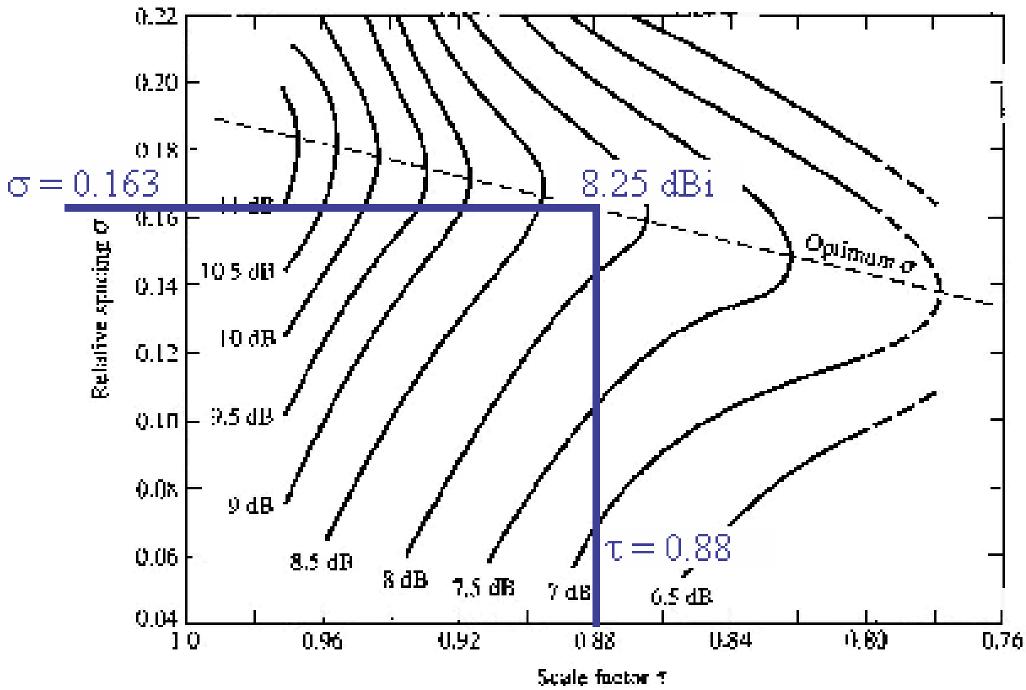
EE 483/583 Antennas for Wireless Communications
Examination #2 (Spring 2024)

Name KEY

Instructions: Open book/notes. Work alone. Place answers in indicated spaces. Use notation as given in class for coordinates & variables. **Show/label all** work for credit. Where applicable, let $c = 2.998 \times 10^8$ m/s.

1) For an **optimum** LPDA with a desired directivity of 8.25 dBi and frequency range of 192-210 MHz (CH 10-13), find the relative spacing σ, scale factor τ, apex angle 2α (deg), lengths l_1 & l_2 and locations R_1 & R_2 of the longest two LPDA elements, and estimated length of shortest element l_N. Show & label work on given figures. Express all lengths/distances in centimeters.



$$\alpha = \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right) = \tan^{-1}\left(\frac{1-0.88}{4(0.163)}\right) = 10.42853° \Rightarrow \underline{2\alpha = 20.85706°}$$

$$\lambda_{max} = \frac{c}{f_{low}} = \frac{2.998\times10^8}{192\times10^6} = 1.56146 \text{ m} \Rightarrow \lambda_{max} = 156.1458 \text{ cm}$$

Using fig. on next page (top) & τ, $l_1 = 0.553\lambda_{max} = 0.553(156.1458) \Rightarrow \underline{l_1 = 86.3486 \text{ cm}}$

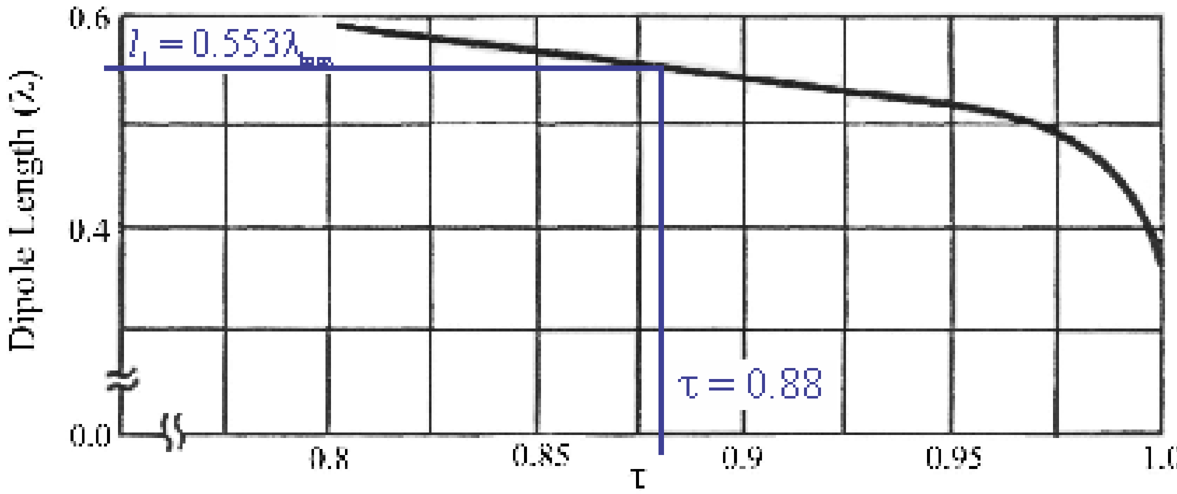
$l_2 = \tau l_1 = 0.88(86.3486) \Rightarrow \underline{l_2 = 75.9868 \text{ cm}}$

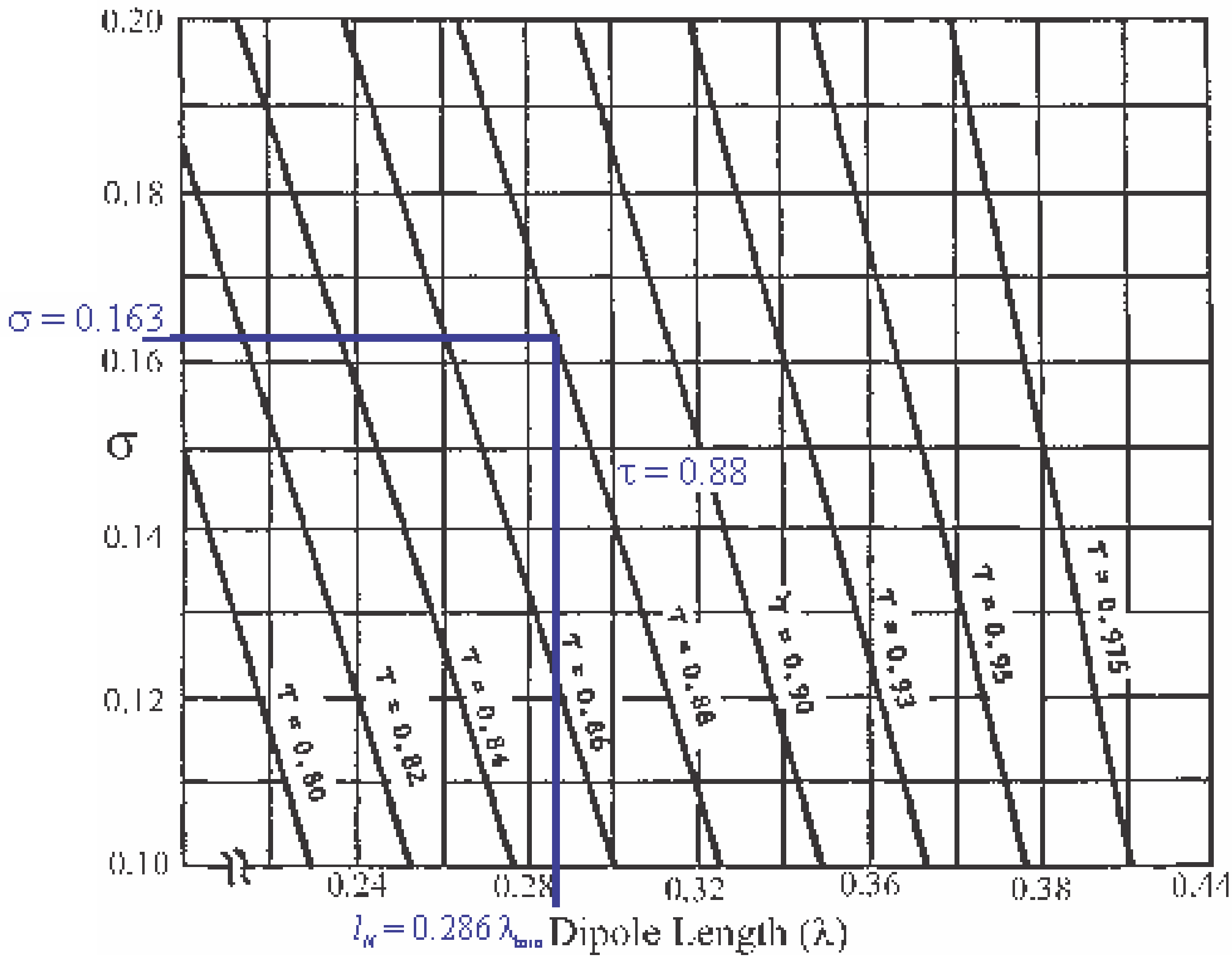
$$\lambda_{min} = \frac{c}{f_{high}} = \frac{2.998\times10^8}{210\times10^6} = 1.42762 \text{ m} \Rightarrow \lambda_{min} = 142.762 \text{ cm}$$

Using fig. on next page (bottom) w/ τ & σ,
$l_N = 0.286\lambda_{min} = 0.286(142.762) \Rightarrow \underline{l_N = 40.8299 \text{ cm}}$

$R_1 = \frac{l_1}{2}\cot(\alpha) = \frac{86.3486}{2}\cot(10.4285°) \Rightarrow \underline{R_1 = 234.5803 \text{ cm}}$ and

$$R_2 = \tau R_1 = 0.88(234.5803) \quad \Rightarrow \quad \underline{R_2 = 206.4307 \text{ cm}}$$





σ = **0.163** τ = **0.88** 2α = **20.857°** l_N = **40.83 cm**

l_1 = **86.349 cm** l_2 = **75.987 cm** R_1 = **234.580 cm** R_2 = **206.431 cm**

2) A UHF Yagi-Uda antenna is needed to operate at 642 MHz with a minimum directivity of 11 dBi. Due to available stock, you are required to use 5/32" diameter pipe for the elements and a 7/16" diameter boom. Using the standard Yagi-Uda antennas presented in class, select (i.e., circle on Table 10.6) the **smallest** antenna design that can satisfy the requirements. Then, design and **separately** list all the elements lengths (l_1, l_2, …) & spacings (s_{12}, s_{23}, …) in the table **after** all corrections are made (show & label work on figures). Make the driven element length the simple average of the reflector and first director.

11 dBi – 2.15 dB = 8.85 dBd

$\Rightarrow$ Per Table 10.6, a **5 element Yagi-Uda** (9.2 dBd or 11.35 dBi) will work.

$$\lambda = \frac{c}{f} = \frac{2.998\times 10^8}{642\times 10^6} = 0.466978 \text{ m} \quad \Rightarrow \quad \lambda = 46.6978 \text{ cm}$$

d/λ = 5/32(2.54)/46.6978 = **0.0084988 ~ 0.0085**

$\Rightarrow$ No length adjustments needed from Fig 10.25

D/λ = 7/16(2.54)/46.6978 = **0.0237966 ~ 0.0238**

$\Rightarrow$ From Fig 10.26, add **0.0171λ** to all elements to compensate for boom.

Using Table 10.6 and above length corrections-

reflector $l_1 = 0.482\lambda + 0.0171\lambda = 0.4991\lambda = 0.4991(46.6978) \Rightarrow$ **l_1 = 23.307 cm**

driven $l_2 = (l_1 + l_3)/2 = (0.4991 + 0.4451)\lambda/2 = 0.4721\lambda = 0.4721(46.6978) \Rightarrow$ **l_2 = 22.046 cm**

director $l_3 = l_5 = 0.428\lambda + 0.0171\lambda = 0.4451\lambda = 0.4451(46.6978) \Rightarrow$ **$l_3 = l_5$ = 20.785 cm**

director $l_4 = 0.424\lambda + 0.0171\lambda = 0.4411\lambda = 0.4411(46.6978) \Rightarrow$ **l_4 = 20.598 cm**

Using Table 10.6, the element spacings are-

$s_{12} = s_{ij} = 0.2\lambda = 0.2(46.6978) \Rightarrow$ **$s_{12} = s_{ij}$ = 9.340 cm**

List of element lengths (in λ & cm)	List of element spacings (in λ & cm)
Reflector $l_1 = 0.4991\lambda = 23.307$ **cm**	$s_{12} = 0.2\lambda = 9.340$ **cm**
driven $l_2 = 0.4721\lambda = 22.046$ **cm**	$s_{23} = 0.2\lambda = 9.340$ **cm**
director #1 $l_3 = 0.4451\lambda = 20.785$ **cm**	$s_{34} = 0.2\lambda = 9.340$ **cm**
director #2 $l_4 = 0.4411\lambda = 20.598$ **cm**	$s_{45} = 0.2\lambda = 9.340$ **cm**
director #3 $l_5 = 0.4451\lambda = 20.785$ **cm**	

Table 10.6 OPTIMIZED UNCOMPENSATED LENGTHS OF PARASITIC ELEMENTS FOR YAGI-UDA ANTENNAS OF SIX DIFFERENT LENGTHS

$d/\lambda = 0.0085$, $s_{12} = 0.2\lambda$		LENGTH OF YAGI-UDA (IN WAVELENGTHS)					
		0.4	**0.8**	1.20	2.2	3.2	4.2
LENGTH OF REFLECTOR (l_1/λ)		0.482	**0.482**	0.482	0.482	0.482	0.475
LENGTH OF DIRECTORS, λ	l_3	0.442	**0.428**	0.428	0.432	0.428	0.424
	l_4		**0.424**	0.420	0.415	0.420	0.424
	l_5		**0.428**	0.420	0.407	0.407	0.420
	l_6			0.428	0.398	0.398	0.407
	l_7				0.390	0.394	0.403
	l_8				0.390	0.390	0.398
	l_9				0.390	0.386	0.394
	l_{10}				0.390	0.386	0.390
	l_{11}				0.398	0.386	0.390
	l_{12}				0.407	0.386	0.390
	l_{13}					0.386	0.390
	l_{14}					0.386	0.390
	l_{15}					0.386	0.390
	l_{16}					0.386	
	l_{17}					0.386	
SPACING BETWEEN DIRECTORS (s_{ij}/λ)		0.20	**0.20**	0.25	0.20	0.20	0.308
DIRECTIVITY RELATIVE TO HALF-WAVE DIPOLE (dBd)		7.1	**9.2**	10.2	12.25	13.4	14.2
DESIGN CURVE (SEE FIGURE 10.25)		*(A)*	***(B)***	*(B)*	*(C)*	*(B)*	*(D)*

SOURCE: Peter P. Viezbicke, *Yagi Antenna Design*, NBS Technical Note 688, December 1976.

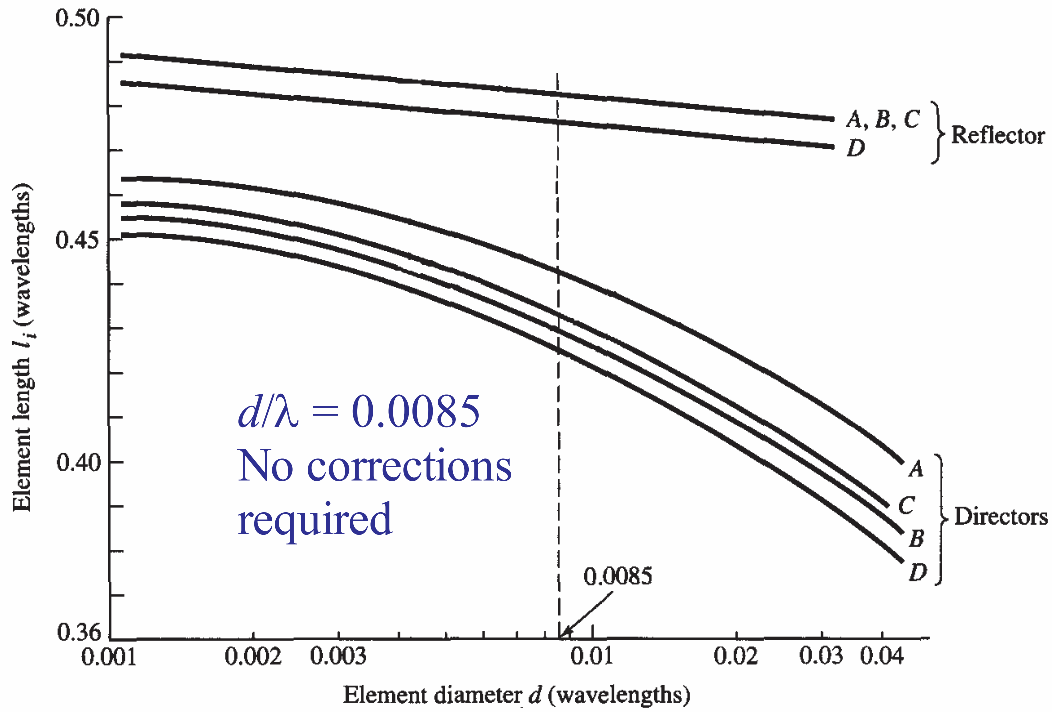


Figure 10.25 Design curves to determine element lengths of Yagi-Uda arrays. (SOURCE: P. P. Viezbicke, "Yagi Antenna Design," NBS Technical Note 688, U.S. Department of Commerce/National Bureau of Standards, December 1976)

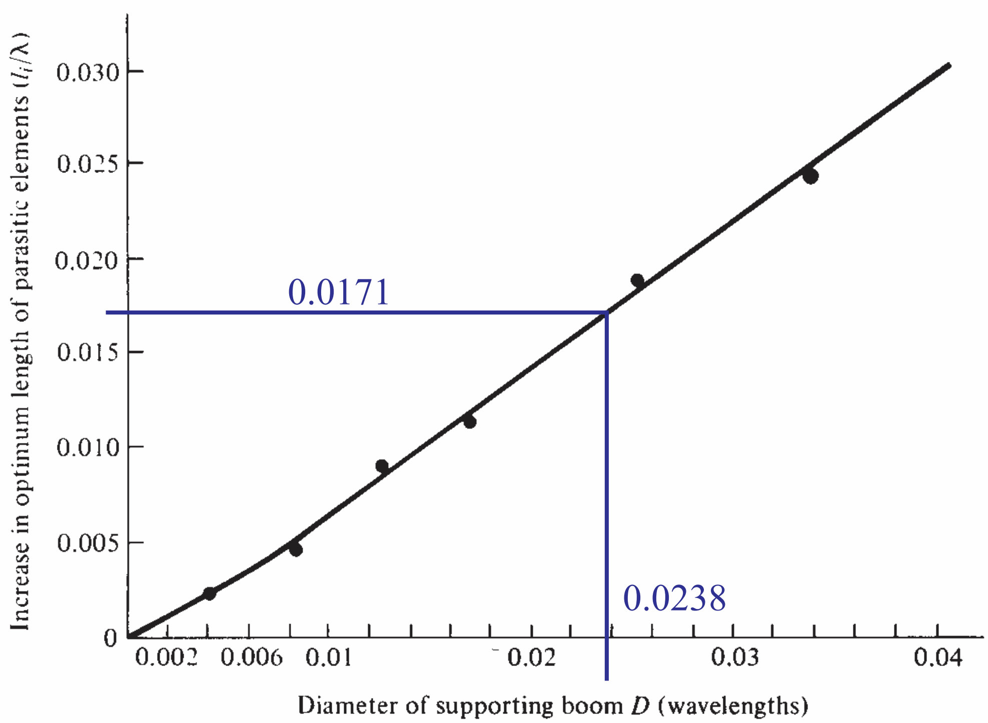


Figure 10.26 Increase in optimum length of parasitic elements as a function of metal boom diameter. (SOURCE: P. P. Viezbicke, "Yagi Antenna Design," NBS Technical Note 688, U.S. Department of Commerce/National Bureau of Standards, December 1976)

3) At 500 MHz, match an antenna with $Z_{in} = 19.5 - j\,15\ \Omega$ to a feeding transmission line ($Z_0 = 75\ \Omega$, $u = c$) using a quarterwave transformer (QWT) with characteristic impedance higher than Z_0. Draw a fully labeled sketch of the final circuit design with all dimensions in centimeters. [Note: Assume the QWT has a phase velocity of 2.4×10^8 m/s.]

- The wavelengths are $\lambda = c / f = 2.998 \times 10^8 / 500 \times 10^6 \Rightarrow$ $\underline{\lambda = 0.5996 \text{ m} = 59.96 \text{ cm}}$, and $\lambda' = u / f = 2.4 \times 10^8 / 2 \times 10^9 \Rightarrow$ $\underline{\lambda' = 0.48 \text{ m} = 48 \text{ cm}}$.

Steps

1) Calculate the normalized impedance for the antenna $z_A = Z_A / Z_0 = (19.5 - j\,15) / 75 \Rightarrow$ $\mathbf{\underline{z_A = 0.26 - j\,0.2\ \Omega/\Omega}}$ and plot on **Smith chart**.

2) Draw circle, centered on Smith chart, through z_A point. This circle of constant $|\Gamma|$ includes the locus of all possible z_{in} (and y_{in}) along the transmission line with this load.

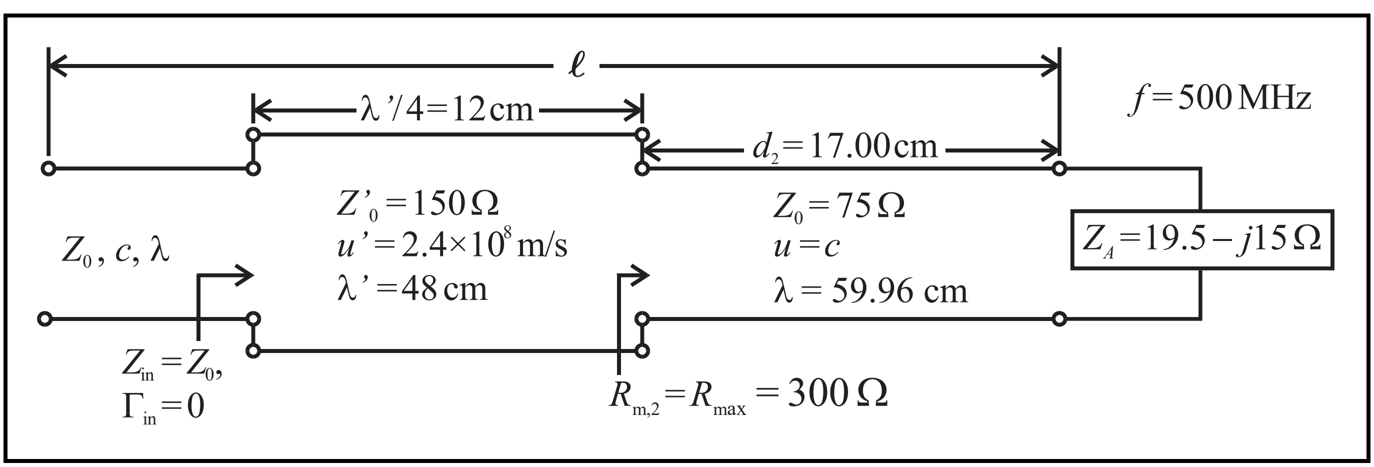
3) The two match points where the circle of constant $|\Gamma|$ intersects the real axis are $r_{m,1} = r_{min} = 0.25$ and $r_{m,2} = r_{max} = 4$ [or $R_{max} = 4(75) = 300\ \Omega$]. Select the $r_{m,2} = r_{max}$ match point as it will give Z_0' higher than Z_0.

4) Find the distance d_2 from z_A to $r_{m,2} = r_{max}$ using scales on Smith chart, $d_2 = 0.0336\lambda + 0.25\lambda \Rightarrow$ $\mathbf{\underline{d_2 = 0.2836\lambda}}$ or $d_2 = 0.2836(59.96) \Rightarrow$ $\mathbf{\underline{d_2 = 17.005 \text{ cm}}}$.

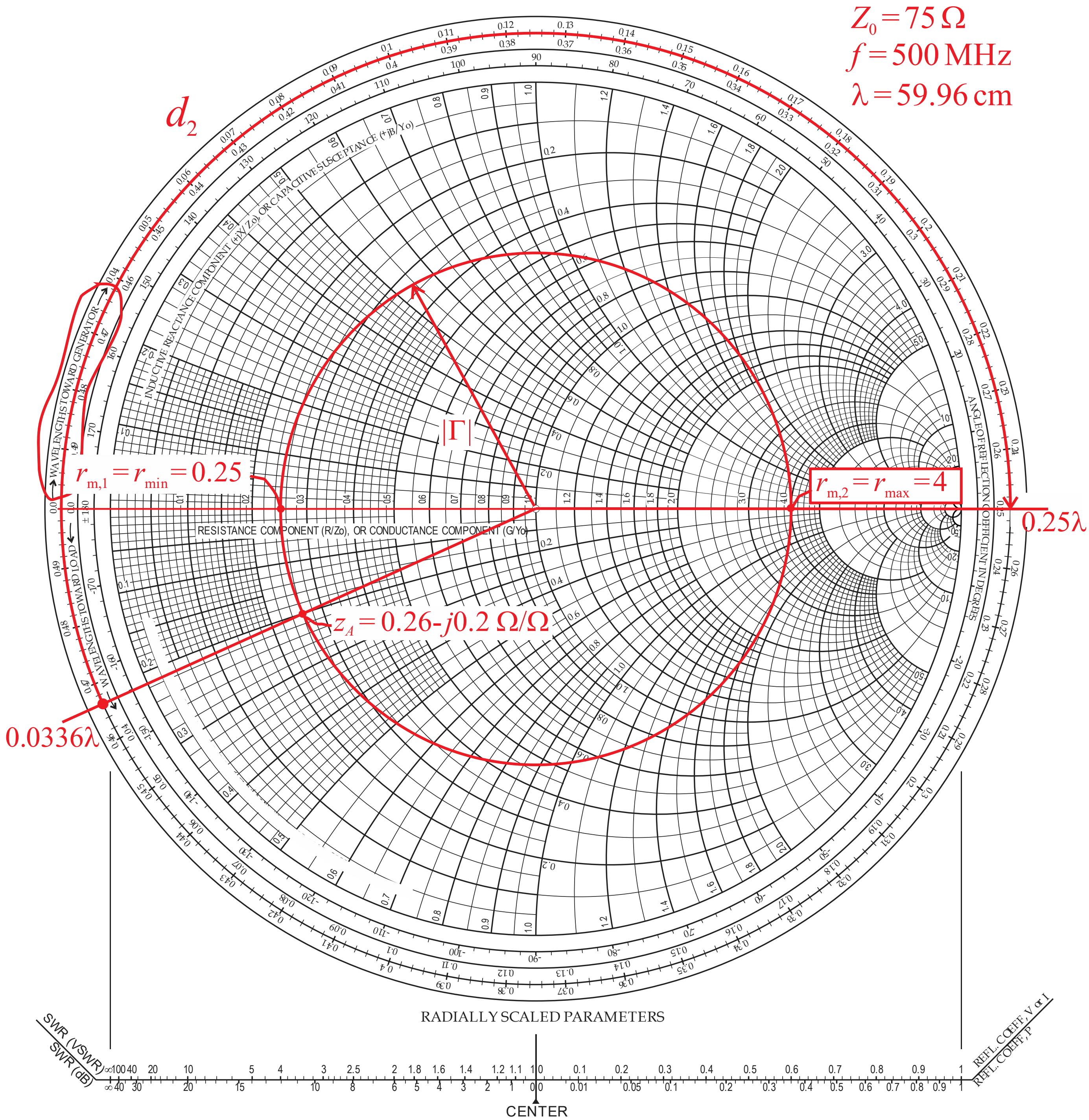
5) Starting at d_2 from z_A, insert the QWT. The QWT will have a characteristic impedance $Z'_{0,min} = \sqrt{Z_0 R_{max}} = \sqrt{75(300)} \Rightarrow$ $\mathbf{\underline{Z_0' = 150\,\Omega}}$ and has length $\lambda'/4 = 48/4 \Rightarrow$ $\mathbf{\underline{\lambda'/4 = 12 \text{ cm}}}$.

6) As shown on circuit design sketch, everywhere toward the source from the QWT sees an input impedance of $Z_{in} = Z_0 = 75\ \Omega$.

final circuit design



Simple Smith Chart



4) Bruce Banner has made a center-fed dipole antenna operating at 2.34 GHz (Sirius XM) in air. It is located and centered on the *z*-axis. Calculate the operating wavelength λ, length of the dipole as a fraction of a wavelength (ℓ/λ) and the wavenumber *k*. Is this an infinitesimal, small, halfwave, or finite-length dipole? Then, find the radiation resistance, loss resistance, and radiation efficiency (%) for the dipole if it is made from a green-glowing radium-plutonium alloy wire ($\sigma = 8.4\times10^5$ S/m, μ_0, length 4 mm, diameter 0.8 mm).

Wavelength is $\lambda = c/f = 2.998\times10^8/2.34\times10^9 \Rightarrow \underline{\lambda = 0.12812\text{ m} = 12.812\text{ cm}}$.

$\ell/\lambda = 4/128.12 \quad \Rightarrow \quad \underline{\boldsymbol{\ell/\lambda = 1/32.03 = 0.03122}}$ (small dipole!)

Wave number $k = 2\pi/\lambda = 2\pi/0.12812 \quad \Rightarrow \quad \underline{\boldsymbol{k = 49.0415\text{ rad/m}}}$

Per (4-37), $R_r = \eta\frac{\pi}{6}\left(\frac{\ell}{\lambda}\right)^2 = 376.7303\frac{\pi}{6}(0.03122)^2 \quad \Rightarrow \quad \underline{\boldsymbol{R_r = 0.19227\ \Omega}}$

Per (2-90b),

$$R_{hf} = \frac{\ell}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{\ell}{\pi d}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{0.004}{\pi(0.0008)}\sqrt{\frac{2\pi(2.34\bullet10^9)4\pi\bullet10^{-7}}{2(8.4\bullet10^5)}} \quad \Rightarrow \quad \underline{\boldsymbol{R_{hf} = 0.166905\ \Omega}}$$

From notes, $R_L = \frac{R_{hf}}{3} = \frac{0.166905}{3} \quad \Rightarrow \quad \underline{\boldsymbol{R_L = 0.055635\ \Omega}}$

Per (2-90) $e_{cd} = R_r/(R_r + R_L) = 0.19227/(0.19227+0.055635)*100\% \quad \Rightarrow \quad \underline{\boldsymbol{e_{cd} = 77.558\ \%}}$

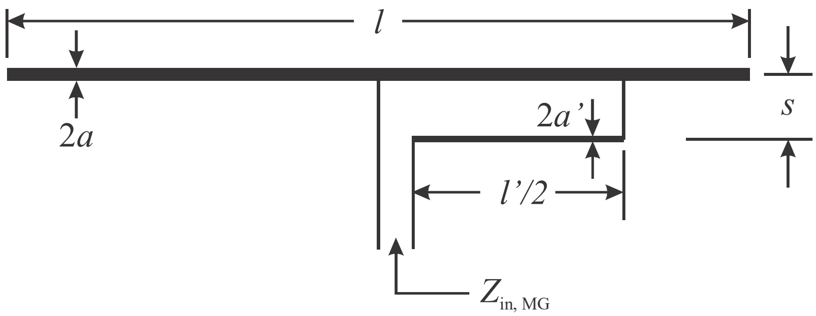
$\lambda =$ __**12.812 cm**__ $\qquad \ell/\lambda =$ __**1/32.03 = 0.03122**__ $\qquad k =$ __**49.0415 rad/m**__

Is this radiant antenna an infinitesimal, **[small]**, halfwave, or finite-length dipole? (circle correct answer)

Why? __**$1/50 < \ell/\lambda = 1/32.03 < 1/10$**__

$R_r =$ __**0.19227 Ω**__ $\qquad R_L =$ __**0.055635 Ω**__ $\qquad$ radiation efficiency = __**77.558 %**__

5) The driven element of a Yagi-Uda antenna, operating at 400 MHz, is driven using a modified Gamma-match (below). Given $l = 35.6$ cm, $a = 6$ mm, $a' = 3$ mm, $s = 2.8$ cm, and $l'/2 = 12$ cm, find the characteristic impedance Z_0 of the transmission line formed by the modified Gamma-match section, current divisor factor, transmission line mode input impedance Z_t, T-Match equivalent radius a_e, and the overall input impedance Z_{in}. Assume the input impedance of the antenna mode has been found to be $24 + j\,16\ \Omega$.



$$\lambda = \frac{c}{f} = \frac{2.998 \bullet 10^8}{400 \bullet 10^6} = 0.7495 \text{ m} \quad \& \quad k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{2\pi(400 \times 10^6)}{2.998 \times 10^8} = 8.38317 \text{ rad/m}$$

$$Z_0 = \frac{\eta}{2\pi}\cosh^{-1}\left(\frac{s^2 - a^2 - a'^2}{2aa'}\right) = \frac{376.7303}{2\pi}\cosh^{-1}\left(\frac{0.028^2 - 0.006^2 - 0.003^2}{2(0.006)0.003}\right)$$

$$\Rightarrow \quad \underline{Z_0 = 222.70577\,\Omega}$$

$$Z_t = jZ_0 \tan\left(\frac{kl'}{2}\right) = j222.706\tan(8.3832 \bullet 0.12) \quad \Rightarrow \quad \underline{Z_t = j351.4489\,\Omega}$$

$u = a/a' = 6/3 = 2$ and $v = s/a' = 28/3 = 9.33333$

$$\alpha = \frac{\cosh^{-1}\left(\frac{v^2 - u^2 + 1}{2v}\right)}{\cosh^{-1}\left(\frac{v^2 + u^2 - 1}{2vu}\right)} = \frac{\cosh^{-1}\left(\frac{9.3333^2 - 2^2 + 1}{2(9.3333)}\right)}{\cosh^{-1}\left(\frac{9.3333^2 + 2^2 - 1}{2(9.3333)2}\right)} \quad \Rightarrow \quad \underline{\alpha = 1.430316}$$

$$a_e = a'e^{\frac{(u^2 \ln u + 2u \ln v)}{(1+u)^2}} = 0.003e^{\frac{(2^2 \ln 2 + 2(2)\ln 9.33333)}{(1+2)^2}} \quad \Rightarrow \quad \underline{a_e = 1.10164 \text{ cm}}$$

$$Z_{\text{in}} = \frac{(1+\alpha)^2 Z_{\text{ant}} Z_t}{(1+\alpha)^2 Z_{\text{ant}} + 2Z_t} = \frac{(1+1.430316)^2(24 + j16)(j351.449)}{(1+1.430316)^2(24 + j16) + 2(j351.449)}$$

$$\Rightarrow \quad \underline{Z_{\text{in}} = 53.38578 + j51.14197\,\Omega}$$

a_e = **1.1016 cm** current divisor factor = **1.430316**

Z_0 = **222.706 Ω** Z_t = ***j* 351.449 Ω** Z_{in} = **53.386 + *j* 51.142 Ω**