EE 483/583 Antennas for Wireless Communications
Quiz #6 (Spring 2024)

Name Key

Instructions: Open book/notes. Put answers in indicated spaces & show all work for credit. $c = 2.998 \times 10^8$ m/s.

A 2.38 m long, thin, center-fed, linear dipole is located along the z-axis, centered on the origin, in free space. When operated at 44 MHz, calculate the length of the dipole as a fraction of a wavelength (ℓ/λ) and the wavenumber k. Is this dipole considered infinitesimal, finite, half-wavelength, or small? If the antenna is driven by a phasor input current of $0.24\angle 0^\circ$ A, find the far-field vector electric and magnetic fields with all know quantities used. Then, find the vector radiated time-average power density.

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8}{44 \times 10^6} = 6.81\overline{36}\text{ m}$$

$$\frac{\ell}{\lambda} = \frac{2.38}{6.81\overline{36}} = \underline{0.3493}$$

Too long to be small or infinitesimal. Too short for $\lambda/2$ $\Rightarrow$ finite length dipole

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6.81\overline{36}} = \underline{0.922148611 \ \tfrac{\text{rad}}{\text{m}}}$$

Far-field

(4-62a) $$E_\theta \simeq j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right]$$

$$\frac{k\ell}{2} = \frac{0.92215(2.38)}{2} = 1.097356847 \text{ rad}$$

$$\cos\left(\tfrac{k\ell}{2}\right) = \cos(1.09736) = 0.45595$$

$$\eta = \eta_0 = 376.7303\,\Omega \quad \& \quad I_0 = 0.24\angle 0^\circ \text{ A}$$

$$\bar{E} = \hat{a}_\theta\, j\,\frac{376.7303(0.24\angle 0^\circ)}{2\pi}\,\frac{e^{-j0.922r}}{r}\left[\frac{\cos(1.097\cos\theta) - 0.456}{\sin\theta}\right]$$

$$\bar{E} = \hat{a}_\theta \, j \, 14.3900375 \, \frac{e^{-j0.922r}}{r} \left[\frac{\cos(1.097\cos\theta) - 0.456}{\sin\theta} \right] \left(\frac{V}{m}\right)$$

$r > 0, \; 0 \le \phi < 2\pi \; + \; 0 \le \theta < \pi$

(4-62b) $H_\phi = \frac{E_\theta}{\eta}$

$$\bar{H} = \hat{a}_\phi \, j \, \underbrace{\frac{14.3900375}{376.7303}}_{0.0381972} \, \frac{e^{-j0.922r}}{r} \left[\frac{\cos(1.097\cos\theta) - 0.456}{\sin\theta} \right] \left(\frac{A}{m}\right)$$

(4-63) $\bar{W}_{ave} = \bar{W}_{rad} = \hat{a}_r \frac{1}{2\eta} |E_\theta|^2$

$$|E_\theta|^2 = \bar{E} \cdot \bar{E}^* = (j)(-j) \, 14.39^2 \, \frac{e^0}{r^2} \left[\frac{\cos(1.097\cos\theta) - 0.456}{\sin\theta} \right]^2$$

$$= \frac{207.07318}{r^2} [\quad]^2$$

$$\bar{W}_{rad} = \frac{207.07318}{2(376.7303) r^2} [\quad]^2 \hat{a}_r = \hat{a}_r \frac{0.27483}{r^2} \left[\frac{\cos(1.097\cos\theta) - 0.456}{\sin\theta} \right]^2 \left(\frac{W}{m^2}\right)$$

$k =$ $0.92215 \frac{rad}{m}$

infinitesimal, finite, half-wavelength, or small? (circle correct)

$\ell/\lambda =$ 0.3493

$\bar{E}_{FF} =$ $\hat{a}_\theta \, j \, 14.39 \, \frac{e^{-j0.92215r}}{r} \left[\frac{\cos(1.09736\cos\theta) - 0.456}{\sin\theta} \right] \frac{V}{m}$

$\bar{H}_{FF} =$ $\hat{a}_\phi \, j \, 38.197 \, \frac{e^{-j0.92215r}}{r} \left[\frac{\cos(1.09736\cos\theta) - 0.456}{\sin\theta} \right] \frac{mA}{m}$

$\bar{W}_{rad} =$ $\hat{a}_r \, \frac{0.27483}{r^2} \left[\frac{\cos(1.09736\cos\theta) - 0.456}{\sin\theta} \right]^2 \frac{W}{m^2}$

$r > 0, \; 0 \le \phi < 2\pi, \; + \; 0 \le \theta < \pi$