

$\Rightarrow$ **$HPBW_H$ = 58.122°**

$HPBW_E$ = **110.245°** $HPBW_H$ = **58.122°**

3) Design an optimum LPDA to have an 800 - 1200 MHz frequency range and 9.5 dBi directivity. The booms are to be made with 5/8" (OD) copper pipe. Find the relative spacing, scale factor, apex angle (deg), length l_1 (cm) & location R_1 of the longest element, total bandwidth, and estimated integer number of elements N_{est} & length L_{est} (cm) of the LPDA. Show all work on given figures.





$\alpha = \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right) = \tan^{-1}\left(\frac{1-0.9293}{4(0.1742)}\right) = 5.7936°$ ⇒ apex angle **$2\alpha = 11.5872°$**

$$\lambda_{max} = \frac{c}{f_{low}} = \frac{2.998 \bullet 10^8}{800 \bullet 10^6} = 0.37475 \text{ m} = 37.475 \text{ cm}$$

Using bottom fig. on prior page & τ, $l_1 = 0.5278\lambda_{max} = 0.5278(37.475)$ ⇒ **$l_1 = 19.779$ cm**

$R_1 = 0.5\, l_1 \cot(\alpha) = 0.5\,(19.7793)\cot(5.7936°)$ ⇒ **$R_1 = 97.4701$ cm**

specified bandwidth (BW)- $B = f_{high} / f_{low} = 1200/800 = 1.5$

active region BW- $B_{ar} = 1.1 + 7.7\,(1 - \tau)^2 \cot(\alpha) = 1.1 + 7.7\,(1 - 0.9293)^2 \cot(5.7936°)$
⇒ $B_{ar} = 1.47933$

Total BW- $B_S = B_{ar}\, B = (1.47933)\, 1.5$ ⇒ **$B_S = 2.219$**

Estimated # elements- $N_{est} = 1 + \log(B_S)/\log(1/\tau) = 1 + \log(2.219)/\log(1/0.9293) = 11.87$
⇒ **$N_{est} = 12$**

Estimated length- $L_{est} = 0.5\, l_1\,(1 - 1/B_S)\cot(\alpha) = 0.5(19.7793)\,(1 - 1/2.219)\cot(5.7936°)$
⇒ **$L_{est} = 53.545$ cm**

σ = **0.1742** τ = **0.9293** apex angle = **$2\alpha = 11.587°$** l_1 = **19.78 cm**

R_1 = **97.47 cm** total BW = **$B_S = 2.219$** N_{est} = **12** L_{est} = **53.545 cm**