Example- Rectangular Microstrip Design

1) Problem statement/specification- Design a rectangular microstrip antenna to operate at a frequency of 5 GHz on a RT/Duroid ($\varepsilon_r = 2.2$, $h = 1.588$ mm = 1/16"). The antenna is to be matched to a 50 Ω microstrip transmission line on this substrate. Accurately sketch final design.

$fr := 5 \cdot 10^9$ Hz $\varepsilon r := 2.2$ $h := 1.588 \cdot 10^{-3}$ m

$Zfd := 50$ Ω $Yfd := \frac{1}{Zfd}$ $Yfd = 0.02$ S

Define some constants

$c := 2.9979 \cdot 10^8$ m/s $\eta 0 := 376.7303$ Ω

$\lambda 0 := \frac{c}{fr}$ $\lambda 0 = 0.05996$ m $k0 := \frac{2 \cdot \pi}{\lambda 0}$ $k0 = 104.793$ rad/m

2) Calculate/estimate width of patch:

$$West := \frac{c}{2 \cdot fr} \cdot \sqrt{\frac{2}{\varepsilon r + 1}}$$ $West \cdot 100 = 2.37005$ cm

$\frac{West}{h} = 14.925$ $W := West$ $W \cdot 100 = 2.37$ cm

Note: To operate ONLY in the TM_{010} mode, W must be less than L. If $W > L$, the dominant mode is the TM_{001}. However, if the rectangular patch is properly excited, only the TM_{010} mode will be active. Regardless, keep $W < 2L$ (where the mode that will be excited by the microstrip feed occurs).

3) Calculate the effective permittivity

$$\varepsilon r\_eff := \frac{\varepsilon r + 1}{2} + \frac{\varepsilon r - 1}{2} \cdot \left(1 + 12 \cdot \frac{h}{W}\right)^{-0.5}$$ $\varepsilon r\_eff = 2.04671$

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4) Calculate the fringing length ΔL

$$\Delta L := 0.412 \cdot h \cdot \frac{\varepsilon r\_eff + 0.3}{\varepsilon r\_eff - 0.258} \cdot \frac{\frac{W}{h} + 0.264}{\frac{W}{h} + 0.8}$$

$\Delta L = 8.291 \times 10^{-4}$ m

$\Delta L \cdot 1000 = 0.8291$ mm

5) Calculate effective patch length L_{eff} and guided wavelength λ

$$Leff := \frac{c}{2 \cdot fr \cdot \sqrt{\varepsilon r\_eff}}$$

$Leff = 0.02096$ m, or $Leff \cdot 100 = 2.0955$ cm

$\lambda := 2 \cdot Leff$

$\lambda = 0.04191$ m, or $\lambda \cdot 100 = 4.19101$ cm

6) Calculate physical patch length L, L/λ, and check if $W/L > 2$?

$L := Leff - 2 \cdot \Delta L$

$L = 0.019297$ m

$L \cdot 100 = 1.9297$ cm

$\frac{L}{\lambda} = 0.4604$

$\frac{W}{L} = 1.228$ < 2 (will be OK)

7) Calculate the slot conductances $G_1 = G_2$ and susceptances $B_1 = B_2$

Slot conductances and susceptances based on infinite slot approximation

$$G1est := \frac{W}{120 \cdot \lambda 0} \cdot \left[1 - \frac{1}{24} \cdot (k0 \cdot h)^2\right]$$

$G1est = 3.29 \times 10^{-3}$ S

$$B1est := \frac{W}{120 \cdot \lambda 0} \cdot (1 - 0.636 \cdot \ln(k0 \cdot h))$$

$B1est = 7.051 \times 10^{-3}$ S

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7) continued

Slot conductance based on cavity model (more accurate)

$$G1 := \frac{1}{\pi \cdot \eta 0} \cdot \int_0^{\pi} \left(\frac{\sin\left(\frac{k0 \cdot W}{2} \cdot \cos(\theta)\right)}{\cos(\theta)} \right)^2 \cdot (\sin(\theta))^3 \, d\theta$$

$G2 := G1$ $\qquad G1 = 1.5735 \times 10^{-3}$ S

Estimate true slot susceptance

$$B1 := \frac{G1}{G1est} \cdot B1est$$

$B2 := B1$ $\qquad B1 = 3.37206 \times 10^{-3}$ S

Slot admittances and impedances

$Y1 := G1 + j \cdot B1$ $\qquad Y2 := Y1$ $\qquad Y1 = 1.574 \times 10^{-3} + 3.372i \times 10^{-3}$ S

$Z1 := \frac{1}{Y1}$ $\qquad Z2 := Z1$ $\qquad Z1 = 113.638 - 243.527i$ Ω

8) Calculate the characteristic impedance $Z_{c,ant}$ and admittance $Y_{c,ant}$ of microstrip antenna (treated as a microstrip transmission line)

Since $W / h > 1$

$$Zcant := \frac{\eta 0}{\sqrt{\varepsilon r\_eff} \cdot \left(\frac{W}{h} + 1.393 + 0.667 \cdot \ln\left(\frac{W}{h} + 1.44 \right) \right)}$$

$Zcant = 14.483$ Ω $\qquad Ycant := \frac{1}{Zcant}$ $\qquad Ycant = 0.069$ S

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9) Use Smith chart and equations to verify that the translated slot 2 admittance Y_{2t} is approximately the complex conjugate of the slot 1 admittance Y_1.

Smith chart approach- normalize slot 2 admittance and plot

$y2 := Y2 \cdot Zcant \qquad y2 = 0.023 + 0.049i \quad$ S/S

move $L/\lambda = 0.4603$ toward the generator, on circle of constant $|\Gamma|$, and read

$y2t := 0.023 - j \cdot 0.2 \quad$ S/S — partial cancellation of imaginary part, real part unchanged.

Analytic approach-

$$z2t := \frac{Z2 + j \cdot Zcant \cdot \tan\left(2 \cdot \pi \cdot \frac{L}{\lambda}\right)}{Zcant + j \cdot Z2 \cdot \tan\left(2 \cdot \pi \cdot \frac{L}{\lambda}\right)} \qquad \frac{1}{z2t} = 0.024 - 0.202i$$

About the same!

From these results, we see that the approximation that $G_{2t} = G_1$ is very good. However, the approximation that $B_{2t} = -B_1$ is only roughly true. There is the issue of what distance is appropriate to use for the translation- L/λ, L_{eff}/λ, somewhere in between?

In any case, the calculation of B_1 is very approximate and using the inset feed would further change its actual value. However, we should see some cancellation/minimization of the input susceptance when Y_{2t} is added to Y_1.

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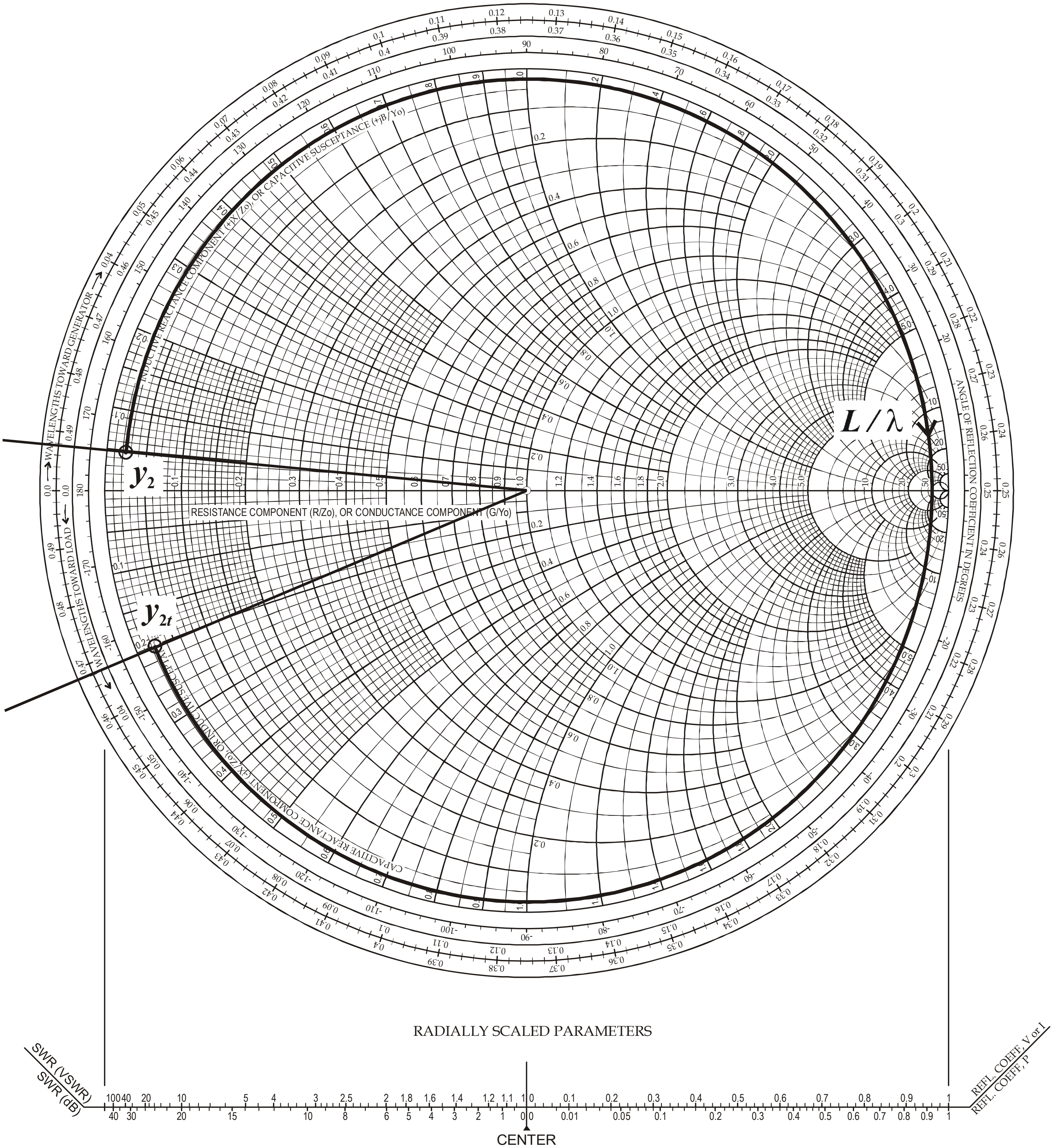
Simple
Smith Chart



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10) Calculate the mutual conductance between the slots.

$$G12 := \frac{1}{\pi \cdot \eta 0} \cdot \int_0^{\pi} \left(\frac{\sin\left(\frac{k0 \cdot W}{2} \cdot \cos(\theta)\right)}{\cos(\theta)} \right)^2 \cdot J0(k0 \cdot L \cdot \sin(\theta)) \cdot (\sin(\theta))^3 \, d\theta$$

$$G12 = 5.163 \times 10^{-4} \quad S$$

11) Calculate expected input impedance (assumed to be real) at the edge of the rectangular patch

$$Rin := \frac{1}{2 \cdot (G1 + G12)} \qquad Rin = 239.2541 \quad \Omega$$

Not a good match to 50 Ω.

12) Calculate length to inset microstrip feed

Check to see if conditions met for approximation- $\frac{G1}{Yfd} = 0.079$ $\ll 1$ OK

$\frac{B1}{Yfd} = 0.169$ $\ll 1$? So-so

Estimate inset length as start point for iterative solution- $y0est := \frac{L}{\pi} \cdot acos\left(\sqrt{\frac{Zfd}{Rin}}\right)$

$y0est = 0.0067$ m. Is it $< L/2$? $0.5 \cdot L = 0.0096$ m. OK

$\frac{L}{4} = 0.0048$ m is another possible starting value

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12) continued

$$Rinset(y0) := Rin\cdot\left[\left(\cos\left(\frac{\pi\cdot y0}{L}\right)\right)^2 + \frac{\left(G1^2 + B1^2\right)}{Yfd^2}\cdot\left(\sin\left(\frac{\pi\cdot y0}{L}\right)\right)^2 - \frac{B1}{Yfd}\cdot\sin\left(\frac{2\pi\cdot y0}{L}\right)\right]$$

$Rinset(y0est) = 23.749$ Ω   Too low (overshot), try a smaller/shorter inset.

Iteratively select y0-

$y01 := 0.0048$	m	$Rinset(y01) = 84.341$	Ω
$y02 := 0.006$	m	$Rinset(y02) = 43.209$	Ω
$y03 := 0.0055$	m	$Rinset(y03) = 59.195$	Ω
$y0 := 0.00578013$	m	$Rinset(y0) = 50$	Ω

$\boxed{y0\cdot 100 = 0.578}$ cm

13) Calculate width of 50 ohm feeding microstrip:

$$\varepsilon r\_fd(W0) := \frac{\varepsilon r + 1}{2} + \frac{\varepsilon r - 1}{2}\cdot\left(1 + 12\cdot\frac{h}{W0}\right)^{-0.5}$$

$$Zcfd(W0) := \frac{\eta 0}{\sqrt{\varepsilon r\_fd(W0)}\cdot\left(\frac{W0}{h} + 1.393 + 0.667\cdot\ln\left(\frac{W0}{h} + 1.444\right)\right)}$$

Iteratively select W0 (in terms of substrate thickness *h*)-

$W0 := 3.1037\cdot h$ $Zcfd(W0) = 50$ Ω

$\varepsilon r\_fd(W0) = 1.872$ $W0 = 0.004929$ m $\boxed{W0\cdot 100 = 0.4929}$ cm

$\frac{W0}{W} = 0.208$ Microstrip feed only blocks about 20% of slot 1, should be OK.

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14) Select a notch width n is needed. Arbitrarily select $n \sim W_0/4 = 0.125$ cm.

15) Draw resulting design