EE 483/583 Antennas for Wireless Communications Quiz #10 (Spring 2022)

Name ___**KEY**___

Instructions: Open book & notes. Place answers in indicated spaces and **show all** work for credit.

A 7-element uniform linear array of isotropic sources, operating at 91.3 MHz, has elements spaced at 1.1 m intervals. Calculate the element spacing as a fraction of a wavelength and progressive phase shift (radians) required for ordinary end-fire ($\theta_{MB} = 0°$) operation. Then, find an expression for the phase factor ψ. Calculate the location of the first null θ_{n1} (deg) and the first null beamwidth FNBW (deg). Estimate the maximum directivity D_{max} of the array (unitless and dBi). Assume $c = 2.998 \times 10^8$ m/s.

wavelength = $\lambda = c/f = 2.998 \times 10^8 / 91.3 \times 10^6$ ⇒ **λ = 3.28368 m**

element spacing as fraction of wavelength = $d/\lambda = 1.1/3.28368$

⇒ **d/λ = 0.33499**

(6-20a) progressive phase shift = $\beta = -kd = -(2\pi/\lambda)d = -(2\pi/3.2837)1.1$

⇒ **β = -2.1048 rad**

(6-7a) phase factor = $\psi = kd\cos(\theta) + \beta$ ⇒ **ψ = 2.1048 cos(θ) - 2.1048 rad**

From Table 6.3, nulls are located @ $\theta_n = \cos^{-1}[1 - n\lambda/(Nd)] = \cos^{-1}[1 - n/(N d/\lambda)]$ where $n = 1, 2, 3 \ldots$ but $n \neq N, 2N, \ldots$

First null, let $n = 1$ to get- $\theta_{n1} = \cos^{-1}[1 - 1/(7 \cdot 0.33499)]$ ⇒ **θ_{n1} = 55.002°**

First null beamwidth = FNBW = $2|\theta_{MB} - \theta_{n1}| = 2|0 - 55.002°|$

⇒ **FNBW = 110.004°**

Per (6-49) or Table 6.8, the maximum directivity D_0 is estimated as

$D_0 \cong 4N(d/\lambda) \cong 4(7)\,0.33499$ ⇒ **D_0 = 9.37972,**

or, in decibels, **$D_0 = 10\log_{10}(9.37972)$** ⇒ **$D_0$ = 9.7219 dBi**

d/λ = **0.33499** β = **-2.1048 rad** ψ = **2.1048 cos(θ) - 2.1048 rad**

θ_{n1} = **55.002°** FNBW = **110.004°** D_{max} = **9.3797** = **9.7219 dBi**

☺ ☼ Friday, April 29, 2022 ✍ ♫

$f := 91.3 \cdot 10^{6}$ Hz $\quad c := 2.998 \cdot 10^{8}$ m/s $\quad \lambda := \frac{c}{f}$ $\quad \lambda = 3.28368$ m

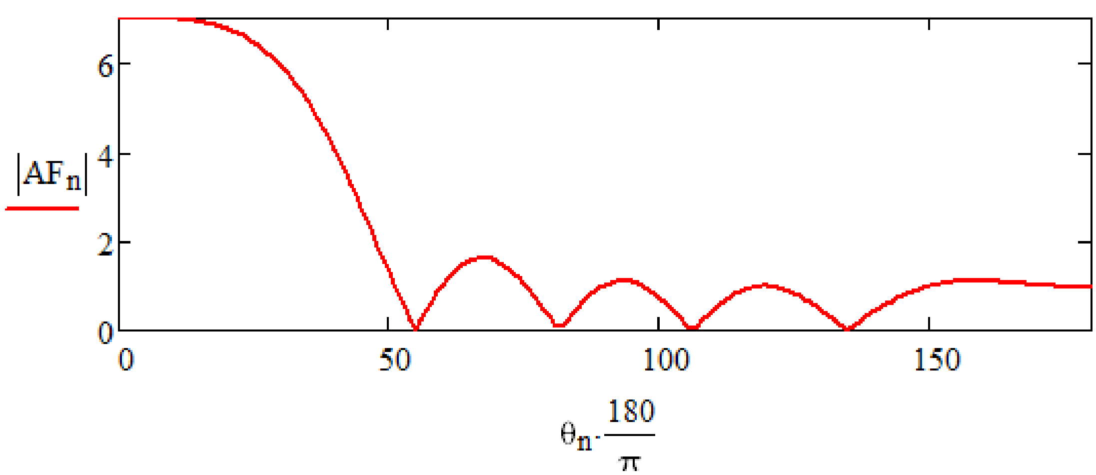
$N := 7 \quad I0 := 1 \quad d := 1.1$ m $\quad d\lambda := \frac{d}{\lambda} \quad d\lambda = 0.33499$

$kd := 2 \cdot \pi \cdot d\lambda \quad \boxed{kd = 2.1048}$ **(6-20a)** $\beta := -kd \quad \boxed{\beta = -2.104804}$ rad

$\boxed{\beta \cdot \frac{180}{\pi} = -120.5964}$ deg

$\psi(\theta, \beta) := kd \cdot \cos(\theta) + \beta \qquad n := 0..180 \qquad \theta_n := \frac{\pi}{180} \cdot n - 0.0001$

$$AF_n := I0 \cdot \frac{\sin\left(\frac{N}{2} \cdot \psi(\theta_n, \beta)\right)}{\sin\left(\frac{1}{2} \cdot \psi(\theta_n, \beta)\right)}$$



First null (Table 6.3) & FNBW $\quad \theta n1 := \text{acos}\left(1 - \frac{1}{N \cdot d\lambda}\right) \quad \boxed{\theta n1 \cdot \frac{180}{\pi} = 55.002}$ deg

Check $\quad \theta n := 55.002 \cdot \frac{\pi}{180} \quad \left(\left|\frac{\sin\left(\frac{N}{2} \cdot \psi(\theta n, \beta)\right)}{N \cdot \sin\left(\frac{1}{2} \cdot \psi(\theta n, \beta)\right)}\right|\right)^{2} = 2.05704 \times 10^{-13}$

$FNBWdeg := 2 \cdot \theta n \cdot \frac{180}{\pi} \qquad \boxed{FNBWdeg = 110.004}$ deg

Max directivity est. (6-49) $\quad Dmax := 4 \cdot N \cdot d\lambda \qquad \boxed{Dmax = 9.37972}$

$\boxed{10 \cdot \log(Dmax) = 9.7219}$ dBi