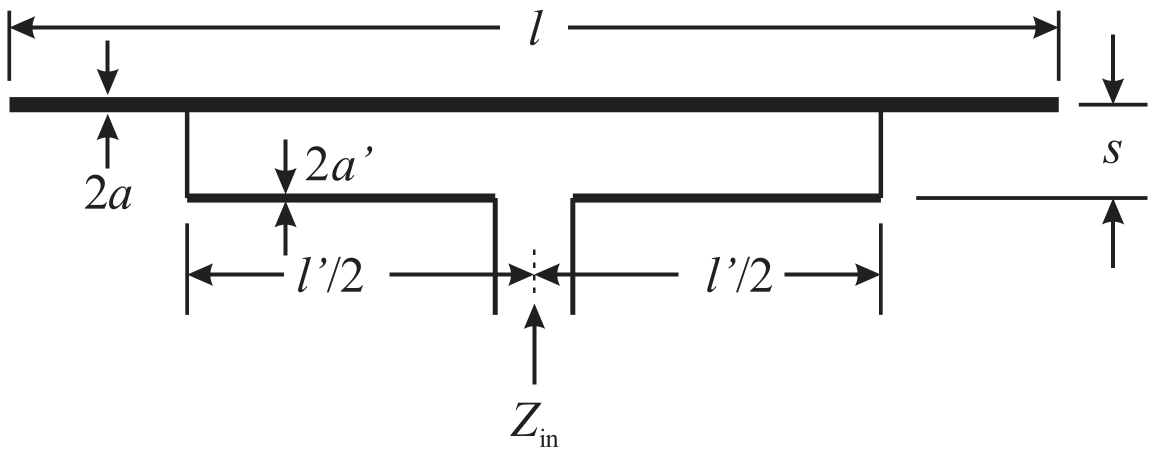
Matching Techniques For Driving Yagi-Uda Antennas: T-Match

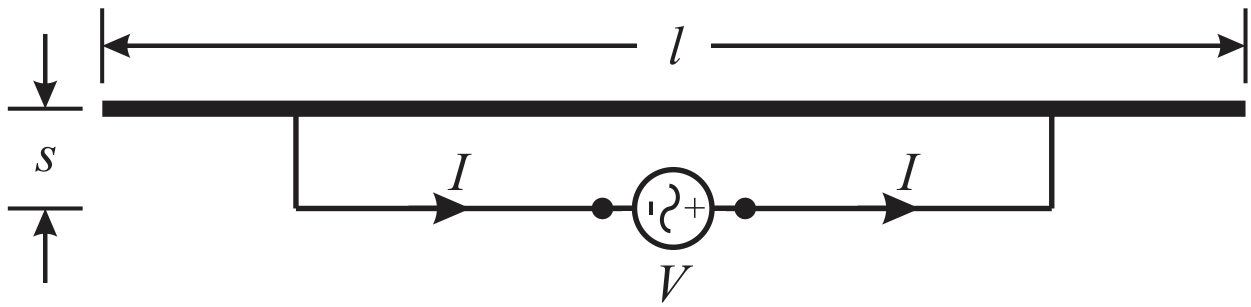
[Sections 9.5, 9.6, & 9.8 of *Antenna Theory, Analysis and Design* (4e) by Balanis)]

T-Match:



- The T-Match is a shunt-matching technique that can be used to feed a dipole or the **driven element** (l_2) of a Yagi-Uda antenna. It uses a second shorter dipole that is placed a small distance s ($s << \lambda$) from the main element and connected at the ends. In a Yagi-Uda antenna, it is parallel to and centered in the plane of the antenna and between the driven element and reflector.
- As it is symmetrical and balanced, it is typically used to connect twin-lead transmission lines to dipoles or Yagi-Uda antennas.
- Design analysis and procedure follows that for the folded dipole.
- Due to mutual coupling with the reflector and director elements, the design of the T-Match is approximate. In practice, length adjustments will usually be required.
- The characteristic impedance of the transmission line portion of the T-Match is given by $Z_0 = \frac{\eta}{2\pi}\cosh^{-1}\left(\frac{s^2 - a^2 - a'^2}{2aa'}\right)$, where $\eta = \sqrt{\mu / \varepsilon_{\text{eff}}}$ is the intrinsic impedance of the material wherein the T-Match exists.

Model:



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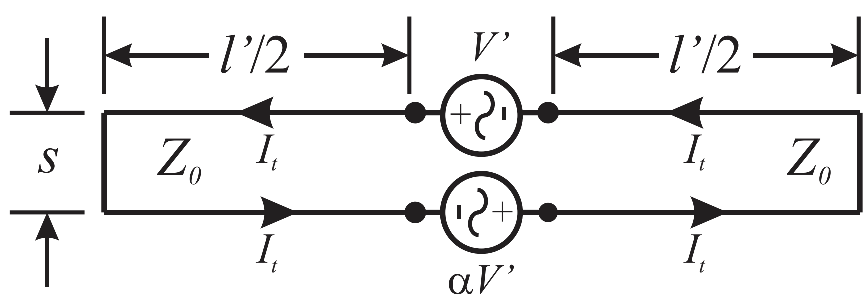
Transmission line mode



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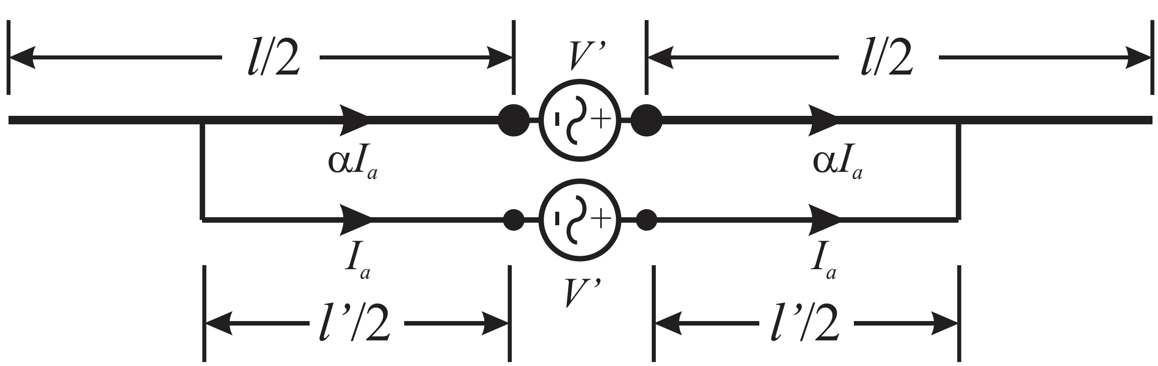
Antenna mode



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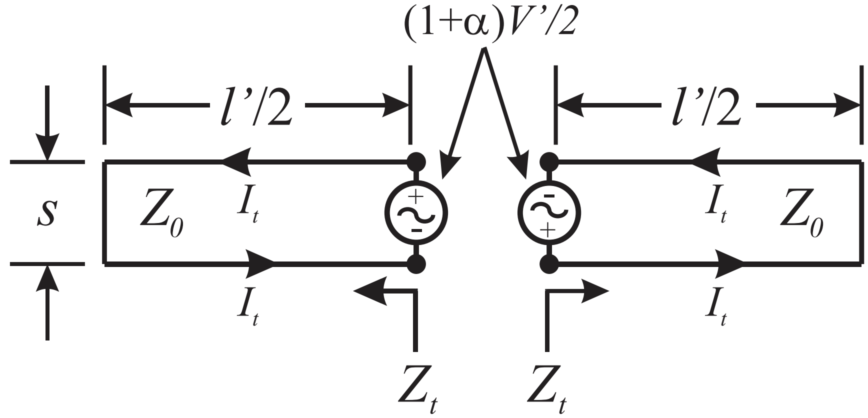
Transmission line mode



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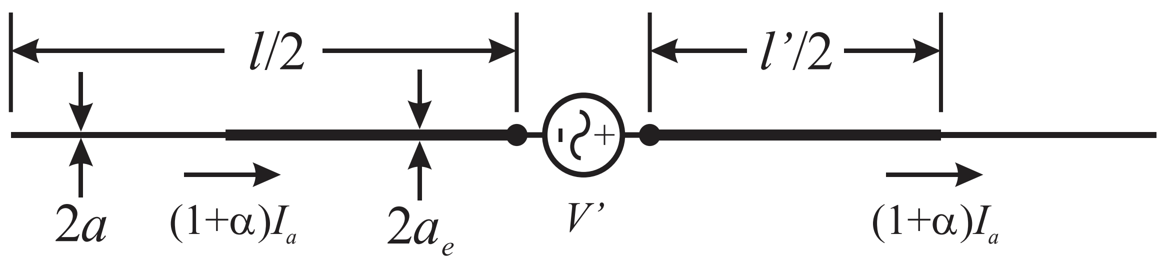
Antenna mode (symmetric about feed)



Above $V' = \dfrac{V}{1+\alpha}$, and we define a **current divisor factor α–**

$$\alpha = \frac{\cosh^{-1}\left(\dfrac{v^2 - u^2 + 1}{2v}\right)}{\cosh^{-1}\left(\dfrac{v^2 + u^2 - 1}{2vu}\right)} \qquad \begin{array}{l} \alpha > 1 \text{ when } a > a' \\ \alpha = 1 \text{ when } a = a' \\ \alpha < 1 \text{ when } a < a' \end{array}$$

where $u = \dfrac{a}{a'}$ and $v = \dfrac{s}{a'}$. The current divisor factor α has a big impact on the magnitude of Z_{in} (i.e., when α increases $|Z_{in}|$ increases and vice versa). In addition to the relationship between *a* and *a*' (above), α is **inversely** related to the spacing *s* (i.e., if *s* decreases, then α increases, and vice versa).

Transmission line mode impedance

Definition of transmission line mode input impedance

$$Z_t = \frac{\left(\frac{1+\alpha}{2}\right)V'}{I_t} = jZ_0 \tan\left(kl'/2\right)$$

where $k = \beta = 2\pi/\lambda$.

Notes:

- For $0 < l' < 0.5\lambda$, we get $Z_0 \tan(kl'/2) > 0$, i.e., **inductive** reactance. This is nearly always the case encountered when using a T-Match with a dipole or the driven element in a Yagi-Uda antenna.
- If $l' = 0.5\lambda$, then $Z_t = jZ_0 \tan(\pi/2) \to \infty$.

Antenna mode impedance and current

- The antenna impedance Z_a of the dipole or driven element is usually found numerically using a Method of Moments (MoM) program for a dipole that has radius a_e (the equivalent radius of the two wires) over the inner length l', corresponding to location of T-Match, and radius a for the tips of the dipole extending beyond the T-Match. **Note**: If T-Match is used with the driven element of a Yagi-Uda antenna, Z_a should be determined with the dipole (driven element) modeled as part of the overall Yagi-Uda antenna.
- Per Table 9.3, the equivalent radius for two closely spaced (center-to-center distance s) wires of radii a and a' is determined by-

$$\ln(a_e) \approx \ln(a') + \frac{1}{(1+u)^2}\left(u^2 \ln u + 2u \ln v\right) \quad \Rightarrow \quad \underline{a_e \approx a' e^{\frac{u^2 \ln u + 2u \ln v}{(1+u)^2}}}.$$

- Definition of antenna mode input impedance-

$$Z_a = \frac{V'}{(1+\alpha)I_a}$$

<u>Total impedance and current for T-Match</u>

The current at the terminals of the T-Match is

$$I = I_t + I_a = \frac{\left(\frac{1+\alpha}{2}\right)V'}{Z_t} + \frac{V'}{(1+\alpha)Z_a}$$

$$I = V'\left[\frac{1+\alpha}{2Z_t} + \frac{1}{(1+\alpha)Z_a}\right] = \frac{V}{1+\alpha}\left[\frac{1+\alpha}{2Z_t} + \frac{1}{(1+\alpha)Z_a}\right]$$

$$= V\left[\frac{1}{2Z_t} + \frac{1}{(1+\alpha)^2 Z_a}\right] = V\left[\frac{(1+\alpha)^2 Z_a + 2Z_t}{2(1+\alpha)^2 Z_a Z_t}\right]$$

Solving for the input admittance and impedance, yields

$$Y_{\text{in}} = \frac{I}{V} = \frac{1}{2Z_t} + \frac{1}{(1+\alpha)^2 Z_a} \quad \Rightarrow \quad \underline{Y_{\text{in}} = \frac{Y_t}{2} + \frac{Y_a}{(1+\alpha)^2}}$$

and

$$Z_{\text{in}} = \frac{V}{I} = \frac{1}{Y_{\text{in}}} \quad \Rightarrow \quad \underline{Z_{\text{in}} = \frac{2(1+\alpha)^2 Z_a Z_t}{(1+\alpha)^2 Z_a + 2Z_t}}.$$

For the case that $l' \approx \lambda/2$ (half-wave dipole), the transmission line impedance $|Z_t| >> |Z_a|$. Then, the input impedance becomes

$$Z_{\text{in}} \approx (1+\alpha)^2 Z_a$$

If $a = a'$, the current division factor $\alpha = 1$ and we get

$$Z_{\text{in}} \approx 4Z_a$$

as was the case for the folded dipole.

Note: If Z_a has an inductive reactance (i.e., $X_a > 0$), it may not be possible to achieve a realizable match using a standard T-Match as Z_t will also have an inductive reactance. In that case, either the length <u>l needs to be shortened</u> to make Z_a have a capacitive reactance (i.e., $X_a < 0$) or a modified T-Match may be used.

Design Process for T-Match in a Yagi-Uda antenna

- We desire to match a given Yagi-Uda antenna to a feeding transmission line characteristic impedance $Z_{0,\text{feed}}$. Typically, a specification in terms of the VSWR is given.

1) Select a driven element length l_2 (takes the place of l used on prior pages) so that $l_1 < l_2 < l_3$ as well as values for a', s, and l'. These values may be changed later.
   - Diameter of feed $2a'$- usually you will want this to be less than the Yagi-Uda element diameters $2a$ to make $\alpha > 1$.
   - T-Match spacing s- make less than $s_{12}/4$, more than 1 cm (practical construction), and less than 5 cm (don't want the characteristic impedance Z_0 of the T-Match section to be too large).
   - T-Match length l'- make less than half of the initial driven element length l_2, i.e., $l' \leq l_2/2$, to avoid overly disturbing the current distribution on l_2.

2) Calculate the characteristic impedance Z_0 of the transmission line portion of the T-Match-

$$Z_0 = \frac{\eta}{2\pi}\cosh^{-1}\left(\frac{s^2 - a^2 - a'^2}{2aa'}\right)$$

where $\eta = \sqrt{\mu/\varepsilon_{\text{eff}}}$ is the intrinsic impedance of the material wherein the T-Match exists.

3) Calculate the transmission line mode input impedance Z_t -

$$Z_t = jZ_0 \tan\left(kl'/2\right) \text{ where } k = \beta = 2\pi/\lambda.$$

4) Calculate the parameters u, v, and α-

$$u = \frac{a}{a'}, \quad v = \frac{s}{a'}, \quad \text{and} \quad \alpha = \frac{\cosh^{-1}\left(\dfrac{v^2 - u^2 + 1}{2v}\right)}{\cosh^{-1}\left(\dfrac{v^2 + u^2 - 1}{2vu}\right)} \quad \begin{array}{l} \alpha > 1 \text{ when } a > a' \\ \alpha = 1 \text{ when } a = a'. \\ \alpha < 1 \text{ when } a < a' \end{array}$$

5) Calculate the equivalent radius a_e for the section of the driven element corresponding to the T-Match-

$$a_e \approx a' e^{\frac{u^2 \ln u + 2u \ln v}{(1+u)^2}}.$$

6) Find the input impedance (i.e., use NEC-2) of the antenna mode Z_a of the driven element as part of the Yagi-Uda antenna. It is modeled as a dipole that has radius a_e over the inner length l', corresponding to location of T-Match, and radius a for the tips extending beyond the T-Match.

7) Find the overall input impedance -

$$Z_{\text{in}} = \frac{2(1+\alpha)^2 Z_a Z_t}{(1+\alpha)^2 Z_a + 2Z_t}.$$

8) Determine if Z_{in} meets your specification. If so, stop design process. If not, consider/try:

   - If Z_{in} *and* Z_a have inductive reactances (i.e., $X > 0$), it may be necessary to **shorten l_2** to make Z_a have a smaller inductive reactance or capacitive reactance (i.e., $X_a < 0$) to achieve an acceptable Z_{in}.
   - Consider changing l' *toward* the length suggested by

   $$l' = \frac{2}{k} \tan^{-1}\left[\frac{1}{2Z_0 \operatorname{Im}\left(\frac{Y_a}{(1+\alpha)^2}\right)}\right]$$

   to better offset the antenna mode reactance, and repeat steps 2) through 8). If necessary, l_2, a', and s can be varied.
   - Remember, the magnitude of the input impedance $|Z_{\text{in}}|$ is greatly affected by α (i.e., when α increases, $|Z_{\text{in}}|$ increases, and vice versa). In turn, α is inversely related to s (i.e., if s decreases, α increases, and vice versa).
   - See http://montoya.sdsmt.edu/ee483_583/notes/matching_tips.pdf for further tips.