EE 483/583 Examination #1 (Spring 2022)

Name Key

Instructions: Place answers in indicated spaces, use notation as given in class for coordinates & vectors, and show all work for credit. Insert equation sheet and hand-in with exam. Assume $c = 2.9979 \times 10^8$ m/s.

Handy integrals: $\int \cos^n(ax)\sin(ax)\,dx = -\frac{\cos^{n+1}(ax)}{(n+1)a}$, $\int \sin^n(ax)\cos(ax)\,dx = \frac{\sin^{n+1}(ax)}{(n+1)a}$

$\int \cos^2(ax)\,dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$, $\int \cos^3(ax)\,dx = \frac{\sin(ax)}{a} - \frac{\sin^3(ax)}{3a}$, $\int \cos^4(ax)\,dx = \frac{3x}{8} + \frac{\sin(2ax)}{4a} - \frac{\sin(4ax)}{32a}$

$\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$, $\int \sin^3(ax)\,dx = \frac{-\cos(ax)}{a} + \frac{\cos^3(ax)}{3a}$, $\int \sin^4(ax)\,dx = \frac{3x}{8} - \frac{\sin(2ax)}{4a} + \frac{\sin(4ax)}{32a}$

1) a) An X-Band police radar system operates at 10.4 GHz and has a vertically-polarized horn antenna that is well-matched to the feeding transmission line. When tested on a range against a spherical calibration target with a radar cross section (RCS) of 0.3 m^2 at a distance of 33 m, determine the antenna gain G (unitless & dBi) if the receiver detects 1.6 nW of power when the transmitter outputs 20 W.

$$(2\text{-}125)\ P_r/P_t = e_{cd_t} e_{cd_r}(1-|\Gamma_t|^2)(1-|\Gamma_r|^2)\,\sigma \frac{D_t D_r}{4\pi}\left(\frac{\lambda}{4\pi R_1 R_2}\right)^2 |\hat{\rho}_w \cdot \hat{\rho}_r|^2$$

where $|\Gamma_t| = |\Gamma_r| = 0$ (well matched), $G = G_t = G_r = e_{cd} D$, $R = R_1 = R_2$,

$PLF = |\hat{\rho}_w \cdot \hat{\rho}_r|^2 = 1$ (sphere), $\lambda = c/f = 2.9979\times10^8/10.4\times10^9 = 0.028826$ m

$$G = \sqrt{\frac{P_r}{P_t}\left(\frac{4\pi R^2}{\lambda}\right)^2 \frac{4\pi}{\sigma}} = \sqrt{\frac{1.6\times10^{-9}}{20}\left(\frac{4\pi\, 33^2}{0.028826}\right)^2 \frac{4\pi}{0.3}} = \sqrt{755.24} = 27.48168$$

$G(\text{dBi}) = 10\log_{10} 27.482 = 14.39$ dBi

$G =$ 27.482 $=$ 14.39 dBi

b) Sonic the SD Mines student is headed for class after eating 6 muffins and drinking 3 Red Bulls. As usual, Sonic (RCS of 0.44 m^2) is late and passes a traffic patrol at a distance of 56 m doing 80 mph on St. Joseph Street. If Sonic's unique running stance causes the reflected radar signal to be tilted 18° from vertical, find the received power (in nW) from the harried Sonic. If the minimum power required for target velocity lock is 0.25 nW, will Sonic make it to class in time and without a ticket?

Per (2-125), w/ $G = 27.482$, $\sigma = 0.44\,m^2$, $\lambda = 0.028826$ m, $|\Gamma| = 0$, $R = 56$ m,

$P_t = 20$ W, and $PLF = \cos^2 18° = 0.9045$

$$P_{sonic} = 20(1)(1)\,0.44\,\frac{27.482^2}{4\pi}\left(\frac{0.028826}{4\pi\,56^2}\right)^2 0.9045$$

$$= 2.55957\times10^{-10}\ W = 0.255957\ nW > 0.25\ nW$$

Busted!

$P_{\text{Sonic}} =$ 0.256 nW

In time?/Ticket? yes ☺ / no ☹ (circle correct answer)

2) The vector magnetic potential for a small circular loop antenna operating in free space is
$$\bar{A} = \hat{a}_\phi \frac{a^2 \mu_0 I_0 \sin\theta}{4}\left(jk + \frac{1}{r}\right)\frac{e^{-jkr}}{r} \text{ (Wb/m)}.$$
First, determine the general **far-zone** phasor vector electric and magnetic fields. Then, evaluate these **far-zone** phasor vector electric and magnetic fields (put in polar form, e.g., $\bar{C} = \hat{a}\, C\angle\theta°$) at the point $V(r = 4\text{ m}, \theta = \pi/3, \phi = \pi/2)$ when $k = 2\pi/\lambda = 2.4\text{ m}^{-1}$, loop radius $a = 4$ cm, and the loop is driven with a $12\angle 0°$ A current. Assume vector electric potential $\bar{F} = 0$.

For far-zone fields, drop term $\propto \frac{1}{r^2}$. $\bar{A}_{FF} = \hat{a}_\phi \frac{a^2 \mu_0 I_0 \sin\theta\, jk}{4}\frac{e^{-jkr}}{r}$

Per (3-58a), $\bar{E}_{FF} = -j\omega \bar{A}_{FF} = \hat{a}_\phi \frac{+\omega k a^2 \mu_0 I_0 \sin\theta}{4}\frac{e^{-jkr}}{r}$ (V/m)

Per (3-58b), $\bar{H}_{FF}$ includes: $H_r \simeq 0$, $H_\theta = j\frac{\omega}{\eta_0}A_\phi$, + $H_\phi = -j\frac{\omega}{\eta_0}\underbrace{A_\theta}_{\to 0}$

$$\bar{H}_{FF} = \hat{a}_\theta \frac{j\omega}{\eta_0} A_{\phi,FF} = \hat{a}_\theta \frac{-\omega k a^2 \mu_0 I_0 \sin\theta}{4\eta_0}\frac{e^{-jkr}}{r} \text{ (A/m)}$$

$k = \frac{2\pi}{\lambda} = 2.4\text{ m}^{-1} \Rightarrow \lambda = \frac{2\pi}{2.4} = 2.618\text{ m} \Rightarrow f = c/\lambda = \frac{2.9979\times10^8}{2.618} = 114.511$ MHz

$= \omega/c$ $\qquad \omega = 2\pi f = 7.19496\times10^8$ rad/s

@ $V(r = 4\text{m}, \theta = \pi/3 = 60°, \phi = \pi/2 = 90°)$

$$\bar{E}_{FF} = \hat{a}_\phi \frac{7.19496\times10^8(2.4)0.04^2(4\pi\times10^{-7})(12\angle0°)\sin 60°}{4}\frac{e^{-j2.4(4)}}{4}$$

$$= \hat{a}_\phi\, 2.255077\, e^{-j\underbrace{9.6}_{-550.04°}} \text{ ← radians} = \hat{a}_\phi\, 2.2551\angle 169.96° \text{ V/m}$$

$$\bar{H}_{FF} = \hat{a}_\theta \frac{-7.19496\times10^8(2.4)0.04^2(4\pi\times10^{-7})(12\angle0°)\sin 60°}{4(376.7303)}\frac{e^{-j2.4(4)}}{4}$$

$$= -\hat{a}_\theta\, 0.00598592\, e^{-j9.6} = \hat{a}_\theta\, 5.9859\angle{-10.04°} \text{ mA/m}$$

$\bar{E}_{FF} = \hat{a}_\phi \frac{\omega k a^2 \mu_0 I_0 \sin\theta}{4}\frac{e^{-jkr}}{r}$ (V/m) $\qquad \bar{E}_{FFV} = \hat{a}_\phi\, 2.2551\angle 169.96°$ (V/m)

$\bar{H}_{FF} = -\hat{a}_\theta \frac{\omega k a^2 \mu_0 I_0 \sin\theta}{4\eta_0}\frac{e^{-jkr}}{r}$ (A/m) $\qquad \bar{H}_{FFV} = \hat{a}_\theta\, 5.9859\angle{-10.04°}$ (mA/m)

3) The radiation intensity of a 510 MHz antenna operating in free space is $\lambda = c/f = 0.587823 5_m$

$$U(\theta,\phi) = \begin{cases} 4.32\sin^3\theta\sin^2\phi \text{ (W/Sr)} & 0 \le \theta \le 180°,\ 0 \le \phi \le 180° \\ 0 & \text{elsewhere} \end{cases}.$$

Determine the radiated power and directivity $D(\theta, \phi)$. Then, find the half-power beamwidths in the azimuthal (wrt ϕ) & elevation (wrt θ) directions, maximum effective area, and maximum directivity D_{max} & gain G_{max} (unitless & dBi) as well as the θ & ϕ angles at which they occur (in degrees) assuming the antenna is 96% efficient and is matched to its feed and the incoming EM wave.

Per (2-13), $P_{rad} = \oiint U\, d\Omega = 4.32\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} \sin^3\theta\sin^2\phi\sin\theta\, d\theta\, d\phi$

$P_{rad} = 4.32\int_{\phi=0}^{\pi}\sin^2\phi\, d\phi\int_{\theta=0}^{\pi}\sin^4\theta\, d\theta = 4.32\left(\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right)\Big|_{\phi=0}^{\pi}\left(\frac{3\theta}{8} - \frac{\sin(2\theta)}{4} + \frac{\sin(4\theta)}{32}\right)\Big|_{\theta=0}^{\pi}$

$= 4.32\left[\left(\frac{\pi}{2} - 0\right) - (0-0)\right]\left[\left(\frac{3\pi}{8} - 0 + 0\right) - (0 - 0 + 0)\right] = 4.32\left(\frac{3\pi^2}{16}\right) = \underline{7.99438\text{ W}}$

Per (2-16), $D(\theta,\phi) = \frac{4\pi U}{P_{rad}} = \frac{4\pi\, 4.32\sin^3\theta\sin^2\phi}{7.99438} = \underline{6.7906105\sin^3\theta\sin^2\phi}$

For $0 \le \theta \le 180°$ & $0 \le \phi \le 180°$, $\underline{D_{max} = 6.79061 = 8.31909\text{ dBi}}$

$\underline{@\ \theta = \phi = 90°}$ Same

(2-49a), $G_{max} = e_{cd} D_{max} = 0.96(6.7906) = \underline{6.51899 = 8.1418\text{ dBi}}$

Azimuthal (set $\theta = 90°$) $U_{Az} = 4.32(1)\sin^2\phi \Rightarrow \sin^2\phi_H = 0.5$

$\phi_H = \sin^{-1}\sqrt{0.5} = 45°$ or $135° \Rightarrow$ Az. HPBW $= 135° - 45° = 90°$

Elevation (set $\phi = 90°$), $U_{El} = 4.32\sin^3\theta\,(1) \Rightarrow \sin^3\theta_H = 0.5$

$\theta_H = \sin^{-1} 0.5^{1/3} = 52.53269°$ or $127.46731° \Rightarrow$ El HPBW $= \underline{74.9346°}$

(2-111) $A_{em} = G_{max}(\lambda^2/4\pi) = 6.519\left(\frac{0.5878^2}{4\pi}\right) = \underline{0.179252\text{ m}^2}$

P_{rad} = 7.99438 W

$$D(\theta,\phi) = \begin{cases} 6.7906\sin^3\theta\sin^2\phi & 0 \le \theta \le 180°, 0 \le \phi \le 180° \\ 0 & \text{elsewhere} \end{cases}$$

D_{max} = 6.7906 = 8.319 dBi G_{max} = 6.519 = 8.142 dBi at angles $\theta = \phi = 90°$

Azimuthal HPBW = 90° Elevation HPBW = 74.9346° A_{em} = 0.17925 m^2