EE 483/583 Antennas for Wireless Communications
Examination #2 (Spring 2022)

Name **KEY**

Instructions: Open book/notes. Place answers in provided spaces. Use notation as given in class for coordinates & variables. **Show/label all** work for partial credit. Where applicable, let $\boldsymbol{c} = \mathbf{2.998 \times 10^8}$ **m/s**.

1) Snow Miser needs a Yagi-Uda antenna operating at 749.5 MHz with a minimum directivity of 14 dBi to communicate with his minions in the battle against Heat Miser. They must ensure that Rapid City gets snow in April.

a) Using the standard Yagi-Uda antennas presented in class, select the *smallest* design (i.e., circle on Table 10.6) that can satisfy the requirements. ⇒ 14 dBi - 2.15 dB = **11.85 dBd**

Table 10.6 OPTIMIZED UNCOMPENSATED LENGTHS OF PARASITIC ELEMENTS FOR YAGI-UDA ANTENNAS OF SIX DIFFERENT LENGTHS

$d/\lambda = 0.0085$ $\boldsymbol{s_{12} = 0.2\lambda}$		LENGTH OF YAGI-UDA (IN WAVELENGTHS)					
		0.4	0.8	1.20	**2.2**	3.2	4.2
LENGTH OF REFLECTOR (l_1/λ)		0.482	0.482	0.482	**0.482**	0.482	0.475
LENGTH OF DIRECTORS, λ	l_3	0.442	0.428	0.428	**0.432**	0.428	0.424
	l_4		0.424	0.420	**0.415**	0.420	0.424
	l_5		0.428	0.420	**0.407**	0.407	0.420
	l_6			0.428	**0.398**	0.398	0.407
	l_7				**0.390**	0.394	0.403
	l_8				**0.390**	0.390	0.398
	l_9				**0.390**	0.386	0.394
	l_{10}				**0.390**	0.386	0.390
	l_{11}				**0.398**	0.386	0.390
	l_{12}				**0.407**	0.386	0.390
	l_{13}					0.386	0.390
	l_{14}					0.386	0.390
	l_{15}					0.386	0.390
	l_{16}					0.386	
	l_{17}					0.386	
SPACING BETWEEN DIRECTORS (s_{ij}/λ)		0.20	0.20	0.25	**0.20**	0.20	0.308
DIRECTIVITY RELATIVE TO HALF-WAVE DIPOLE (dBd)		7.1	9.2	10.2	**12.25**	13.4	14.2
DESIGN CURVE (SEE FIGURE 10.25)		*(A)*	*(B)*	*(B)*	***(C)***	*(B)*	*(D)*

SOURCE: Peter P. Viezbicke, *Yagi Antenna Design*, NBS Technical Note 688, December 1976.

b) Determine the reflector-driven element spacing (cm) as well as the director-director spacings (cm).

$\lambda = c/f = 2.998 \times 10^8 / 749.5 \times 10^6 \Rightarrow \lambda = 0.4$ m = **40 cm**

From Table 10.6, $s_{12} = s_{ij} = 0.2\lambda = 0.2(40) \Rightarrow$ **$s_{12} = s_{ij} = 8$ cm**

reflector-driven element spacing = **8 cm** director- director spacings = **8 cm**

c) Determine the overall length of the Yagi-Uda array, i.e., distance from center of reflector to last director (cm).

From Table 10.6, the length ℓ of this 12 element Yagi-Uda antenna is

$\ell = 2.2\,\lambda = (12 - 1)\,0.2\lambda = 2.2(40) \Rightarrow$ **$\ell = 88$ cm**

overall length = **$\ell = 88$ cm**

d) Per Snow Miser's specific instructions, you are required to use niobium–titanium (Nb-Ti) alloy elements (8 mm diameter) and boom (13.6 mm diameter) cooled to 9K (below critical temperature for superconductivity). Determine the length of the reflector (cm) as well as the length of the first director (cm) **after** all corrections are made (show & label all work on design figures).

Element compensation

$d = 8$ mm, $d / \lambda = 0.008/0.4 \Rightarrow$ **$d / \lambda = 0.02$** (w/in range of Fig. 10.25).

Using design curve C, read off element-compensated lengths-

reflector $l_1' = 0.479\,\lambda = 0.479(40) \Rightarrow$ **$l_1' = 19.16$ cm**

director $l_3' = 0.4124\lambda = 0.4124(40) \Rightarrow$ **$l_3' = 16.496$ cm**

Boom compensation

$D = 13.6$ mm, $D / \lambda = 0.0136/0.4 \Rightarrow$ **$D / \lambda = 0.034$** (w/in range of Fig. 10.26).

Read off boom compensation to be $0.025\,\lambda$. Adding that to the element-compensated lengths yields-

reflector $l_1 = (0.479+0.025)\,\lambda = 0.504\,\lambda = 0.504(40) \Rightarrow$ **$l_1 = 20.16$ cm**

director $l_3 = (0.4124+0.025)\,\lambda = 0.4374\,\lambda = 0.4374(40) \Rightarrow$ **$l_3 = 17.496$ cm**

reflector length = **$l_1 = 20.16$ cm** director length = **$l_3 = 17.496$ cm**

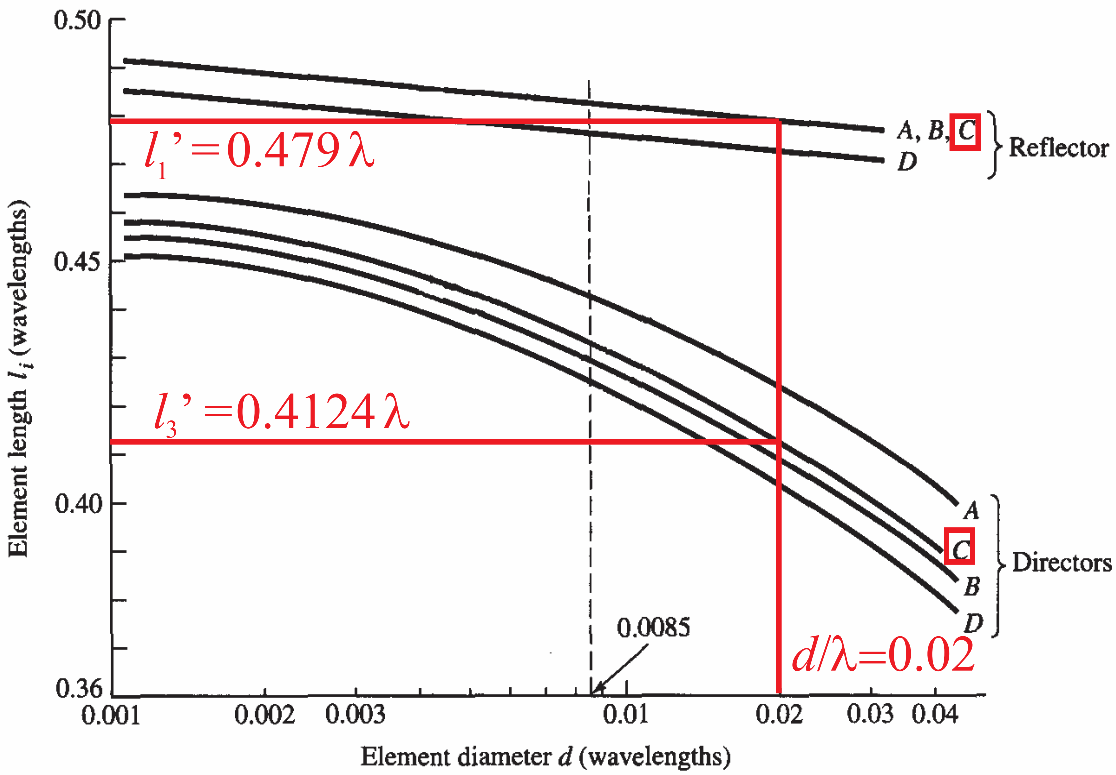


Figure 10.25 Design curves to determine element lengths of Yagi-Uda arrays. (SOURCE: P. P. Viezbicke, "Yagi Antenna Design," NBS Technical Note 688, U.S. Department of Commerce/National Bureau of Standards, December 1976)

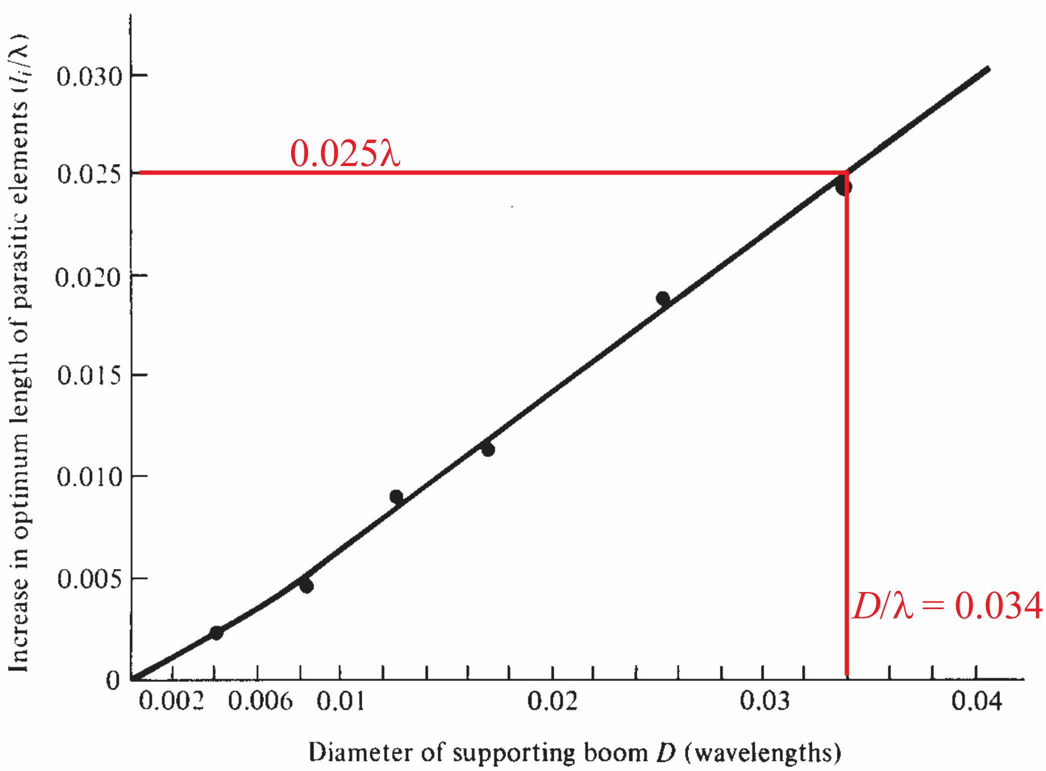
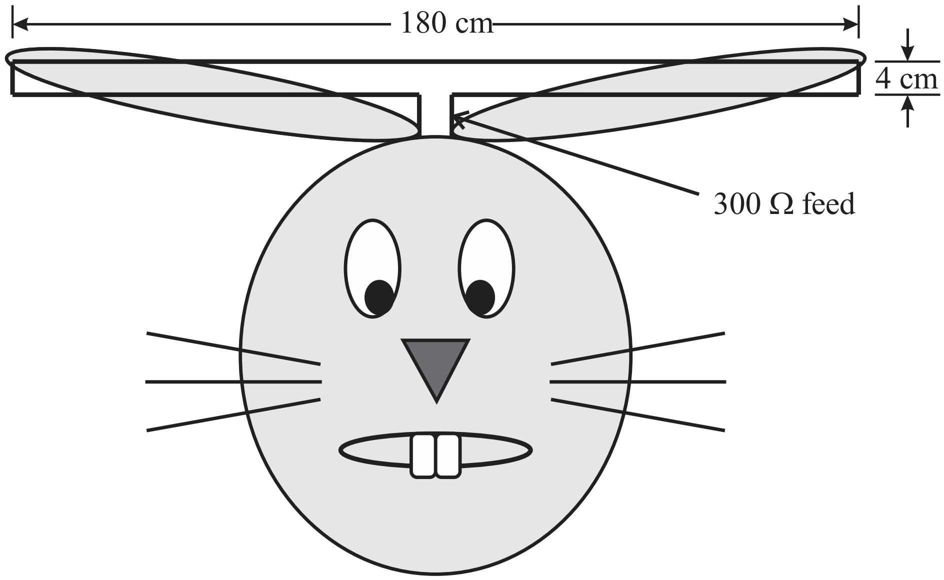


Figure 10.26 Increase in optimum length of parasitic elements as a function of metal boom diameter. (SOURCE: P. P. Viezbicke, " Yagi Antenna Design," NBS Technical Note 688, U.S. Department of Commerce/National Bureau of Standards, December 1976)

2) An enormous Peter Rabbit has an ear-mounted folded dipole (see below) made of 10 AWG (diameter = 2.588 mm) wires fabricated from a special conductive chocolate blend. If Peter uses his ear-mounted folded dipole at 75 MHz to communicate Mr. McGregor's location to his sisters Flopsy, Mopsy and Cotton-tail, find the transmission line mode input impedance. Then, calculate the effective radius of the antenna mode dipole and select the appropriate antenna mode input impedance. Last, determine the overall input impedance of the folded dipole. Assume free space conditions.



$\lambda = c/f = 2.998 \times 10^8 / 75 \times 10^6 \Rightarrow$ **$\lambda = 3.9973333$ m**

$Z_{0t} = \eta/\pi \cosh^{-1}(s/2a) = 376.7303/\pi \cosh^{-1}(0.04/0.002588) \Rightarrow$ **$Z_{0t} = 411.3264\ \Omega$**

$Z_{in,t} = jZ_{0t} \tan(k\,l/2) = jZ_{0t} \tan(\pi\, l/\lambda) = j\,411.3264 \tan(\pi 1.8/3.997333) \Rightarrow$ **$Z_{in,t} = j\,2612.9596\ \Omega$**

$a_e = \sqrt{(a\,s)} = [0.5(0.002588)\ 0.04]^{0.5} \Rightarrow$ **$a_e = 0.0071944$ m $= 7.194$ mm**

$Z_{in,exact} = 4\, Z_A\, Z_{int} / (2\, Z_A + Z_{int}) = 4\,(62.7 - j31.9)(j2612.96)/[2\,(62.7 - j31.9) + j2612.96]$

$\Rightarrow$ **$Z_{in,t} = 262.875 - j\,117.862\ \Omega$**

transmission line mode input impedance = **$j\,2612.96\ \Omega$** effective radius = **7.194 mm**

(Circle one): Z_{ant}(radius = 42.588 mm) = 60.5 - j 23.6 Ω Z_{ant}(radius = 7.19 mm) = 62.7 – j 31.9 Ω

Z_{ant}(radius = 10.17 mm) = 61.8 - j 28.4 Ω Z_{ant}(radius = 5.09 mm) = 64.5 – j 37.2 Ω

overall input impedance = **262.875 - j 117.862 Ω**

3) A Baroque musician has commissioned a Venetian artisan to build a Vivaldi tapered slot antenna for her cello. If the antenna has an input impedance of $Z_A = 20 - j\,20\ \Omega$ at 3 GHz, she needs you to match it to a lossless transmission line ($Z_0 = 50\ \Omega$, $u = 2.4 \times 10^8$ m/s) using a discrete capacitor connected in parallel as close to antenna as possible. As part of the solution process, calculate the normalized input impedance and admittance for the antenna. Then, find the normalized y_M & un-normalized Y_M admittances of the appropriate match point and its distance d_M (cm) from the antenna. Calculate the necessary capacitance C of the parallel capacitor. Show, with labels, all work done on Smith chart. Draw a fully labeled sketch of final match.

- The wavelength is $\lambda = c\,/\,f = 2.4 \times 10^8/3 \times 10^9 = 0.08$ m = 8 cm.

Steps

1) Calculate normalized impedance $z_A = Z_A\,/Z_0 = (20 - j\,20)/50 \Rightarrow$ **$z_A = 0.4 - j\,0.4\ \Omega/\Omega$** and plot on **Smith chart** (see Figure 2).

2) Draw circle, centered on Smith chart, through z_A point. This circle of constant $|\Gamma|$ includes the locus of all possible z_{in} (and y_{in}) along the transmission line with this load.

3) Go $\lambda/4$ (180°) around the circle of constant $|\Gamma|$ from z_A point to **$y_A = 1.25 + j\,1.25$ S/S** point and plot.

4) Note, the two match points are $y_{m,i} = 1 \pm j1.14$ S/S. In order to use a discrete capacitor for matching, select **$y_M = 1 - j1.14$ S/S** as it has an inductive susceptance. Note, then un-normalized admittance is $Y_M = y_M\,(Y_0) = y_M\,/\,Z_0 = (1 - j1.14)/50 \Rightarrow$ **$Y_M = 0.02 - j\,0.0228$ S**.

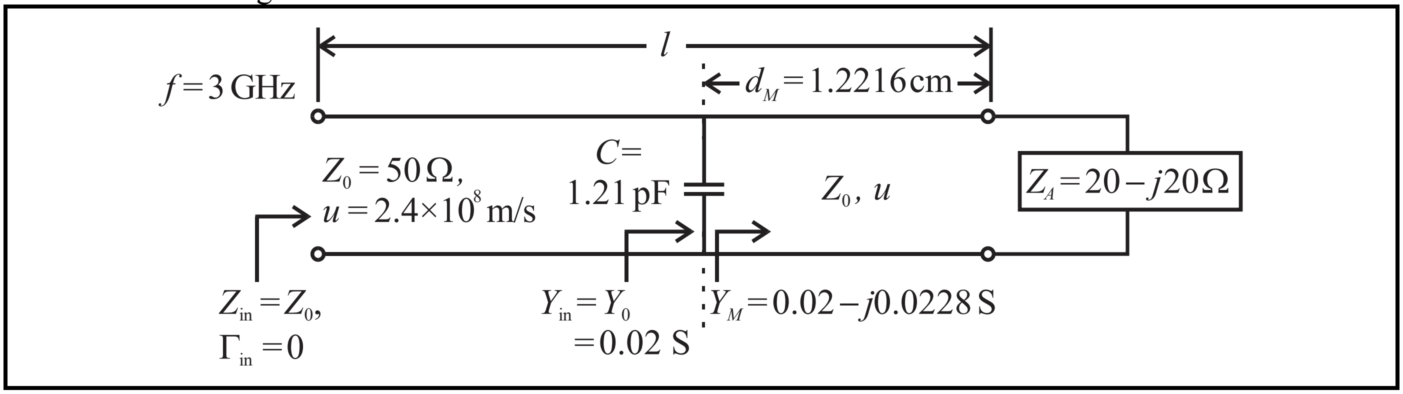
5) Find distance d_M from y_A to y_M using scales on Smith chart, $d_M = 0.334\lambda - 0.1813\lambda \Rightarrow$ **$d_M = 0.1527\lambda$** or, in meters, $d_M = 0.1527(0.08) \Rightarrow$ **$d_M = 0.012216$ m = 1.2216 cm**.

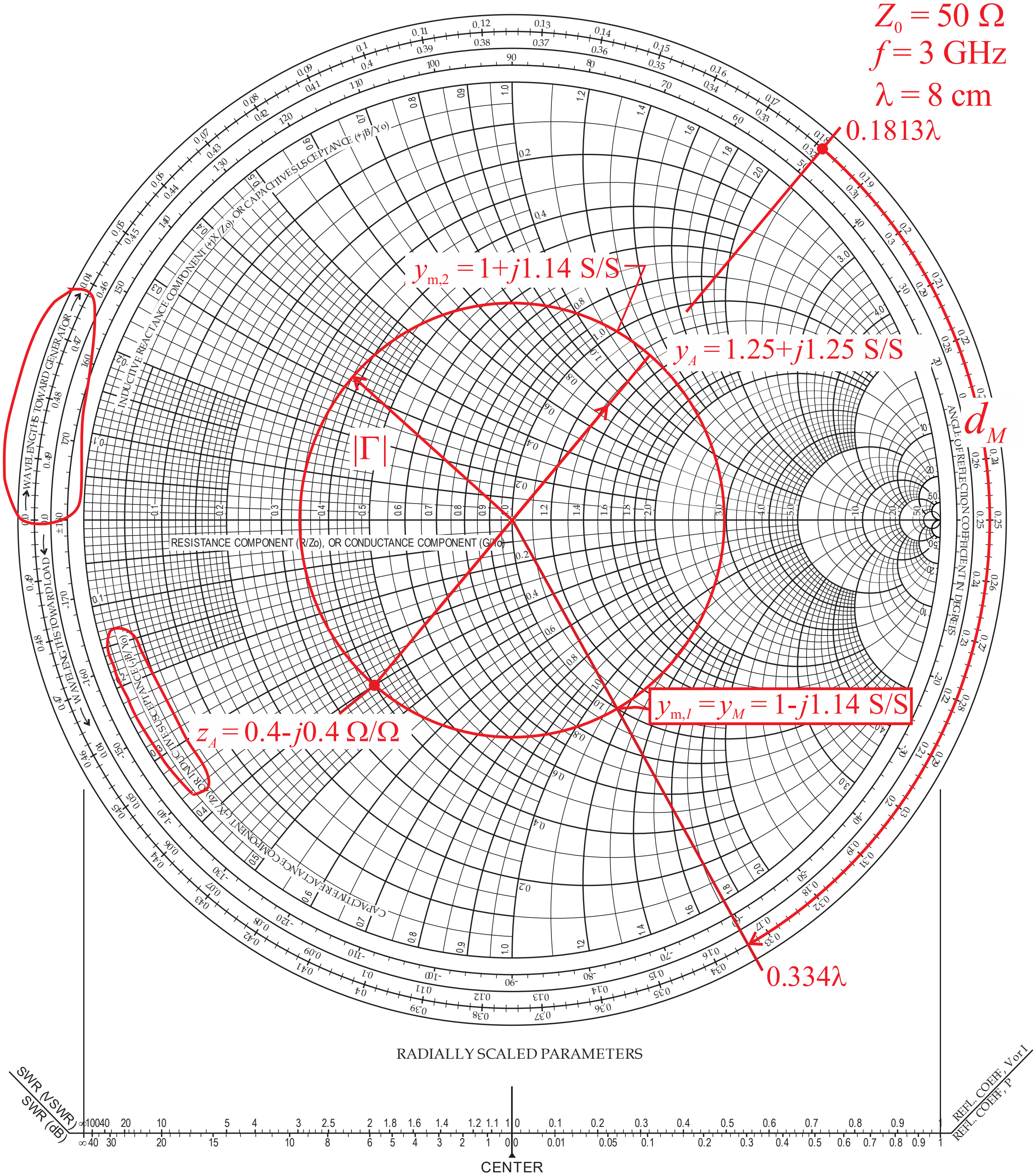
6) At d_1, add a discrete capacitor in parallel with susceptance $Y_{\text{cap}} = j\omega C = j0.0228$ S. Solving for C yields $C = 0.0228/(2\pi\,3\times10^9) = 1.209577\times10^{-12}$ F $\Rightarrow$ **$C = 1.2096$ pF**.

7) As shown on Figure 1, everywhere toward the source from the location of C will be matched, i.e., $Z_{\text{in}} = 50\ \Omega$.

z_A = **0.4 - j 0.4 Ω/Ω** y_A = **1.25 + j 1.25 S/S** y_M = **1 - j 1.14 S/S**

Y_M = **0.02 - j 0.0228 S** d_M = **1.2216 cm** C = **1.2096 pF** final circuit design





4) While suffering from sleep deprivation, a loopy student builds glow-in-the-dark dipole, centered on the z-axis in free space, using radium-uranium alloy ($\sigma = 2 \times 10^6$ S/m) wire of length 3.996 cm and a diameter of 0.8 mm. If it is operated at 749.5 MHz, calculate the length of the dipole as a fraction of a wavelength (ℓ/λ) and the wavenumber k. Is this dipole considered infinitesimal, finite, half-wavelength, or small? At the point $A(r = 7.9$ mm, $\theta = 90°$, $\phi = 120°)$ are we in the reactive near-field, radiating near-field (Fresnel), or far-field (Fraunhofer) region? Why? Also, find the radiation resistance, loss resistance, and efficiency (%) of this uniquely luminescent, mutation causing dipole.

$\lambda = c/f = 2.998 \times 10^8 / 749.5 \times 10^6 \quad \Rightarrow \quad \lambda = 0.4 \text{ m} = 40 \text{ cm}$

$k = 2\pi/\lambda = 2\pi/0.4 \quad \Rightarrow \quad$ **$k = 15.7079633$ rad/m**

$\ell/\lambda = 3.996 \times 10^{-2}/0.4 \quad \Rightarrow \quad$ **$\ell/\lambda = 0.0999$** $< 1/10 = 0.1 \quad \Rightarrow$ **small dipole**

Per (4-54), reactive near-field if $r_A < 0.62\sqrt{l^3/\lambda} = 0.62\sqrt{0.03996^3/0.4}$

$\Rightarrow r_A = 7.9 \text{ mm} > 7.83 \text{ mm}$ **NOT** reactive near-field

Per (4-53), point A is in the radiating near-field (Fresnel) if $0.62\sqrt{l^3/\lambda} < r_A < 2l^2/\lambda$

$7.83 \text{ mm} < r_A = 7.9 \text{ mm} < 2l^2/\lambda = 2(0.03996^2)/0.4 = 7.984 \text{ mm} \quad \Rightarrow$ **Fresnel**

Per (4-37), $R_r = \eta\frac{\pi}{6}(l/\lambda)^2 = 376.7303\frac{\pi}{6}(0.0999)^2 \quad \Rightarrow$ **$R_r = 1.9686121\ \Omega$**

Per (2-90b), the high frequency resistance is

$$R_{hf} = \frac{l}{P}\sqrt{\frac{\omega\mu}{2\sigma}} = \frac{0.03996}{\pi 0.0008}\sqrt{\frac{2\pi(749.5 \bullet 10^6)4\pi \bullet 10^{-7}}{2(2 \bullet 10^6)}} = 0.6115561\ \Omega$$

Per notes, for a small dipole, $R_{loss} = R_{hf}/3 = 0.611556/3 \quad \Rightarrow$ **$R_{loss} = 0.203852036\ \Omega$**

Per (2-90), the efficiency is

$$e_{cd} = \frac{R_r}{R_r + R_l} = \frac{1.9686121}{1.9686121 + 0.203852} \quad \Rightarrow \quad \mathbf{e_{cd} = 0.9061655 = 90.617\ \%}$$

$\ell/\lambda =$ **0.0999** $\quad k =$ **15.708 rad/m** $\quad$ infinitesimal, finite, half-wavelength, or **small**? (circle)

Point A in the near-field, **Fresnel**, or far-field region? (circle correct) Why? $0.62\sqrt{l^3/\lambda} < r_A < 2l^2/\lambda$

$R_{rad} =$ **1.96861 Ω** $\quad R_{loss} =$ **0.20385 Ω** $\quad$ efficiency = **0.9061655 = 90.617 %**