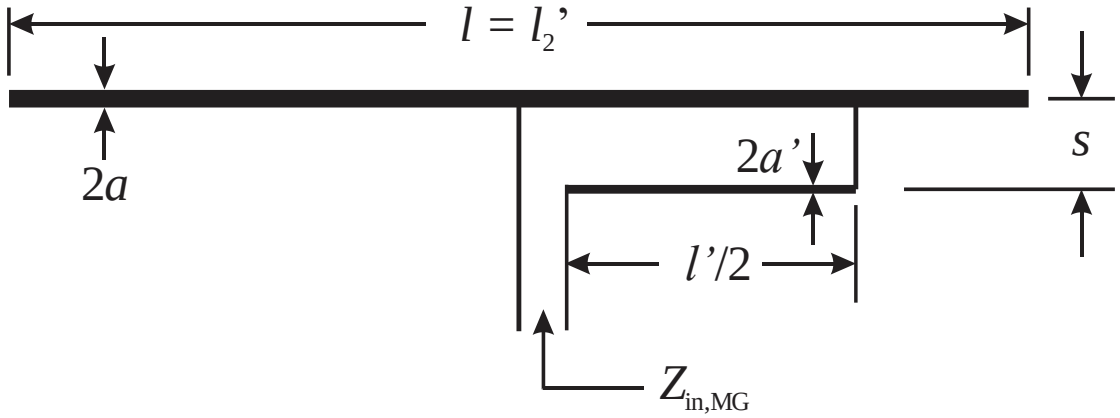
Example: Use a modified Gamma-Match to drive the previously designed 5-element UHF Channel 43 Yagi-Uda antenna without boom with a 50 Ω coaxial transmission line. The matching specification is that the VSWR ≤ 1.1.

- To save time, I'll use the final Gamma-Match design as a starting point.



First attempt at modified Gamma-Match:

element diameters $d = 2a = 0.25" = 0.635$ cm $\Rightarrow$ $a = 0.125" = 0.3175$ cm

element spacings $s_{ij} = 0.2\lambda = 9.274$ cm

reflector length $\boldsymbol{l_1' = 22.7}$ **cm** (3 mm longer to make up for no capacitor.)

driven element length $l_2' = 20.8$ cm

director lengths $l_3' = 19.1$ cm

director length $l_4' = 19.09$ cm

director length $l_5' = 19.41$ cm

Gamma-Match diameter $2a' = 0.125" = 0.3175$ cm $\Rightarrow$ $a' = 0.15875$ cm

Gamma-Match spacing $s = 2$ cm

Gamma-Match length $l'/2 = 5.08$ cm

Using NEC-2 and MathCad (see attached pages)-

- Using MathCad for modified Γ-match: $Z_0 = 260.329\ \Omega$, current division factor $\alpha = 1.36748$, effective radius $a_e = 0.666$ cm, & transmission line mode input impedance $Z_t = j214.36\ \Omega$.
- From NEC: $Z_a = 18.2244 + j\,1.28708\ \Omega$ & Gain = 11.5057 dBi
- Using MathCad: $Z_{in,MG} = 46.826 + j\ 14.52\ \Omega$. With this input impedance, $|\Gamma| = 0.152$, and VSWR = 1.358 > 1.1 (high).

NEC input file for first modified Gamma-Match attempt

```
CM Yagi-Uda Antenna for UHF channel 43 (NO BOOM)
CM
CM THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN
CM ELEMENT OF A 5 ELEMENT ANTENNA. CENTER FREQUENCY IS 647 MHz
CM W/ WAVELENGTH OF 0.46335 m.
CM
CM THE DIMENSIONS ARE:
CM element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in,
CM equivalent radius of Gamma-Match portion of driven element
CM is ae=0.00666 m and has a length of l'/2=0.0508 m  < l2/2
CM
CM l1=0.227 m, l2=0.4489 l= 0.208m, l3=0.191 m, l4=0.1909m, l5=0.1941 m
CM ELEMENT SPACINGS Sij=0.2 l = 0.09267m
CM SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
CE THE DRIVEN SEGMENT IS #1 on GW3 of l2.
GW 1 17 -0.1135  0.0 0.0      0.1135  0.0 0.0      0.003175 !Reflector
GW 2  8 -0.104   0.0 0.09267  0.0     0.0 0.09267  0.003175 !Driven tip
GW 3  4  0.0     0.0 0.09267  0.0508  0.0 0.09267  0.00666  !Driven mid
GW 4  4  0.0508  0.0 0.09267  0.104   0.0 0.09267  0.003175 !Driven tip
GW 5 15 -0.0955  0.0 0.18534  0.0955  0.0 0.18534  0.003175 !Director 1
GW 6 15 -0.09545 0.0 0.27801  0.09545 0.0 0.27801  0.003175 !Director 2
GW 7 15 -0.09705 0.0 0.37068  0.09705 0.0 0.37068  0.003175 !Director 3
GE 0 0
FR 0 1 0 0 647 0
EX 0 3 1 0 1.0 0.0
RP 0 2 3 0000 0.0 0.0 180.0 90.0
PT -1
XQ 0
EN
```

NEC output file excerpt for first modified Gamma-Match attempt

```
<snip>

 FREQUENCY= 6.4700E+02 MHZ    WAVELENGTH= 4.6337E-01 METERS

    - - - ANTENNA INPUT PARAMETERS - - -

TAG SEG.  VOLTAGE (V) CURRENT (A)    IMPEDANCE (OHMS) ADMITTANCE (MHOS) POWER
NO. NO.  REAL  IMAG.   REAL  IMAG.     REAL  IMAG.      REAL  IMAG.   (WATTS)
 3  26  1.0E+00 0.0 5.46E-02-3.856E-03 1.82244E+01 1.28708E+00 5.46E-02 -
3.86E-03 2.72995E-02

    - - - RADIATION PATTERNS - - -

  THETA   PHI      MAJOR
 DEGREES  DEGREES   DB
  0.00    0.00    11.50571
  180.00  0.00    -0.71554

<snip>
```

MathCad file for first modified Gamma-Match attempt

Modified Gamma-Match equations- First Try

$c := 2.9979 \cdot 10^{8}$ m/s $fc := 647 \cdot 10^{6}$ Hz $\lambda := \frac{c}{fc}$ $\lambda = 0.46335$ m

$k := \frac{2 \cdot \pi}{\lambda}$ $k = 13.56023$ rad/m $Zdesired := 50$ Ω

$d := 0.635 \cdot 10^{-2}$ m $a := d \cdot 0.5$ $a = 0.003175$ m $s := 2. \cdot 10^{-2}$ m

$dprime := 0.3175 \cdot 10^{-2}$ m $aprime := dprime \cdot 0.5$ $aprime = 0.0015875$ m

$Z0 := \frac{376.7303}{2 \cdot \pi} \cdot acosh\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right]$ $Z0 = 260.329$ Ω $u := \frac{a}{aprime}$ $v := \frac{s}{aprime}$

$u = 2$ $v = 12.5984$

$\alpha := \frac{acosh\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{acosh\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]}$ $\alpha = 1.36748$ $ae := aprime \cdot e^{\frac{u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)}{(1+u)^2}}$

$ae = 0.00666$ $ae \cdot 100 = 0.66609$ cm

$lprime := 10.16 \cdot 10^{-2}$ m $lprime \cdot 0.5 = 0.0508$ m

$Zt := j \cdot Z0 \cdot tan\left(\frac{k \cdot lprime}{2}\right)$ $Zt = 214.3602i$ Ω $Yt := \frac{1}{Zt}$ $Yt = -4.665i \cdot 10^{-3}$

Za from NEC (a MoM program)

$Za := 18.2244 + j \cdot 1.28708$ Ω $Ya := \frac{1}{Za}$ $Ya = 0.0546 - 0.00386i$ Mhos

$Yin := Yt + \frac{Ya \cdot 2}{(1 + \alpha)^2}$ $Yin = 0.019 - 6.041i \times 10^{-3}$ S $\frac{1}{Zdesired} = 0.02$ Mhos

$ZinMG := \frac{1}{Yin}$ $ZinMG = 46.826 + 14.52i$ Ω $Zdesired = 50$ Ω

$\Gamma := \frac{(ZinMG - Zdesired)}{ZinMG + Zdesired}$ $\Gamma = -0.01 + 0.151i$ $|\Gamma| = 0.152$

$VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$ $VSWR = 1.358$ **First Try- VSWR > 1.1**

$lsuggested2 := \frac{1}{k} \cdot atan\left[\frac{1}{Z0 \cdot Im\left[\frac{2Ya}{(1 + \alpha)^2}\right]}\right]$ $lsuggested2 = -0.0905$ m, NOT realizable

Second attempt at modified Gamma-Match:

element diameters $d = 2a = 0.25" = 0.635$ cm $\Rightarrow$ $a = 0.125" = 0.3175$ cm

element spacings $s_{ij} = 0.2\lambda = 9.274$ cm

reflector length $l_1' = 22.7$ cm

Shorten driven element length $\boldsymbol{l_2' = 20.56}$ **cm** (down from 20.8 cm to try to eliminate/minimize reactance)

director lengths $l_3' = 19.1$ cm

director length $l_4' = 19.09$ cm

director length $l_5' = 19.41$ cm

Gamma-Match diameter $2a' = 0.125" = 0.3175$ cm $\Rightarrow$ $a' = 0.15875$ cm

Gamma-Match spacing $s = 2$ cm

Gamma-Match length $l'/2 = 5.08$ cm

Using NEC-2 and MathCad (see attached pages)-

- Using MathCad for modified Γ-match: $Z_0 = 260.329\ \Omega$, current division factor $\alpha = 1.36748$, effective radius $a_e = 0.666$ cm, & transmission line mode input impedance $Z_t = j214.36\ \Omega$.
- From NEC: $Z_a = 17.378 - j\,4.09449\ \Omega$ & Gain = 11.4924 dBi
- Using MathCad: $Z_{in,MG} = 51.404 + j\ 0.216\ \Omega$. With this input impedance, $|\Gamma| = 0.014$, and **VSWR = 1.028** < 1.1 (meets spec $\Rightarrow$ DONE!).

NEC input file for second modified Gamma-Match attempt

```
CM Yagi-Uda Antenna for UHF channel 43 (NO BOOM)
CM
CM THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN
CM ELEMENT OF A 5 ELEMENT ANTENNA. CENTER FREQUENCY IS 647 MHz
CM W/ WAVELENGTH OF 0.46335 m.
CM
CM THE DIMENSIONS ARE:
CM element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in,
CM equivalent radius of Gamma-Match portion of driven element
CM is ae=0.00666 m and has a length of l'/2=0.0508 m  < l2/2
CM
CM l1=0.227 m, l2=0.2056 m, l3=0.191 m, l4=0.1909m, l5=0.1941 m
CM ELEMENT SPACINGS Sij=0.2 l = 0.09267m
CM SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
CE THE DRIVEN SEGMENT IS #1 on GW3 on l2.
GW 1 17 -0.1135  0.0 0.0      0.1135  0.0 0.0      0.003175 !Reflector
GW 2  8 -0.1028  0.0 0.09267  0.0     0.0 0.09267  0.003175 !Driven tip
GW 3  4  0.0     0.0 0.09267  0.0508  0.0 0.09267  0.00666  !Driven mid
GW 4  4  0.0508  0.0 0.09267  0.1028  0.0 0.09267  0.003175 !Driven tip
GW 5 15 -0.0955  0.0 0.18534  0.0955  0.0 0.18534  0.003175 !Director 1
GW 6 15 -0.09545 0.0 0.27801  0.09545 0.0 0.27801  0.003175 !Director 2
GW 7 15 -0.09705 0.0 0.37068  0.09705 0.0 0.37068  0.003175 !Director 3
GE 0 0
FR 0 1 0 0 647 0
EX 0 3 1 0 1.0 0.0
RP 0 2 3 0000 0.0 0.0 180.0 90.0
PT -1
XQ 0
EN
```

NEC output file excerpt for second modified Gamma-Match attempt

```
<snip>

 FREQUENCY= 6.4700E+02 MHZ    WAVELENGTH= 4.6337E-01 METERS

    - - - ANTENNA INPUT PARAMETERS - - -

TAG SEG.  VOLTAGE (V) CURRENT (A)    IMPEDANCE (OHMS) ADMITTANCE (MHOS) POWER
NO. NO.  REAL  IMAG.   REAL  IMAG.     REAL  IMAG.      REAL  IMAG.   (WATTS)
 3  26 1.0E+00 0.0E+00 5.45E-02 1.28E-02 1.7378E+01-4.09449E+00 5.45177E-02
1.28451E-02 2.72588E-02

    - - - RADIATION PATTERNS - - -

  THETA   PHI      MAJOR
 DEGREES  DEGREES   DB
  0.00    0.00    11.49238
  180.00  0.00    -0.70477

<snip>
```

MathCad file for second modified Gamma-Match attempt

Modified Gamma-Match equations- Second Try

$c := 2.9979 \cdot 10^{8}$ m/s $fc := 647 \cdot 10^{6}$ Hz $\lambda := \frac{c}{fc}$ $\lambda = 0.46335$ m

$k := \frac{2 \cdot \pi}{\lambda}$ $k = 13.56023$ rad/m $Zdesired := 50$ Ω

$d := 0.635 \cdot 10^{-2}$ m $a := d \cdot 0.5$ $a = 0.003175$ m $s := 2. \cdot 10^{-2}$ m

$dprime := 0.3175 \cdot 10^{-2}$ m $aprime := dprime \cdot 0.5$ $aprime = 0.0015875$ m

$Z0 := \frac{376.7303}{2 \cdot \pi} \cdot acosh\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right]$ $Z0 = 260.329$ Ω $u := \frac{a}{aprime}$ $v := \frac{s}{aprime}$

$u = 2$ $v = 12.5984$

$\alpha := \frac{acosh\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{acosh\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]}$ $\alpha = 1.36748$ $ae := aprime \cdot e^{\frac{u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)}{(1+u)^2}}$

$ae = 0.00666$ $ae \cdot 100 = 0.66609$ cm

$lprime := 10.16 \cdot 10^{-2}$ m $lprime \cdot 0.5 = 0.0508$ m

$Zt := j \cdot Z0 \cdot tan\left(\frac{k \cdot lprime}{2}\right)$ $Zt = 214.3602i$ Ω $Yt := \frac{1}{Zt}$ $Yt = -4.665i \cdot 10^{-3}$

Za from NEC (a MoM program)

$Za := 17.378 + j \cdot -4.09449$ Ω $Ya := \frac{1}{Za}$ $Ya = 0.05452 + 0.01285i$ Mhos

$Yin := Yt + \frac{Ya \cdot 2}{(1 + \alpha)^2}$ $Yin = 0.019 - 8.159i \times 10^{-5}$ S $\frac{1}{Zdesired} = 0.02$ Mhos

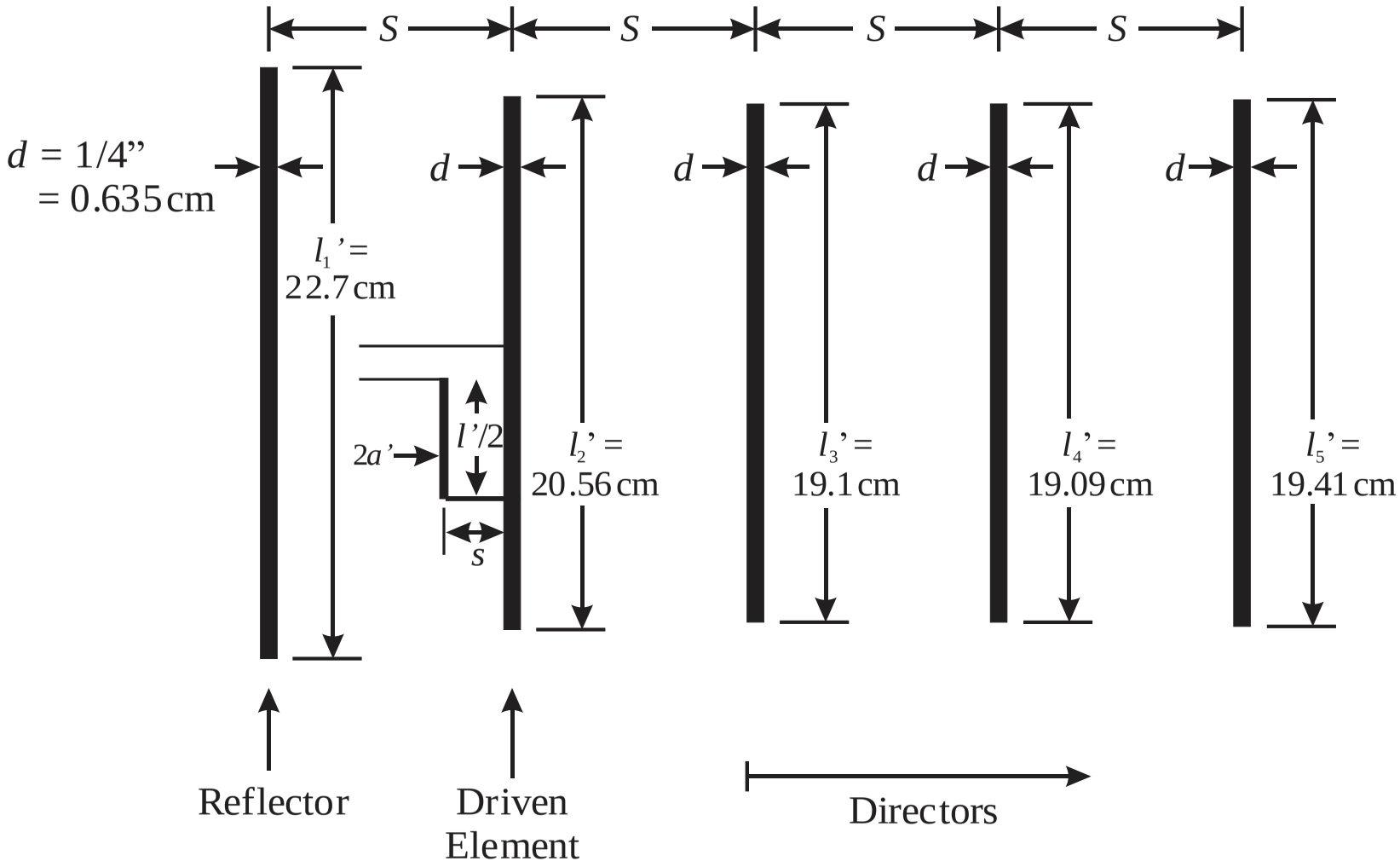
$ZinMG := \frac{1}{Yin}$ $ZinMG = 51.404 + 0.216i$ Ω $Zdesired = 50$ Ω

$\Gamma := \frac{(ZinMG - Zdesired)}{ZinMG + Zdesired}$ $\Gamma = 0.014 + 2.097i \times 10^{-3}$ $|\Gamma| = 0.014$

$VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$ $VSWR = 1.028$ **Second Try- VSWR < 1.1 DONE!**

5 element, CH 43 Yagi-Uda antenna w/ modified Gamma-Match

(element spacing S = 3.651” = 9.274 cm)



Modified Gamma-Match Dimensions: $2a'$ = 1/8” = 0.3175 cm, s = 2 cm, and $l'/2$ = 5.08 cm.