❄ Wednesday, March 30, 2022

EE 483/583 Antennas for Wireless Communications Quiz #7 (Spring 2022)

Name Key

Instructions: Open book/notes/homework. Place answers in indicated spaces and **show all work** for credit.

A dipole (0.7144 cm diameter, 26 cm length), operating at 551 MHz in free space, is driven by a **T-match** (0.3175 cm diameter, 13 cm length, separated by 1.5875 cm from dipole). Calculate the characteristic impedance of transmission line mode of the T-match as well as the input impedance of transmission line mode. Next, calculate the current divisor factor and equivalent radius (cm) of T-match section for the antenna mode. For extra credit, find the exact overall input impedance (rectangular format) if the input impedance of antenna mode is found (NEC-2) as $79.66 + j12.93\ \Omega$. Let $c = 2.998 \times 10^8$ m/s.

$a = \frac{0.007144}{2} = 0.003572\text{ m}$, $\ell = 0.26\text{ m}$, $a' = \frac{0.003175}{2} = 0.0015875\text{ m}$

$s = 0.015875\text{ m}$, $\ell' = 0.13\text{ m}$, $\eta = \eta_0 = 376.7303\ \Omega$

$$Z_0 = \frac{\eta_0}{2\pi}\cosh^{-1}\left[\frac{s^2 - a^2 - a'^2}{2aa'}\right] = \frac{376.73}{2\pi}\cosh^{-1}\left[\frac{0.015875^2 - 0.003572^2 - 0.0015875^2}{2(0.003572)0.0015875}\right]$$

$$= 59.9585\cosh^{-1}(20.87419) = 223.7103\ \Omega$$

$$Z_t = jZ_0\tan\left(\frac{k\ell'}{2}\right) = j223.71\tan\left(\frac{2\pi\,551\times10^6(0.13)}{2.998\times10^8(2)}\right) = j208.66194\ \Omega$$

$u = \frac{a}{a'} = \frac{0.003572}{0.0015875} = 2.25$, $v = \frac{s}{a'} = \frac{0.015875}{0.0015875} = 10$

$$\alpha = \frac{\cosh^{-1}\left[\frac{v^2 - u^2 + 1}{2v}\right]}{\cosh^{-1}\left[\frac{v^2 + u^2 - 1}{2vu}\right]} = \frac{\cosh^{-1}\left[\frac{10^2 - 2.25^2 + 1}{2(10)}\right]}{\cosh^{-1}\left[\frac{10^2 + 2.25^2 - 1}{2(10)2.25}\right]} = \frac{\cosh^{-1}(4.79686)}{\cosh^{-1}(2.31243)}$$

$$= 1.51926$$

$$a_e = a' e^{\frac{1}{(1+u)^2}\left[u^2\ln u + 2u\ln v\right]} = a' e^{\frac{1}{(1+2.25)^2}\left[2.25^2\ln 2.25 + 2(2.25)\ln 10\right]}$$

$$= 0.0015875\, e^{1.369666} = 0.0062453\text{ m} = 0.62453\text{ cm}$$

Extra $Y_{in} = \frac{Y_t}{2} + \frac{Y_a}{(1+\alpha)^2} = \frac{(j208.66)^{-1}}{2} + \frac{(79.66+j12.93)^{-1}}{(1+1.51926)^2} = 0.001927 - j0.0027095$

$Z_{in} = 1/Y_{in} = (0.001927 - j0.0027095)^{-1} = 174.36 + j245.0985\ \Omega$

char. imped. of TL mode = 223.71 Ω input imped. of TL mode = j208.66 Ω

current divisor factor = 1.51926 equiv. radius of ant. mode = 0.6245 cm

(extra credit) $Z_{in,exact}$ = 174.36 + j245.1 Ω