Example- Rectangular Microstrip Cavity Analysis

1) Problem statement/specification- Design a rectangular microstrip antenna to operate at a frequency of 5 GHz on a RT/Duroid (ε_r = 2.2, *h* = 1.588 mm = 1/16”).

The antenna is to be matched to a 50 Ω microstrip transmission line on this substrate. Accurately sketch final design.

$fr := 5 \cdot 10^{9}$ Hz $\varepsilon r := 2.2$ $h := 1.588 \cdot 10^{-3}$ mm

$Zfd := 50$ Ω $Yfd := \frac{1}{Zfd}$ $Yfd = 0.02$ S

Define some constants

$c := 2.9979 \cdot 10^{8}$ m/s $\eta 0 := 376.7303$ Ω

$\lambda 0 := \frac{c}{fr}$ $\lambda 0 = 0.05996$ m $k0 := \frac{2 \cdot \pi}{\lambda 0}$ $k0 = 104.793$ rad/m

2) Calculate/estimate width of patch:

$W := \frac{c}{2 \cdot fr} \cdot \sqrt{\frac{2}{\varepsilon r + 1}}$ $\frac{W}{h} = 14.925$ $\boxed{W \cdot 100 = 2.37}$ cm

3) Calculate the effective permittivity

$$\varepsilon r\_eff := \frac{\varepsilon r + 1}{2} + \frac{\varepsilon r - 1}{2} \cdot \left(1 + 12 \cdot \frac{h}{W}\right)^{-0.5}$$ $\varepsilon r\_eff = 2.04671$

4) Calculate the fringing length ΔL

$$\Delta L := 0.412 \cdot h \cdot \frac{\varepsilon r\_eff + 0.3}{\varepsilon r\_eff - 0.258} \cdot \frac{\frac{W}{h} + 0.264}{\frac{W}{h} + 0.8}$$

$\Delta L = 8.291 \times 10^{-4}$ m

$\Delta L \cdot 1000 = 0.8291$ mm

5) Calculate effective patch length L_{eff} and guided wavelength λ

$Leff := \frac{c}{2 \cdot fr \cdot \sqrt{\varepsilon r\_eff}}$ $Leff = 0.02096$ m, or $Leff \cdot 100 = 2.0955$ cm

$\lambda := 2 \cdot Leff$ $\lambda = 0.04191$ m, or $\lambda \cdot 100 = 4.19101$ cm

Example- Rectangular Microstrip Cavity Analysis cont.

6) Calculate physical patch length L, L/λ, and check if $W/L > 2$?

$L := Leff - 2\cdot\Delta L$ $L = 0.019297$ m $L\cdot 100 = 1.9297$ cm

$\frac{L}{\lambda} = 0.4604$ $\frac{W}{L} = 1.228$ < 2 (will be OK)

7) Calculate the slot conductances $G_1 = G_2$ and susceptances $B_1 = B_2$

$$G1est := \frac{W}{120\cdot\lambda 0}\cdot\left[1 - 24\cdot(k0\cdot h)^2\right] \qquad G1est = 1.105\times 10^{-3} \text{ S}$$

$$B1est := \frac{W}{120\cdot\lambda 0}\cdot(1 - 0.636\cdot\ln(k0\cdot h)) \qquad B1est = 7.051\times 10^{-3} \text{ S}$$

$$G1 := \frac{1}{\pi\cdot\eta 0}\cdot\int_0^{\pi}\left(\frac{\sin\left(\frac{k0\cdot W}{2}\cdot\cos(\theta)\right)}{\cos(\theta)}\right)^2\cdot(\sin(\theta))^3\,d\theta$$

$G2 := G1$ $G1 = 1.5735\times 10^{-3}$ S

$B1 := \frac{G1}{G1est}\cdot B1est$ $B2 := B1$ $B1 = 0.01004$ S

$Y1 := G1 + j\cdot B1$ $Y2 := Y1$ $Y1 = 1.574\times 10^{-3} + 0.01i$ S

$Z1 := \frac{1}{Y1}$ $Z2 := Z1$ $Z1 = 15.227 - 97.186i$ Ω

8) Calculate $Z_{c,ant}$ and admittance $Y_{c,ant}$ of microstrip antenna

$$Zcant := \frac{\eta 0}{\sqrt{\varepsilon r\_eff}\cdot\left(\frac{W}{h} + 1.393 + 0.667\cdot\ln\left(\frac{W}{h} + 1.44\right)\right)}$$

$Zcant = 14.483$ Ω $Ycant := \frac{1}{Zcant}$ $Ycant = 0.069$ S

Example- Rectangular Microstrip Cavity Analysis cont.

9) skip

10) Calculate the mutual conductance between the slots.

$$G12 := \frac{1}{\pi \cdot \eta 0} \cdot \int_0^{\pi} \left(\frac{\sin\left(\frac{k0 \cdot W}{2} \cdot \cos(\theta)\right)}{\cos(\theta)} \right)^2 \cdot J0(k0 \cdot L \cdot \sin(\theta)) \cdot (\sin(\theta))^3 \, d\theta$$

$G12 = 5.163 \times 10^{-4}$ S

11) Calculate expected input impedanceat the edge of the rectangular patch

$Rin := \frac{1}{2 \cdot (G1 + G12)}$ $Rin = 239.2541$ Ω

12) Calculate length to inset microstrip feed

$$Rinset(y0) := Rin \cdot \left[\left(\cos\left(\frac{\pi \cdot y0}{L} \right) \right)^2 + \frac{\left(G1^2 + B1^2 \right)}{Yfd^2} \cdot \left(\sin\left(\frac{\pi \cdot y0}{L} \right) \right)^2 - \frac{B1}{Yfd} \cdot \sin\left(\frac{2\pi \cdot y0}{L} \right) \right]$$

$y0 := 0.0042214$ m $Rinset(y0) = 50$ Ω

$y0 \cdot 100 = 0.422$ cm

13) Calculate width of 50 ohm feeding microstrip:

$$\varepsilon r\_fd(W0) := \frac{\varepsilon r + 1}{2} + \frac{\varepsilon r - 1}{2} \cdot \left(1 + 12 \cdot \frac{h}{W0} \right)^{-0.5}$$

$$Zcfd(W0) := \frac{\eta 0}{\sqrt{\varepsilon r\_fd(W0)} \cdot \left(\frac{W0}{h} + 1.393 + 0.667 \cdot \ln\left(\frac{W0}{h} + 1.444 \right) \right)}$$

$W0 := 3.1037 \cdot h$ $Zcfd(W0) = 50$ Ω

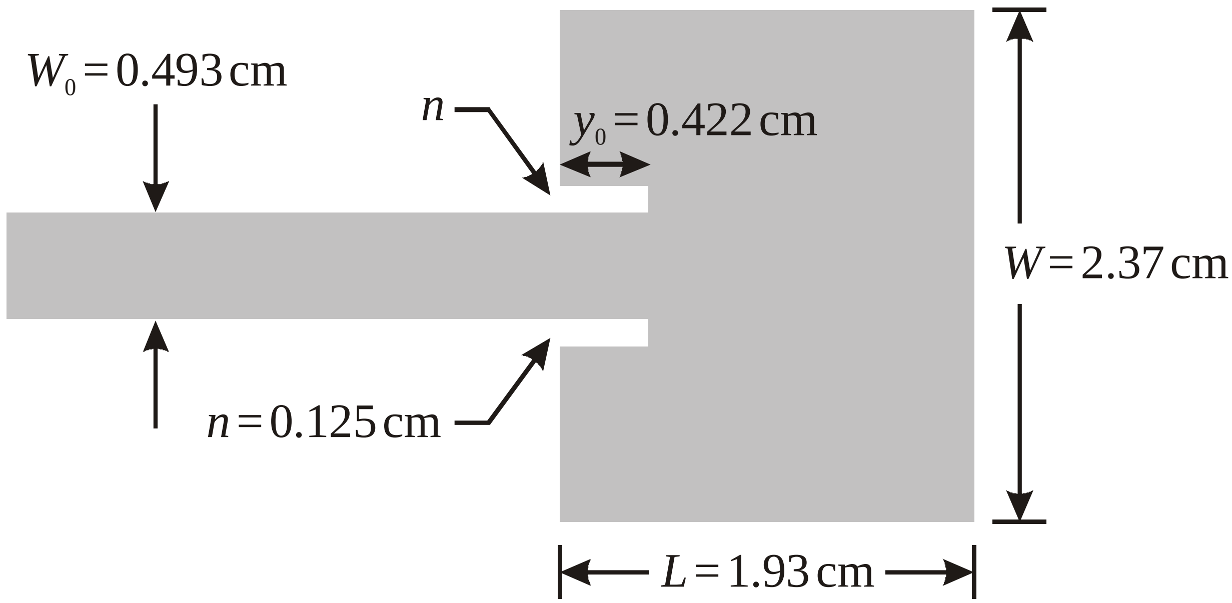
$\varepsilon r\_fd(W0) = 1.872$ $W0 = 0.004929$ m $W0 \cdot 100 = 0.4929$ cm

Example- Rectangular Microstrip Cavity Analysis cont.

14) Select a notch width *n* is needed. Arbitrarily select $n \sim W_0/4 = 0.125$ cm.

15) Draw resulting design

Top View



Find the maximum directivity of this design

Method 1 (assumes $k_0 h << 1$, a fair assumption) $k0 \cdot h = 0.1664$

$$I2 := \int_0^{\pi} \int_0^{\pi} \left(\frac{\sin\left(\frac{k0 \cdot W}{2} \cdot \cos(\theta)\right)}{\cos(\theta)} \right)^2 \cdot (\sin(\theta))^3 \cdot \left(\cos\left(\frac{k0 \cdot Leff}{2} \cdot \sin(\theta) \cdot \sin(\phi) \right) \right)^2 \, d\theta \, d\phi$$

$I2 = 3.625$

$$Dtot1 := \left(\frac{2 \cdot \pi \cdot W}{\lambda 0} \right)^2 \cdot \frac{\pi}{I2}$$

$Dtot1 = 5.346$

$10 \cdot \log(Dtot1) = 7.28$ dBi

Example- Rectangular Microstrip Cavity Analysis cont.

Method 2 (assumes $k_0 h << 1$, a fair assumption) $k0 \cdot h = 0.1664$

$$I1 := \int_0^{\pi} \left(\frac{\sin\left(\frac{k0 \cdot W}{2} \cdot \cos(\theta)\right)}{\cos(\theta)} \right)^2 \cdot (\sin(\theta))^3 \, d\theta \qquad I1 = 1.862$$

First, find directivity of a single slot- $D0 := \left(\frac{2 \cdot \pi \cdot W}{\lambda 0}\right)^2 \cdot \frac{1}{I1}$ $D0 = 3.312$

Then, find overall directivity

$$Dtot2 := D0 \cdot \frac{2}{1 + \frac{G12}{G1}}$$

$Dtot2 = 4.988$

$10 \cdot \log(Dtot2) = 6.9792$ dBi

A bit less than the first method.

Method 3 Use Figure 14.22 in Antenna Theory(4th Edn) by Balanis (valid since $\varepsilon_r = 2.2 \sim 2.25$)

$\frac{W}{\lambda 0} = 0.395$ Plot vertical line on figure at this value (see following page).

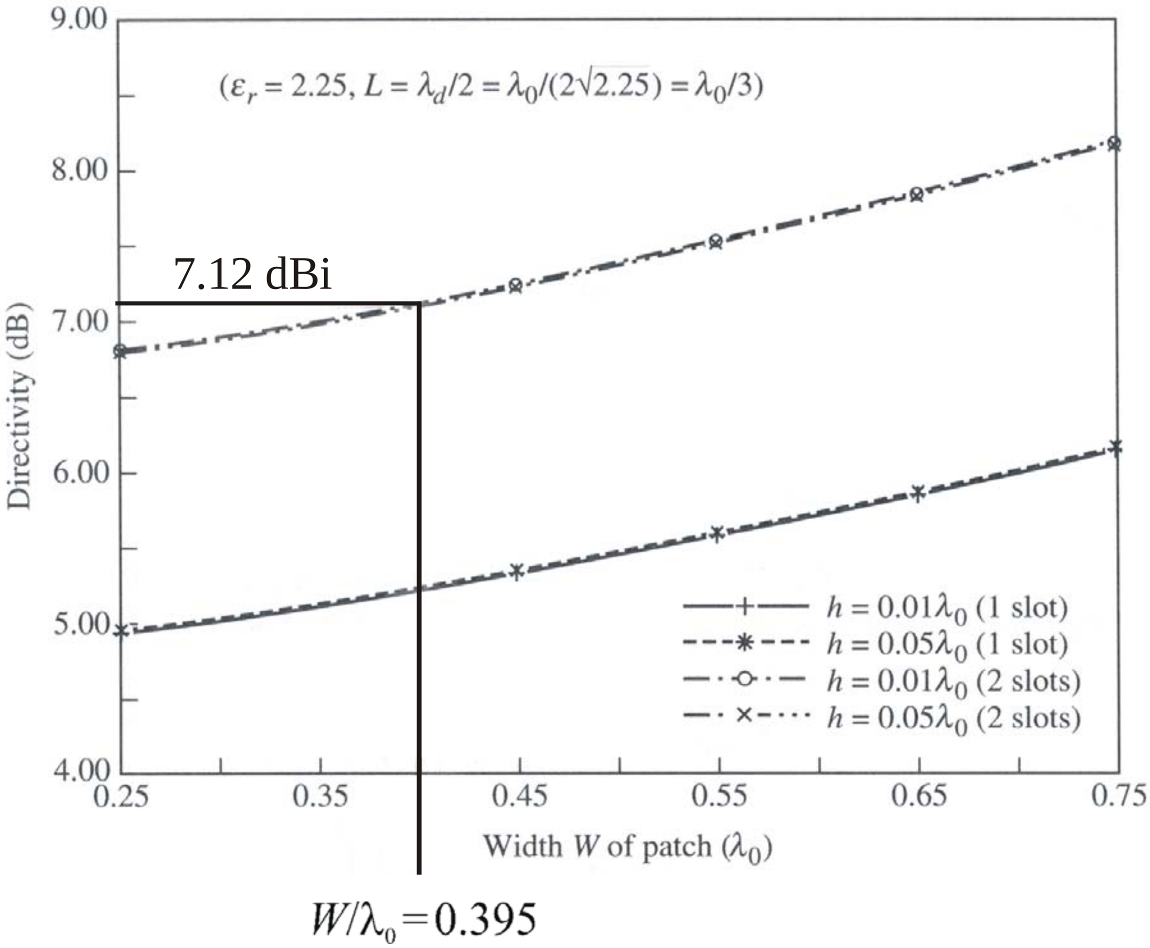
Read directivity (in dBi) as- $Dtot3 := 7.12$ dBi

About halfway in between previous results.

All three methods agree to within 0.3 dB.

Example- Rectangular Microstrip Cavity Analysis cont.

Method 3 cont.



Estimate the HPBWs for this design using (14-58) & (14-59)

$$\Theta E := 2 \cdot \operatorname{asin}\left[\sqrt{\frac{7.03 \cdot \lambda 0^2}{4\left(3 \cdot Leff^2 + h^2\right) \cdot \pi^2}}\right] \qquad \Theta E \cdot \frac{180}{\pi} = 88.283 \quad \text{deg}$$

$$\Theta H := 2 \cdot \operatorname{asin}\left(\sqrt{\frac{1}{2 + k0 \cdot W}}\right) \qquad \Theta H \cdot \frac{180}{\pi} = 56.363 \quad \text{deg}$$

Example- Rectangular Microstrip Cavity Analysis cont.

Define function for normalized far-zone electric field and spherical angles θ & ϕ

$X(\theta,\phi) := \frac{k0\cdot h}{2}\cdot\sin(\theta)\cdot\cos(\phi)$ $\qquad$ $Z(\theta) := \frac{k0\cdot W}{2}\cdot\cos(\theta)$

$$E(\theta,\phi) := \sin(\theta)\cdot\frac{\sin(X(\theta,\phi))}{X(\theta,\phi)}\cdot\frac{\sin(Z(\theta))}{Z(\theta)}\cdot\cos\left(\frac{k0\cdot Leff}{2}\cdot\sin(\theta)\cdot\sin(\phi)\right)$$

$n := 0..180$ $\qquad$ $phi_n := (n-90)\cdot\frac{\pi}{180}$ $\qquad$ $theta_n := n\cdot\frac{\pi}{180}$

Calculate normalized E-plane (*x*-*y* plane for *x* > 0) far-zone electric field & HPBW

$$E\_Eplane_n := \left|E\left(\frac{\pi}{2},phi_n\right)\right|$$

$$E\_Eplane\_dB_n := \text{if}\left(E\_Eplane_n < 0.01, 0, 20\cdot\log\left(E\_Eplane_n\right)+40\right)$$

$\phi1 := 45.63\cdot\frac{\pi}{180}$ $\qquad$ $\left|E\left(\frac{\pi}{2},\phi1\right)\right| = 0.7071$

$\phi2 := -45.63\cdot\frac{\pi}{180}$ $\qquad$ $\left|E\left(\frac{\pi}{2},\phi2\right)\right| = 0.7071$

Symmetric about ϕ = 0 at θ = 90 deg

$HPBW\_Eplane := (\phi1-\phi2)\cdot\frac{180}{\pi}$ $\qquad$ $\boxed{HPBW\_Eplane = 91.26}$ degrees

Estimate from earlier using (14-58) $\qquad$ $\Theta E\cdot\frac{180}{\pi} = 88.28$ degrees

Good agreement

Example- Rectangular Microstrip Cavity Analysis cont.

Calculate normalized H-plane (*x-z* plane for *x* > 0) far-zone electric field & HPBW

$$E\_Hplane_n := \left|E\left(theta_n, 0\right)\right|$$

$$E\_Hplane\_dB_n := \text{if}\left(E\_Hplane_n < 0.01, 0, 20 \cdot \log\left(E\_Hplane_n\right) + 40\right)$$

$$\theta 1 := 51.55 \cdot \frac{\pi}{180} \qquad \left|E(\theta 1, 0)\right| = 0.7071$$

Symmetric about θ = 90 deg at ϕ = 0

$$\theta 2 := 128.45 \cdot \frac{\pi}{180} \qquad \left|E(\theta 2, 0)\right| = 0.7071$$

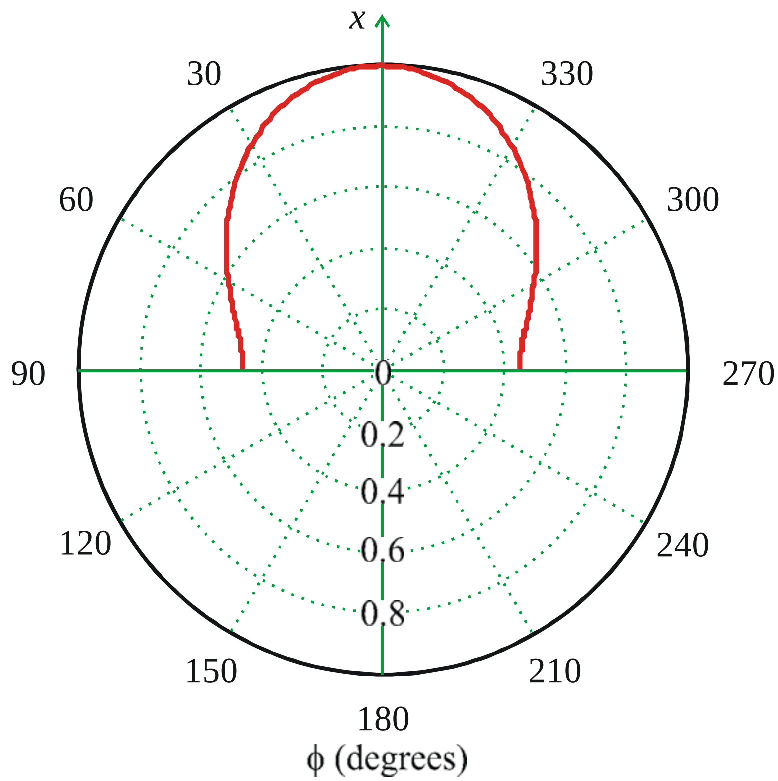
$$HPBW\_Hplane := (\theta 2 - \theta 1) \cdot \frac{180}{\pi} \qquad \boxed{HPBW\_Hplane = 76.9} \quad \text{degrees}$$

Estimate from earlier using (14-59) $\Theta H \cdot \frac{180}{\pi} = 56.36$ degrees

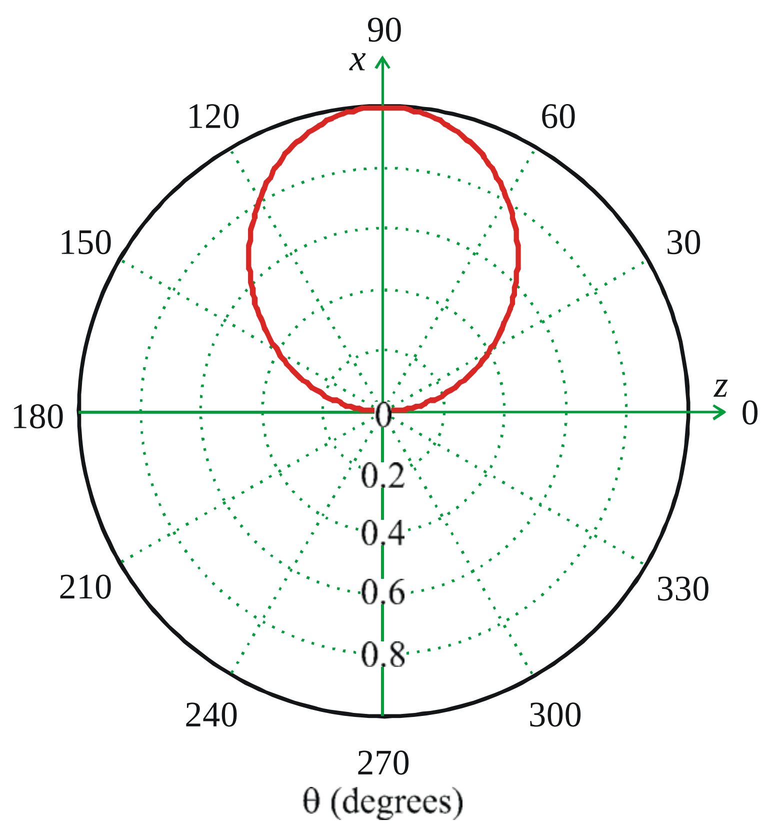
Not very good agreement

Example- Rectangular Microstrip Cavity Analysis cont.

Normalized |E| in E-plane (x-y plane with $x > 0$)

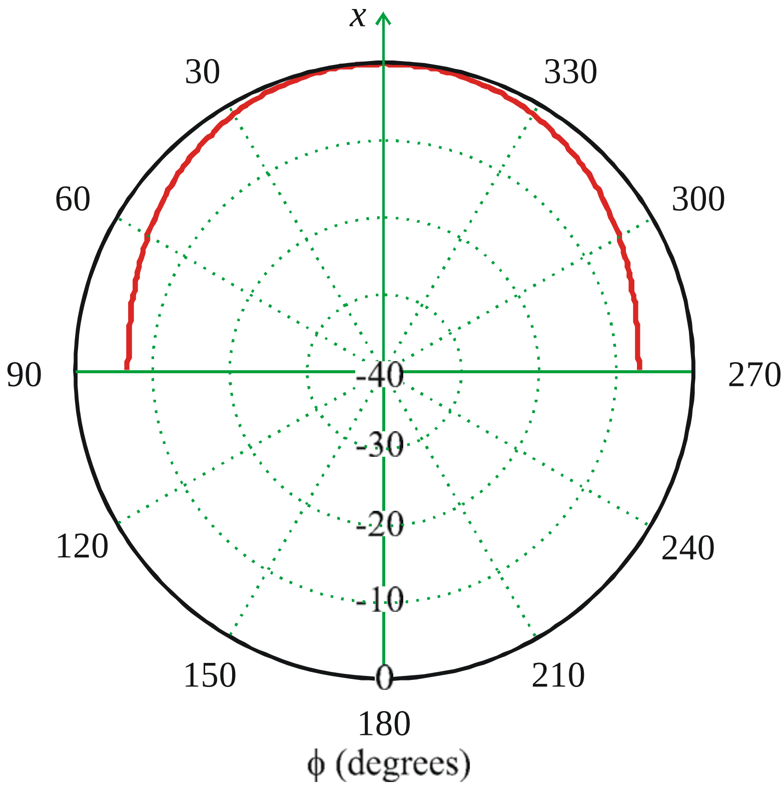


Normalized |E| in H-plane (x-z plane with $x > 0$)

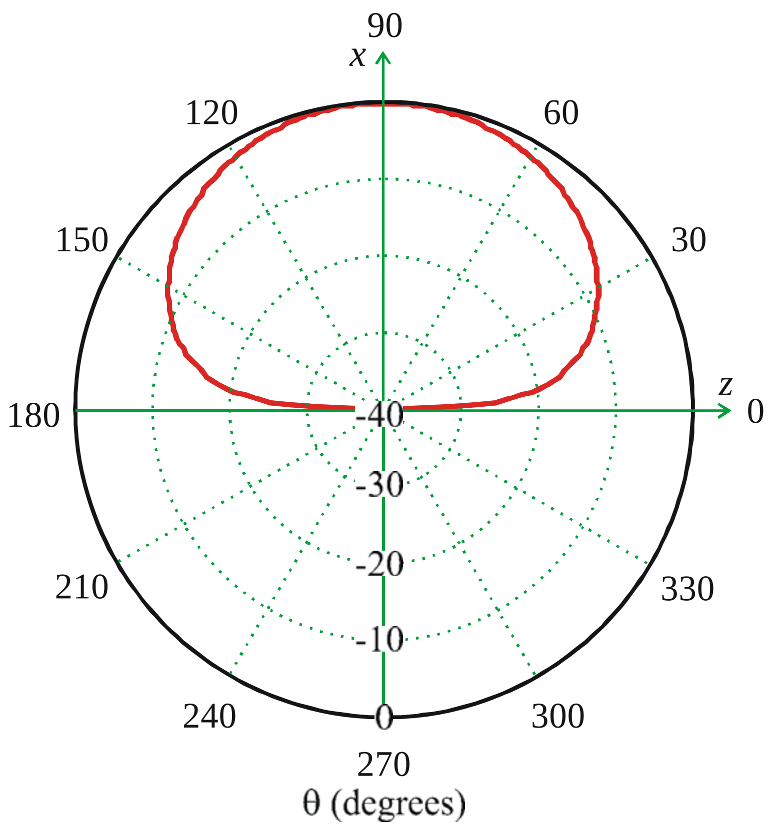


Example- Rectangular Microstrip Cavity Analysis cont.

Normalized |E| (dB) in E-plane (*x*-*y* plane with *x* > 0)

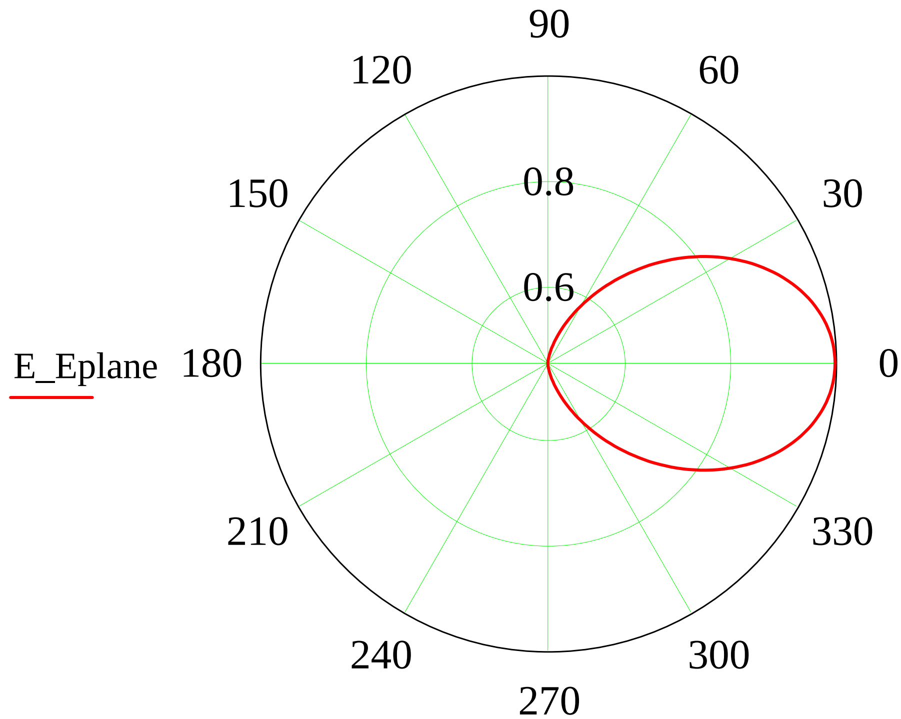


Normalized |E| (dB) in H-plane (*x*-*z* plane with *x* > 0)



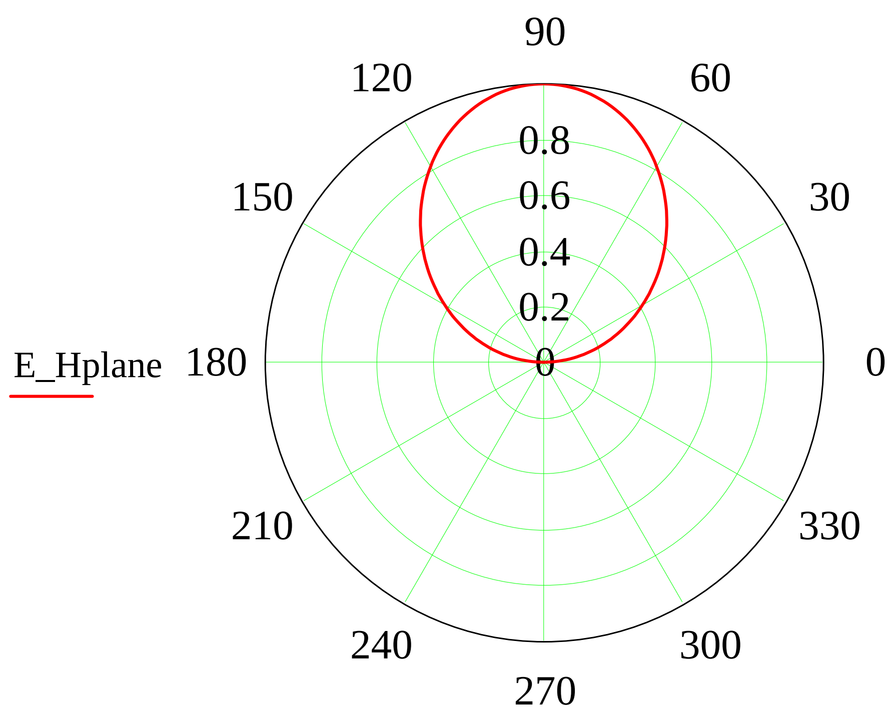
Example- Rectangular Microstrip Cavity Analysis cont.

Normalized E in E-plane



phi

Normalized E in H-plane



theta

Example- Rectangular Microstrip Cavity Analysis cont.