EE 483/583 Examination #1 (Spring 2024)

Name Key

Instructions: Place answers in indicated spaces, use notation as given in class for coordinates & vectors, and show all work for credit. Turn-in equation sheet w/ exam. Assume $c = 2.9979 \times 10^8$ m/s.

Handy integrals: $\int \cos^n(ax)\sin(ax)\,dx = -\dfrac{\cos^{n+1}(ax)}{(n+1)a}$, $\int \sin^n(ax)\cos(ax)\,dx = \dfrac{\sin^{n+1}(ax)}{(n+1)a}$

$\int \cos^2(ax)\,dx = \dfrac{x}{2} + \dfrac{\sin(2ax)}{4a}$, $\int \cos^3(ax)\,dx = \dfrac{\sin(ax)}{a} - \dfrac{\sin^3(ax)}{3a}$, $\int \cos^4(ax)\,dx = \dfrac{3x}{8} + \dfrac{\sin(2ax)}{4a} - \dfrac{\sin(4ax)}{32a}$

$\int \sin^2(ax)\,dx = \dfrac{x}{2} - \dfrac{\sin(2ax)}{4a}$, $\int \sin^3(ax)\,dx = \dfrac{-\cos(ax)}{a} + \dfrac{\cos^3(ax)}{3a}$, $\int \sin^4(ax)\,dx = \dfrac{3x}{8} - \dfrac{\sin(2ax)}{4a} + \dfrac{\sin(4ax)}{32a}$

1) Batman has a communications link with Wayne Manor. He is using a $\lambda/4$ monopole antenna (5 cm long, linear polarization coincident with monopole, $Z_{\text{Bat}} = 36 + j24\ \Omega$, and $G_{\text{Bat}} = 3.7$ dBi) molded into his mask and connected to a 50 Ω transmission line (TL). The base unit at Wayne Manor has a well-matched vertically-polarized antenna array with gain $G_{\text{WM}} = 15$ dBi. As Batman climbs the walls of Arkham Asylum, 9 km from Wayne Manor, the monopole is tipped 20° from vertical. Find the wavelength & frequency of operation, TL mismatch loss factor (unitless & dB), polarization loss factor (unitless & dB), and power (W & dBm) that Wayne Manor must transmit for Batman to receive 20 nW.

$\lambda/4 = 5\text{cm} \rightarrow \lambda = 20\text{cm}$ $\quad f = c/\lambda = \dfrac{2.9979\times10^8}{0.2} = 1.49895\ \text{GHz}$

$\Gamma_{Bat} = \dfrac{Z_{Bat} - Z_0}{Z_{bat} + Z_0} = \dfrac{36+j24-50}{36+j24+50} = 0.31119\angle 104.664^\circ$

$TL\,MM = 1 - |\Gamma_{bat}|^2 = 1 - 0.31119^2 = 0.90316 \overset{10\log}{\Rightarrow} -0.44235\ \text{dB}$

$PLF = |\cos^2\psi| = \cos^2 20^\circ = 0.88302 \overset{10\log}{\Rightarrow} -0.54028\ \text{dB}$

$P_r = P_{bat} = 20\ nW$ $\quad G_{bat} = 10^{3.7/10} = 2.34423$ $\quad G_{WM} = 10^{15/10} = 31.623$

Friis Trans Eqn (2-118) $\dfrac{P_r}{P_t} = [1-|\Gamma_t|^2]^0[1-|\Gamma_r|^2]\left(\dfrac{\lambda}{4\pi R}\right)^2 G_t G_r\, PLF$

$P_{WM} = \dfrac{20\times10^{-9}}{0.90316\left(\dfrac{0.2}{4\pi 9000}\right)^2 2.34423(31.623)0.88302}$

$= 108.17774\ W = 10\log_{10}\dfrac{108.1777}{10^{-3}} = 50.3414\ \text{dBm}$

$\lambda =$ 20cm $\quad f =$ 1.49895 GHz $\quad$ TL mismatch loss factor = 0.90316 = −0.4423 dB

PLF = 0.88302 = −0.5403 dB $\quad P_{\text{WM}} =$ 108.1777 W = 50.341 dBm

2) Riddler's small circular loop antenna, operating in free space, has a vector magnetic potential

$$\bar{A} = \hat{a}_\phi \frac{a^2 \mu_0 I_0 \sin\theta}{4} \frac{e^{-jkr}}{r}\left(jk + \cancel{\frac{1}{r}}\right) \text{ (Wb/m)}.$$

Find the general **far-field** phasor vector electric and magnetic fields given $k = 2\pi/\lambda = 2.12$ rad/m, loop radius $a = 4$ cm, and the loop is driven with a $12\angle 0°$ A current (substitute in known quantities). Evaluate the phasor vector electric and magnetic fields at spherical point $C(r = 5\text{ m}, \theta = \pi/3, \phi = \pi/2)$ in polar form w/ angle in degrees (e.g., $\bar{C} = \hat{a}\, C\angle\theta°$). Assume the vector electric potential $\bar{F} = 0$.

far-field $\rightarrow$ remove term $\propto \frac{1}{r^2} \Rightarrow \bar{A}_{FF} = \hat{a}_\phi \frac{jk a^2 \mu_0 I_0 \sin\theta}{4} \frac{e^{-jkr}}{r}$

Per (3-58a), $\bar{E}_{FF} = \bar{E}_A = -j\omega\bar{A} = \hat{a}_\phi \frac{+\omega k a^2 \mu_0 I_0 \sin\theta}{4} \frac{e^{-jkr}}{r}$

Now, $k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = 2.12 \frac{\text{rad}}{\text{m}} \Rightarrow \omega = 2.12c = 6.355548 \times 10^8 \frac{\text{rad}}{\text{s}}$

$\bar{E}_{FF} = \hat{a}_\phi \frac{6.355548\times10^8 (2.12)\, 0.04^2 (12\angle 0°)(4\pi\times10^{-7})}{4} \sin\theta \frac{e^{-j2.12r}}{r}$

$\underline{\bar{E}_{FF} = \hat{a}_\phi\, 8.127182 \sin\theta \frac{e^{-j2.12r}}{r} \text{ (V/m)}}$

$\bar{E}_{FF,C} = \hat{a}_\phi\, 8.127182 \sin\frac{\pi}{3} \frac{e^{-j2.12(5)}}{5} = 1.40767\, e^{-j10.6} \hat{a}_\phi$ (rad)

$= \underline{\hat{a}_\phi\, 1.40767\angle 112.66470° \text{ (V/m)}}$

r, θ, ϕ

Per (3-58b), $\bar{H}_{A,FF} = \frac{\hat{a}_r}{\eta} \times \bar{E}_A = -\hat{a}_\theta \frac{8.127182}{376.7303} \sin\theta \frac{e^{-j2.12r}}{r}$

$= \underline{-\hat{a}_\theta\, 0.021573 \sin\theta \frac{e^{-j2.12r}}{r}}$

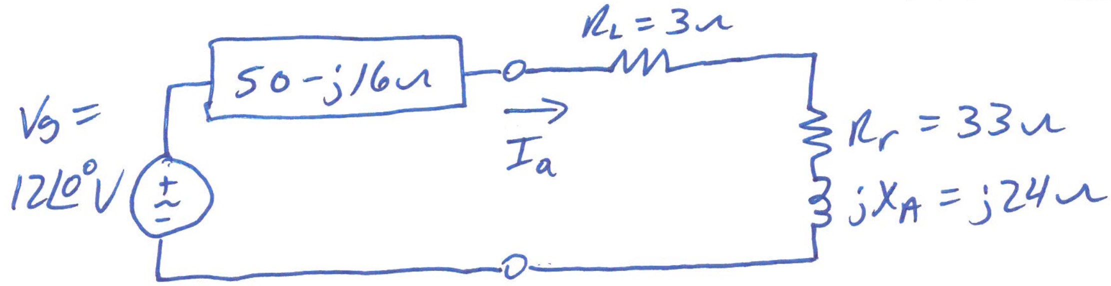
$\bar{H}_{FF,C} = -\hat{a}_\theta \frac{0.021573 \sin(\pi/3)}{5} e^{-j10.6} = \underline{\hat{a}_\theta\, 3.7365\times10^{-3}\angle -67.335°} \text{ (A/m)}$

$\bar{E}_{FF} = \underline{\hat{a}_\phi\, 8.12718 \sin\theta \frac{e^{-j2.12r}}{r} \text{ (V/m)}}$ $\bar{E}_{FF,C} = \underline{\hat{a}_\phi\, 1.4077\angle 112.665° \text{ (V/m)}}$

$\bar{H}_{FF} = \underline{-\hat{a}_\theta\, 21.573 \sin\theta \frac{e^{-j2.12r}}{r} \left(\frac{\text{mA}}{\text{m}}\right)}$ $\bar{H}_{FF,C} = \underline{\hat{a}_\theta\, 3.7365\angle -67.335° \left(\frac{\text{mA}}{\text{m}}\right)}$

$= -\hat{a}_\theta\, 3.7365\angle 112.665° \left(\frac{\text{mA}}{\text{m}}\right)$

3) Alfred is analyzing the power budget (transmit mode) for the $\lambda/4$ monopole antenna embedded in Batman's mask. The generator is characterized by $V_g = 12\angle 0°$ V and $Z_g = 50 - j16\ \Omega$. The antenna is characterized by an overall impedance $Z_{Bat} = 36 + j24\ \Omega$. Due to the carbon fibers in the armored composite of the mask, 3 Ω of the impedance is due to resistive losses. Assuming the generator is directly connected to the antenna, draw the equivalent circuit. Then, calculate the time-average power supplied by the generator, dissipated in the generator impedance, radiated by the antenna, and dissipated/lost by the antenna. Also, find the radiation efficiency (%) of the antenna.



$$I_a = \frac{12\angle 0°}{(50-j16)+(36+j24)} = 0.138935053\angle -5.3145457° \text{ A}$$

$$P_{Vg} = \tfrac{1}{2}\,\mathrm{Re}\{V_g I_a^*\} = \tfrac{1}{2}\,\mathrm{Re}\{(12\angle 0°)(0.1389\angle +5.3145°)\}$$
$$= \mathrm{Re}(0.8300268 + j0.0772) = \underline{0.830027\,\text{W}}$$

$$P_{Zg} = \tfrac{1}{2}|I_a|^2 R_g = \tfrac{1}{2}(0.138935)^2 50 = \underline{0.4825737\ \text{W}}$$

$$P_{rad} = \tfrac{1}{2}|I_a|^2 R_r = \tfrac{1}{2}(0.138935)^2 33 = \underline{0.31849866\,\text{W}}$$

$$P_{loss} = \tfrac{1}{2}|I_a|^2 R_L = \tfrac{1}{2}(0.138935)^2 3 = \underline{0.0289544\,\text{W}}$$

$$\text{(2-90)}\quad e_{cd} = \frac{R_r}{R_r + R_L} = \frac{33}{36} = 0.91\bar{6} \text{ or } \underline{91.\bar{6}\%}$$

P_{Vg} = 0.83003W P_{Zg} = 0.48257W P_{rad} = 0.3185 W

P_{loss} = 0.02895W radiation efficiency = 91.667%

4) The radiation intensity of a 710 MHz antenna operating in free space is

$$U(\theta,\phi)=\begin{cases}3.33\sin^2\theta\sin\phi\ (\text{W/Sr}) & 0\le\theta\le180°,\ 0\le\phi\le180° \\ 0 & \text{elsewhere}\end{cases}.$$

$\lambda = \frac{2.9979\times10^8}{710\times10^6} = 0.42224\,m$

Determine the time-average radiated power and directivity $D(\theta, \phi)$. Next, find the maximum directivity D_{max} as well as half-power beamwidths (deg) in the azimuthal (wrt ϕ) and elevation (wrt θ) directions. Assuming the antenna is 94% efficient and is well matched to the feed as well as the incident EM wave, determine the maximum gain G_{max} and effective area as well as θ & ϕ angles (deg) at which they occur.

(2-13) $P_{rad} = \oiint_\Omega U\,d\Omega = 3.33\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi}\sin^2\theta\sin\phi\sin\theta\,d\theta\,d\phi$

$= 3.33\int_{\phi=0}^{\pi}\sin\phi\,d\phi\int_{\theta=0}^{\pi}\sin^3\theta\,d\theta$

$= 3.33(-\cos\phi)\Big|_0^{\pi}\left[-\cos\theta+\frac{\cos^3\theta}{3}\right]\Big|_0^{\pi} = 3.33(1+1)\left[1-\frac{1}{3}-\left(-1+\frac{1}{3}\right)\right]$

$\underline{P_{rad} = 8.88\,W}$

(2-16) $D(\theta,\phi) = \frac{4\pi U}{P_{rad}} = \frac{4\pi}{8.88}3.33\sin^2\theta\sin\phi$

$= 4.71238898\sin^2\theta\sin\phi \quad \begin{matrix}0\le\theta\le180° \\ 0\le\phi\le180°\end{matrix}$

$\underline{D_{max} = 4.7123889}$ @ $\underline{\theta=\phi=90°}$

$G_{max} = e_{cd}D_{max} = 0.94(4.7124) = \underline{4.42965 = 6.4637\,dB_i}$

Elev. @ $\phi=90°$, $D(\theta_H, 90°) = \frac{4.7124}{2} = 4.7124\sin^2\theta_H(1) \Rightarrow \theta_H = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$ or $135°$

$\Rightarrow HPBW_\theta = 135-45 = \underline{90°}$

@ $\theta=90°$, $D(90°,\phi_H) = \frac{4.7124}{2} = 4.7124\sin^2 90°\sin\phi_H$

$\phi_H = \sin^{-1}(0.5) = 30°$ or $150° \Rightarrow HPBW_\phi = 150-30 = \underline{120°}$

$P_{rad} =$ 8.88 W

$D(\theta,\phi) = \begin{cases}4.7124\sin^2\theta\sin\phi & 0\le\theta\le180°,\ 0\le\phi\le180° \\ 0 & \text{elsewhere}\end{cases}$

$D_{max} =$ 4.7124 = 6.73 dB_i Azimuthal HPBW = 120° Elevation HPBW = 90°

$G_{max} =$ 4.43 = 6.464 dB_i $A_{em} =$ 0.062846 m^2 at angles $\theta=\phi=90°$

(2-111) $A_{em} = e_{cd}D_0\left(\frac{\lambda^2}{4\pi}\right) = 4.42965\,\frac{0.42224^2}{4\pi} = 0.062846\,m^2$