EE 483/583 Examination #1 (Spring 2025)

Name KEY

Instructions: Place answers in indicated spaces, use notation as given in class for coordinates & vectors, and show all work for credit. Attach equation sheet and hand-in with exam.

1) The vector electric potential for an antenna is found to be

$$\bar{F} = \hat{a}_r \frac{F_0 \cos(ka\sin\theta)}{10ka\sin\theta}\frac{e^{-jkr}}{r^2} + \hat{a}_\theta F_0 \cos\theta\cos\phi \frac{e^{-jkr}}{r}\left(2k^2 + \frac{jk}{r} - \frac{1}{r^2}\right) - \hat{a}_\phi F_0 \sin\phi \frac{e^{-jkr}}{r}\left(k^2 + \frac{jk}{r}\right) \text{ (C/m)}.$$

Find the phasor **far-zone** electric and magnetic fields *with all common terms factored out*. Assume the vector magnetic potential $\bar{A} = 0$. For extra credit, evaluate k, η, and ω if the antenna is operated in free space at 800 MHz.

Drop all terms $\propto 1/r^2, 1/r^3, 1/r^4, \ldots$ for far-zone

$$\bar{F}_{FF} = \hat{a}_\theta F_0 \cos\theta\cos\phi \frac{e^{-jkr}}{r}\left(2k^2\right) - \hat{a}_\phi F_0 \sin\phi \frac{e^{-jkr}}{r}\left(k^2\right) \text{ (C/m)}.$$

Per (3-59a), the far-zone magnetic field components are

$$H_r \simeq 0,$$

$$H_\theta \simeq -j\omega F_\theta = -j\omega 2k^2 F_0 \cos\theta\cos\phi \frac{e^{-jkr}}{r} \text{ (A/m), and}$$

$$H_\phi \simeq -j\omega F_\phi = j\omega k^2 F_0 \sin\phi \frac{e^{-jkr}}{r} \text{ (A/m)}.$$

Per (3-59b), the far-zone electric field components are

$$E_r \simeq 0,$$

$$E_\theta \simeq \eta H_\phi = j\omega\eta k^2 F_0 \sin\phi \frac{e^{-jkr}}{r} \text{ (V/m), and}$$

$$E_\phi \simeq -\eta H_\theta = j\omega\eta 2k^2 F_0 \cos\theta\cos\phi \frac{e^{-jkr}}{r} \text{ (V/m)}.$$

$\omega = 2\pi f = 2\pi(800\times10^6)$ $\Rightarrow$ $\underline{\omega = 5.02655\times10^9 \text{ rad/s}}$

Free space, $k = \omega/c = 2\pi f/c = 2\pi(800\times10^6)/2.9979\times10^8$ $\Rightarrow$ $\underline{k = 16.767 \text{ rad/m}}$

Free space, $\underline{\eta = \eta_0 = 376.7303\ \Omega}$

$$\bar{E}_{FF} = \underline{j\omega\eta k^2 F_0 \frac{e^{-jkr}}{r}\left(\hat{a}_\theta \sin\phi + \hat{a}_\phi 2\cos\theta\cos\phi\right) \quad \text{(V/m)}}$$

$$\bar{H}_{FF} = \underline{-j\omega k^2 F_0 \frac{e^{-jkr}}{r}\left(\hat{a}_\theta 2\cos\theta\cos\phi - \hat{a}_\phi \sin\phi\right) \quad \text{(A/m)}}$$

Extra credit: k = $\underline{16.767 \text{ rad/m}}$ η = $\underline{376.7303\ \Omega}$ ω = $\underline{5.02655\times10^9 \text{ rad/s}}$

2) A bistatic radar system is used to track a drone with an RCS of 0.09 m^2. The radar system operates at 12 GHz and uses identical antennas which are well-matched to the feeding transmission lines. If the transmitting and receiving antennas are 2 km and 3 km, respectively, from the target, find the gain of each antenna $G_{t/r}$ (unitless & dBi) if the receiver picks up 32 pW when both are aligned for best reception (i.e., max gain & polarization matched) given a transmitter output power of 600 kW. If the minimum power required for target detection is 12 pW, how far can the transmitter be from the drone if the receiving antenna location remains 3 km from the target? Assume $c = 2.9979 \times 10^8$ m/s.

Use Radar Range equation (2-125)

$$\frac{P_r}{P_t} = e_{cd,r}\, e_{cd,t}\, [1-|\Gamma_r|^2][1-|\Gamma_t|^2]\, \sigma \frac{D_r D_t}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 |\hat{\rho}_w \bullet \hat{\rho}_r|^2$$

where

$e_{cd} D_t = e_{cd} D_t = G$, $|\Gamma_t| = |\Gamma_t| = 0$ (well matched), $R_t = R_1 = 2000$ m,

$R_r = R_2 = 3000$ m, $P_r = 32\times10^{-12}$ W, $P_t = 600\times10^3$ W, $\sigma = 0.09$ m^2, and

$|\hat{\rho}_w \bullet \hat{\rho}_r|^2 = PLF = 1$ (polarization-matched).

The wavelength $\lambda = c / f = 2.9979\times10^8 / 12\times10^9 = 0.0249825$ m. This gives

$$\frac{32\times10^{-12}}{600\times10^3} = [1-0][1-0]\, 0.09 \frac{G^2}{4\pi} \left(\frac{0.0249825}{4\pi(2000)3000} \right)^2 1 \;\Rightarrow\; \underline{\boldsymbol{G = 260.44034 = 24.157\ \text{dBi}}}.$$

If the receive power is changed to $P_r = 12\times10^{-12}$ W, the gain G just computed is used, and we solve for $R_{t,\max} = R_1$, we get

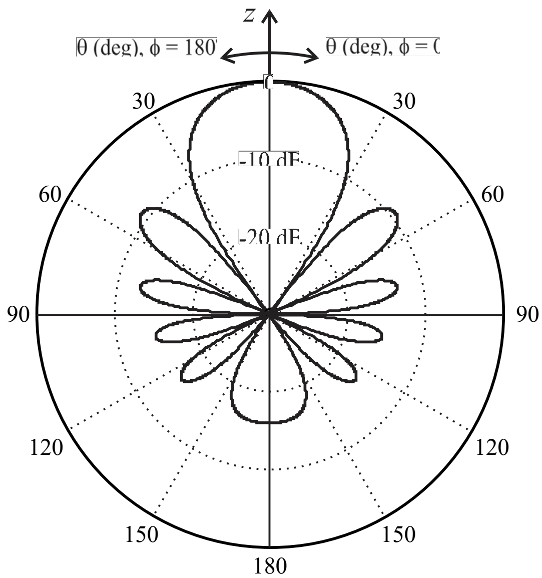
$$\frac{12\times10^{-12}}{600\times10^3} = [1-0][1-0]\, 0.09 \frac{260.440344^2}{4\pi} \left(\frac{0.0249825}{4\pi R_1 3000} \right)^2 1$$

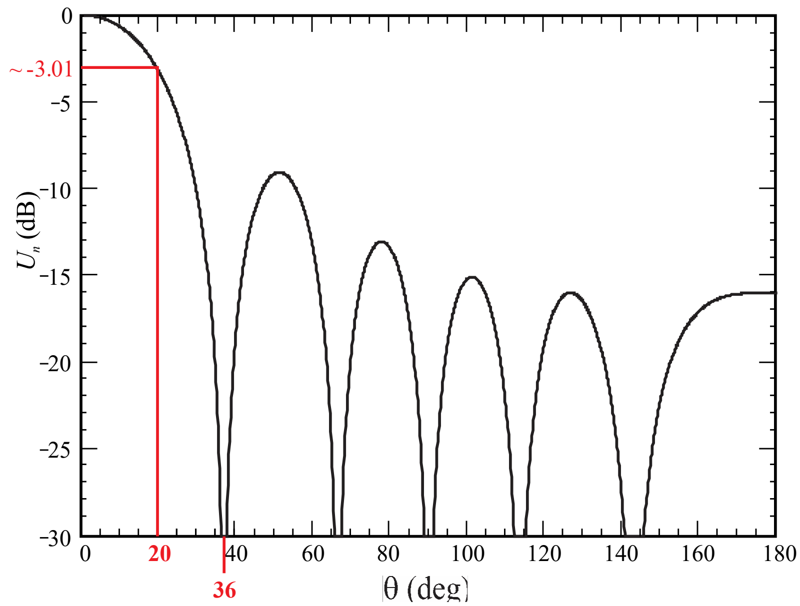
$$R_1 = R_{t,\max} = \sqrt{[1-0][1-0]\, 0.09 \frac{260.440344^2}{4\pi} \left(\frac{0.0249825}{4\pi R_1 3000} \right)^2 1 \left(\frac{600\times10^3}{12\times10^{-12}} \right)}$$

$$\Rightarrow \underline{\boldsymbol{R_{t,\max} = R_1 = 3265.986\ \text{m}}}.$$

$G_{t/r}$ = **260.44034** = **24.157 dBi** $\qquad R_{t,\max}$ = **3265.986 m**

3) The figure shows 2D cuts of the rotationally symmetric (i.e., independent of ϕ) normalized radiation pattern for an endfire antenna array. Find the half-power beamwidth (deg & rad) and first-null beamwidth (deg & rad). If the array is 90% efficient, estimate the beam solid angle (sr), maximum directivity (unitless & dB), and maximum gain (unitless & dB) using the Kraus approximation.





Half power occurs when U (dB) = 10log$_{10}$(0.5) = -3.01 dB. From the right hand graph, $\theta_h \cong 20°$. Therefore, $\theta_{1r} = \theta_{2r} = \text{HPBW} \cong 2(20)$ $\Rightarrow$ **HPBW = 40° = 0.698 rad**.

A null occurs when U (dB) = 10log$_{10}$(0) → -∞. From the right hand graph, $\theta_n \cong 36°$. Therefore, FNBW ≅ 2(36) $\Rightarrow$ **FNBW = 72° = 1.257 rad**.

Use (2-26a) to estimate the beam solid angle

$$\Omega_A \cong \theta_{1r}\,\theta_{2r} = [40(\pi/180)]^2 \quad \Rightarrow \quad \Omega_A \cong 0.4874 \text{ sr.}$$

Use (2-26) to estimate the maximum directivity

$$D_0 = D_{\max} \cong 4\pi / \Omega_A = 4\pi / 0.4874 \quad \Rightarrow \quad D_0 = D_{\max} \cong 25.783 = 14.11 \text{ dBi.}$$

Use (2-49a) to estimate the maximum gain

$$G_0 = G_{\max} = e_{cd} D_0 = 0.9\,(25.783) \quad \Rightarrow \quad G_0 = G_{\max} \cong 23.205 = 13.66 \text{ dBi.}$$

HPBW = **40°** = **0.698 rad** FNBW = **72°** = **1.257 rad** Ω_A = **0.4874 sr**

$D_{\max}$ = **25.783** = **14.11 dBi** $G_{\max}$ = **23.205** = **13.66 dBi**

4) In free space, an antenna has the far-zone electric field $\bar{E} = \hat{a}_\theta \dfrac{800\cos\theta}{4\pi r} e^{-jkr}$ (V/m) for $0 \le \theta \le 90°$ & $0 \le \phi \le 360°$, and is zero elsewhere. Find the radiation intensity $U(\theta, \phi)$ (W/sr) and maximum radiation intensity U_{max} (W/sr). Calculate the time-average power P_{rad} radiated by the antenna. Then, find the directivity $D(\theta, \phi)$ (unitless) and maximum directivity D_{max} (unitless & dBi).

Use (2-12a) to get radiation intensity

$$U(\theta,\phi) = \frac{r^2}{2\eta}|\bar{E}|^2 = \frac{r^2}{2\eta}\bar{E}\bullet\bar{E}^* = \frac{r^2}{2\eta_0}\frac{800^2\cos^2\theta}{(4\pi r)^2}\left(e^{-jkr}e^{+jkr}\right) = \frac{800^2\cos^2\theta}{2(376.7303)(4\pi)^2}$$

$\Rightarrow$ **$\underline{U(\theta) = 5.378977\cos^2\theta \text{ (W/sr)}}$** $0 \le \theta \le 90°$ & $0 \le \phi \le 360°$, 0 elsewhere

For the allowed range of angles, $\Rightarrow$ **$\underline{U_{max} = 5.378977 \text{ (W/sr)}}$** @ $\theta = 0°$ & $0 \le \phi \le 360°$.

Use (2-13) to get radiated power

$$P_{rad} = \oiint_\Omega U\, d\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} 5.37898\cos^2\theta\sin\theta\, d\theta\, d\phi$$

$$= 5.37898\int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2}\cos^2\theta\sin\theta\, d\theta = 5.37898(2\pi - 0)\left(\frac{-\cos^3\theta}{3}\right)\Bigg|_{\theta=0}^{\pi/2}$$

$$= 5.37898(2\pi)\left(0 + \frac{1}{3}\right) \qquad \Rightarrow \quad \underline{P_{rad} = 11.2657 \text{ W}}$$

Use (2-16) to get directivity

$$D = \frac{4\pi U}{P_{rad}} = \frac{4\pi(5.37898\cos^2\theta)}{11.2657}$$

$\Rightarrow$ **$\underline{D(\theta) = 6\cos^2\theta}$** $0 \le \theta \le 90°$ & $0 \le \phi \le 360°$, 0 elsewhere

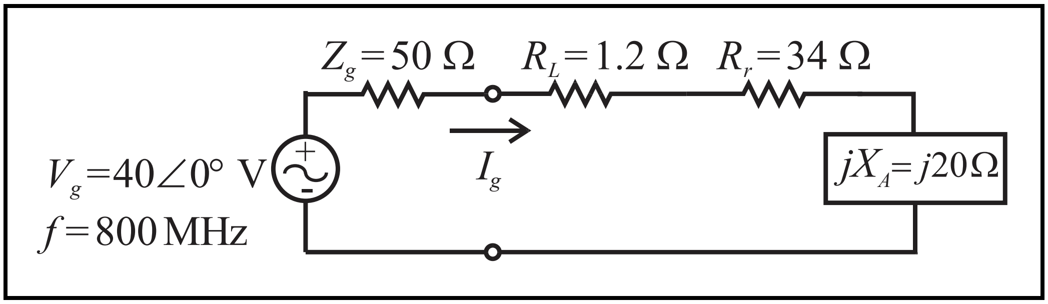
For the allowed range of angles, $\Rightarrow$ **$\underline{D_{max} = 6}$** @ $\theta = 0°$ & $0 \le \phi \le 360°$.

$D_{max} = 10\log_{10}(6)$ $\Rightarrow$ **$\underline{D_{max} = 7.7815 \text{ dBi}}$** @ $\theta = 0°$ & $0 \le \phi \le 360°$.

$U(\theta) =$ **$5.37898\cos^2\theta$ (W/sr)** $0 \le \theta \le 90°$ & $0 \le \phi \le 360°$; **0** elsewhere	$D(\theta) =$ **$6\cos^2\theta$** $0 \le \theta \le 90°$ & $0 \le \phi \le 360°$; **0** elsewhere

U_{max} = **$\underline{5.37898 \text{ (W/sr)}}$** P_{rad} = **$\underline{11.2657 \text{ (W)}}$** D_{max} = **$\underline{6}$** = **$\underline{7.7815 \text{ dBi}}$**

5) A quarter-wave monopole has an impedance $Z_{\lambda/4} = 35.2 + j20\ \Omega$ (includes a loss resistance of $1.2\ \Omega$) when operated at 800 MHz. The monopole is driven by a source with Thevenin equivalent resistance of $50\ \Omega$ and voltage of $40\angle 0°$ V. Sketch the equivalent circuit in the box. Then, find the radiation resistance, time-average dissipated & radiated powers for the antenna, and antenna radiation efficiency (%).



To get the radiation resistance, subtract the loss resistance from the overall antenna resistance, i.e., $R_{\text{rad}} = R_A - R_L = 35.2 - 1.2 \quad \Rightarrow$ $\underline{\boldsymbol{R_{\text{rad}} = 34\ \Omega}}$.

By circuit theory, the phasor current from the source is

$$I_g = \frac{V_g}{Z_g + Z_{\lambda/4}} = \frac{40\angle 0°}{50 + (35.2 + j20)} = 0.45706\angle -13.21°\ \text{A}.$$

By circuit theory, the power dissipated is

$$P_{\text{diss}} = P_{\text{loss}} = 0.5|I_g|^2 R_L = = 0.5|0.45706|^2\, 1.2 \quad \Rightarrow \underline{\boldsymbol{P_{\text{diss}} = 0.125342\ \text{W}}}.$$

By circuit theory, the power radiated is

$$P_{\text{rad}} = 0.5|I_g|^2 R_r = = 0.5|0.45706|^2\, 1.2 \quad \Rightarrow \underline{\boldsymbol{P_{\text{rad}} = 3.550433\ \text{W}}}$$

Per (2-90), the radiation efficiency is

$$e_{cd} = \frac{R_r}{R_r + R_L} = \frac{34}{34 + 1.2} \Rightarrow \underline{\boldsymbol{e_{cd} = 0.96591 = 96.591\%}}.$$

$R_{\text{rad}} = \underline{\mathbf{34\ \Omega}}$ $\quad P_{\text{diss}} = \underline{\mathbf{0.125342\ W}}$ $\quad P_{\text{rad}} = \underline{\mathbf{3.550433\ W}}$ $\quad$ rad. effic. $= \underline{\boldsymbol{e_{cd} = 96.591\%}}$