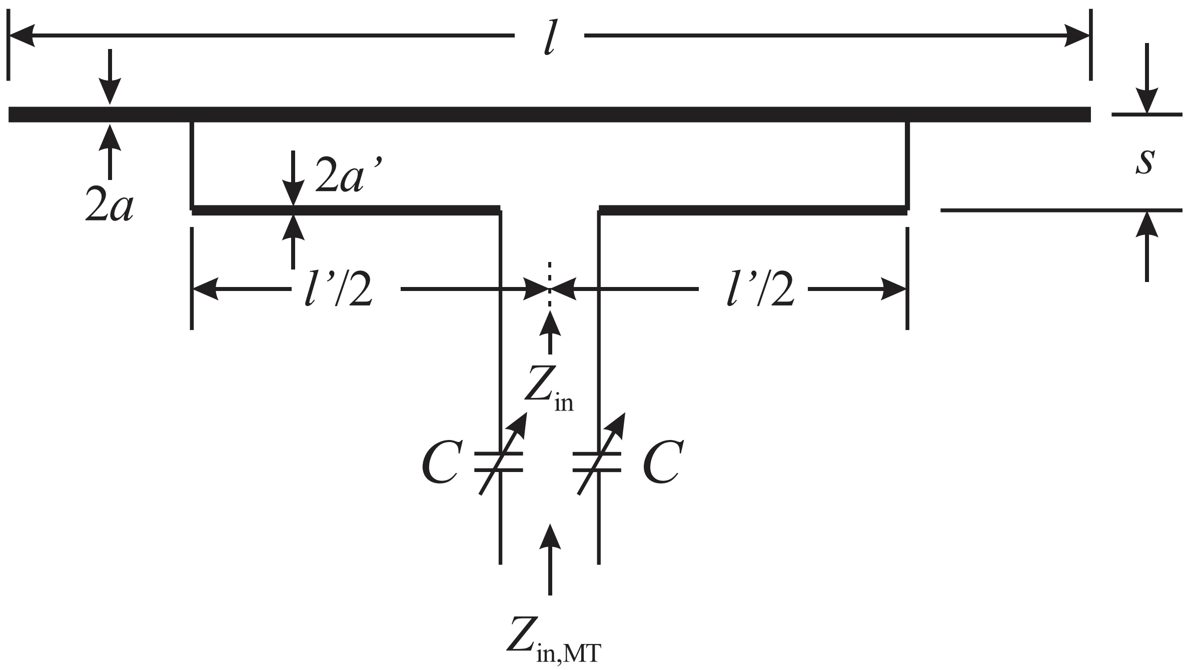
Matching Techniques for Driving Yagi-Uda Antennas: Modified T-Match

[Sections 9.5, 9.6, & 9.8 of *Antenna Theory, Analysis and Design* (4e) by Balanis)]

Modified or Resonant T-Match:



- The modified or resonant T-Match is another shunt-matching technique that can be used to feed a dipole or the driven element of a Yagi-Uda antenna.
- As it is symmetrical and balanced, it is typically used to connect twin-lead transmission lines to Yagi-Uda antennas.
- As shown, a pair of variable/tunable capacitors has been added in **series** with the T-Match.
- This is done because $Z_{\text{in}} = R_{\text{in}} + j\,X_{\text{in}}$ for the T-Match (unmodified) can have an inductive reactance X_{in}, a consequence of $l'/2$ usually being small in terms of the wavelength λ, i.e., $l'/2 < l_2/2 < \lambda/4$.

- The input impedance $Z_{\text{in,MT}}$ of the modified T-Match is equal to that of the T-Match (Z_{in}) plus the impedance of the two series capacitors ($2Z_{\text{cap}}$) -

$$Z_{\text{in,MT}} = 2Z_{\text{cap}} + Z_{\text{in}} \quad \Rightarrow \quad \underline{Z_{\text{in,MT}} = \frac{2}{j\omega C} + \frac{2(1+\alpha)^2 Z_a Z_t}{(1+\alpha)^2 Z_a + 2Z_t}}$$

where C is the capacitance required for the two series capacitors to cancel the inductive reactance of Z_{in}. I.e., $Z_{\text{in,MT}} = R_{\text{in}} \approx Z_{0,\text{feed}}$ where $Z_{0,\text{feed}}$ is the characteristic impedance of the feeding transmission line.

- Knowing $Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}}$, to achieve $Z_{\text{in,MT}} = R_{\text{in}}$ for the modified T-match, the capacitance is calculated as-

$$C = \frac{1}{\pi f X_{\text{in}}}.$$

<u>Design Process</u>

- We desire to match a given Yagi-Uda antenna to $Z_{0,\text{feed}}$. Typically, a specification in terms of the VSWR is given.

1) Select a driven element length l_2 (this corresponds to l in T-Match) so that $l_1 < l_2 < l_3$ as well as a', s, and l' (usually $l' < l_2/2$). These values may be changed later.
   - Diameter of feed $2a'$- usually you will want this to be less than the Yagi-Uda element diameters $2a$ to make $\alpha > 1$.
   - T-Match spacing s- make less than $s_{12}/4$, more than 1 cm (practical construction), and less than 5 cm (don't want the characteristic impedance Z_0 of the T-Match section to be too large).
   - T-Match length l'- make less than half of the initial driven element length l_2, i.e., $l' \leq l_2/2$, to avoid overly disturbing the current distribution on l_2.
2) Calculate the transmission line mode characteristic impedance –

$$Z_0 = \frac{\eta}{2\pi}\cosh^{-1}\left(\frac{s^2 - a^2 - a'^2}{2aa'}\right)$$

where $\eta = \sqrt{\mu / \varepsilon_{\text{eff}}}$ is the intrinsic impedance of the material wherein the T-Match exists.

3) Calculate the transmission line mode input impedance –

$$Z_t = jZ_0 \tan\left(kl'/2\right)$$

where $k = \beta = 2\pi/\lambda$.

4) Calculate the parameters u, v, and α –

$$u = \frac{a}{a'},\quad v = \frac{s}{a'},\quad \text{and}\quad \alpha = \frac{\cosh^{-1}\left(\dfrac{v^2 - u^2 + 1}{2v}\right)}{\cosh^{-1}\left(\dfrac{v^2 + u^2 - 1}{2vu}\right)} \quad \begin{array}{l} \alpha > 1 \text{ when } a > a' \\ \alpha = 1 \text{ when } a = a'. \\ \alpha < 1 \text{ when } a < a' \end{array}$$

5) Calculate the equivalent radius a_e of the T-Match section –

$$a_e \approx a' e^{\frac{u^2 \ln u + 2u \ln v}{(1+u)^2}}.$$

6) Find the input impedance (i.e., use NEC-2) of the antenna mode Z_a of the driven element as part of the Yagi-Uda antenna. It is modeled as a dipole that has radius a_e over the inner length l', corresponding to location of T-Match, and radius a for the tips extending beyond the T-Match.

7) Find overall T-Match input impedance Z_{in} of the driven element as part of the overall Yagi-Uda antenna –

$$Z_{\text{in}} = \frac{2(1+\alpha)^2 Z_a Z_t}{(1+\alpha)^2 Z_a + 2Z_t}.$$

8) Determine if Z_{in} meets your specification, i.e., will a regular T-match work? If so, stop design process (don't need the capacitors). If not, check if $X_{\text{in}} > 0$ (inductive reactance). If so, go to next step 9), else back to step 1) to adjust l_1, l_2, l_3, a', s, and/or l'. Hint: Making l_2 (or reflector l_1) longer can increase inductive reactance.

9) If $X_{\text{in}} > 0$, calculate $C = \dfrac{1}{\pi f X_{\text{in}}}$ and $Z_{\text{in,MT}} = 2Z_{\text{cap}} + Z_{\text{in}} = \dfrac{2}{j\omega C} + Z_{\text{in}}$. Determine if $Z_{\text{in,MT}}$ meets your specification (i.e., calculate VSWR). If not, back to step 1) to adjust l_1, $\boldsymbol{l_2}$, l_3, a', s, and/or l' to get $X_{\text{in}} > 0$.

- Consider changing l' *toward* the length suggested by

$$l' = \frac{2}{k}\tan^{-1}\left[\frac{1}{2Z_0 \operatorname{Im}\left(\dfrac{Y_a}{(1+\alpha)^2}\right)}\right]$$

  to better offset the antenna mode reactance, and repeat steps 2) through 8). If necessary, l_2, a', and s can be varied.

- Remember, the magnitude of the input impedance $|Z_{\text{in}}|$ is greatly affected by α (i.e., when α increases, $|Z_{\text{in}}|$ increases, and vice versa). In turn, α is inversely related to s (i.e., if s decreases, α increases, and vice versa).
- See http://montoya.sdsmt.edu/ee483_583/notes/matching_tips.pdf for further tips.