EE 483/583 Antennas for Wireless Communications
Quiz #4 (Spring 2024)

Name Key

Instructions: Open book & notes. Place answers in indicated spaces and show all work for credit. Per Wikipedia, the speed of light in vacuum is 299,792,458 m/s, $\varepsilon_0 = 8.8541878 \times 10^{-12}$ F/m, & $\mu_0 = 4\pi \times 10^{-7}$ H/m.

An antenna, operating at 663 MHz in free space, has a phasor vector electric potential of $\bar{F} = \hat{a}_z 12 \frac{e^{-jkr}}{r}\left(1 - \frac{jk}{r}\right)$ (nC/m). Calculate the propagation constant (AKA wave number) and intrinsic impedance. Then, find the far-field phasor vector electric and magnetic fields in **spherical coordinates**.

$$K = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{2\pi(663\times10^6)}{299{,}792{,}458} = \underline{13.8954525 \text{ rad/m}}$$

$$\eta = \sqrt{\mu/\varepsilon} = \sqrt{\frac{4\pi\times10^{-7}}{8.8542\times10^{-12}}} = \underline{376.730314\ \Omega}$$

Convert $\hat{a}_z$ to spherical coordinates

$$\bar{F} = (\cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta)\,12\times10^{-9}\frac{e^{-jkr}}{r}\left(1 - \frac{jk}{r}\right)\ (\text{C/m})$$

For far-field, ignore $\hat{a}_r$ component + $-\frac{jk}{r}$ term

$$\bar{F}_{FF} = -\hat{a}_\theta\, 12\times10^{-9}\sin\theta\,\frac{e^{-jkr}}{r}$$

(3-59a) $H_r \simeq 0$, $H_\phi = -j\omega \cancel{F_\phi}^{0} = 0$

$$H_\theta \simeq -j\omega F_\theta = +j\omega\, 12\times10^{-9}\sin\theta\,\frac{e^{-jkr}}{r}$$

$$= +j(2\pi)663\times10^6\, 12\times10^{-9}\sin\theta\,\frac{e^{-jkr}}{r} = \underline{j\,49.989\sin\theta\,\frac{e^{-jkr}}{r}}$$

(3-59b) $E_r \simeq 0$, $E_\theta \simeq -j\omega\eta \cancel{F_\phi}^{0} = 0$

$$E_\phi = j\omega\eta F_\theta = -\eta H_\theta = -j\,376.73(49.989)\sin\theta\,\frac{e^{-jkr}}{r}$$

$$= -j\,18{,}832.38\sin\theta\, e^{-jkr}/r$$

$k =$ 13.8955 rad/m $\qquad \eta =$ 376.73 Ω

$\bar{E}_{FF} =$ $-\hat{a}_\phi\, j\,18{,}832.4\sin\theta\,\frac{e^{-j13.8955r}}{r}$ (V/m)

$\bar{H}_{FF} =$ $\hat{a}_\theta\, j\,49.989\sin\theta\,\frac{e^{-j13.8955r}}{r}$ (A/m)

Cartesian Coordinates $\Leftrightarrow$ Spherical Coordinates

$$(x, y, z) \Leftrightarrow (r, \theta, \phi)$$

Point / variable conversions:

$$r = \sqrt{x^2 + y^2 + z^2} \qquad x = r\sin\theta\cos\phi \qquad \cos\phi = \frac{x}{\sqrt{x^2+y^2}} \qquad \cos\theta = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\phi = \tan^{-1}\frac{y}{x} \qquad y = r\sin\theta\sin\phi$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} \qquad z = r\cos\theta \qquad \sin\phi = \frac{y}{\sqrt{x^2+y^2}} \qquad \sin\theta = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$$

Vector conversions:

$$\overline{A} = \hat{a}_r A_r + A_\theta \hat{a}_\theta + \hat{a}_\phi A_\phi = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

$$\hat{a}_r = \sin\theta\cos\phi\ \hat{a}_x + \sin\theta\sin\phi\ \hat{a}_y + \cos\theta\ \hat{a}_z$$
$$\hat{a}_\theta = \cos\theta\cos\phi\ \hat{a}_x + \cos\theta\sin\phi\ \hat{a}_y - \sin\theta\ \hat{a}_z$$
$$\hat{a}_\phi = -\sin\phi\ \hat{a}_x + \cos\phi\ \hat{a}_y$$

$$\hat{a}_x = \sin\theta\cos\phi\ \hat{a}_r + \cos\theta\cos\phi\ \hat{a}_\theta - \sin\phi\ \hat{a}_\phi$$
$$\hat{a}_y = \sin\theta\sin\phi\ \hat{a}_r + \cos\theta\sin\phi\ \hat{a}_\theta + \cos\phi\ \hat{a}_\phi$$
$$\hat{a}_z = \cos\theta\ \hat{a}_r - \sin\theta\ \hat{a}_\theta$$

$$A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$$
$$A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$$
$$A_\phi = -A_x\sin\phi + A_y\cos\phi$$

$$A_x = A_r\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$$
$$A_y = A_r\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$$
$$A_z = A_r\cos\theta - A_\theta\sin\theta$$

Dot Products:

$$\hat{a}_r \bullet \hat{a}_x = \sin\theta\cos\phi \qquad \hat{a}_r \bullet \hat{a}_y = \sin\theta\sin\phi \qquad \hat{a}_r \bullet \hat{a}_z = \cos\theta$$
$$\hat{a}_\theta \bullet \hat{a}_x = \cos\theta\cos\phi \qquad \hat{a}_\theta \bullet \hat{a}_y = \cos\theta\sin\phi \qquad \hat{a}_\theta \bullet \hat{a}_z = -\sin\theta$$
$$\hat{a}_\phi \bullet \hat{a}_x = -\sin\phi \qquad \hat{a}_\phi \bullet \hat{a}_y = \cos\phi \qquad \hat{a}_\phi \bullet \hat{a}_z = 0$$