Example: Use a Gamma-Match to drive the previously designed 5-element UHF Channel 43 Yagi-Uda antenna without boom with a 50 Ω coaxial transmission line. The matching specification is that the VSWR ≤ 1.1.

Yagi-Uda antenna design summary

f_c = 647 MHz and λ = 46.3 cm

element diameters $d = 2a = 0.25" = 0.635$ cm $\Rightarrow$ $a = 0.125" = 0.3175$ cm

element spacings $s_{ij} = 0.2\lambda = 9.274$ cm

reflector length $l_1' = 0.48\lambda = 22.24$ cm

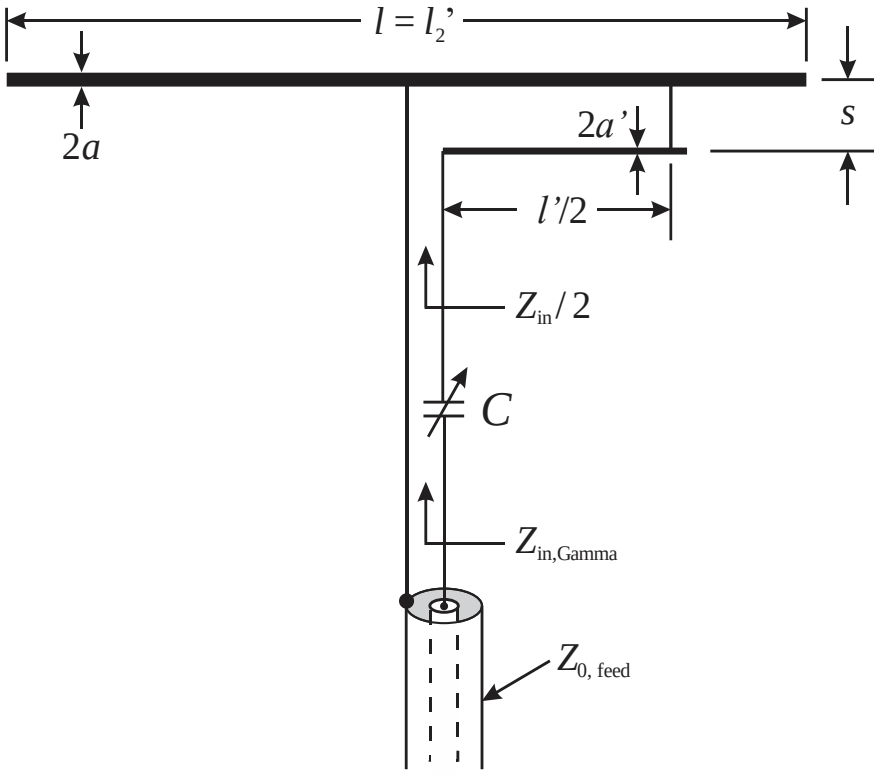
director lengths $l_3' = l_5' = 0.419\lambda = 19.41$ cm

director length $l_4' = 0.412\lambda = 19.09$ cm

selected driven element length $l_2' = (l_1' + l_3')/2 = 0.45\lambda = 20.85$ cm

From NEC-2, $Z_a = 17.35 + j\,20.59\ \Omega$ and Gain = 11.265 dBi w/ no match

Gamma-Match:



First attempt at Gamma-Match (Γ-Match)

Select Γ-Match diameter $2a' = 0.125" = 0.3175\,\text{cm} \Rightarrow a' = 0.15875\,\text{cm}$

Select Γ-Match spacing $s = 2$ cm and length $l'/2 = 5$ cm

Select driven element length $l_2' = 0.47\lambda = 21.8$ cm (want inductive reactance)

Using NEC-2 and MathCad (see attached pages)-

- Using MathCad for Γ-match: $Z_0 = 260.329\ \Omega$, current division factor $\alpha = 1.36748$, effective radius $a_e = 0.666$ cm, & transmission line mode input impedance $Z_t = j209.663\ \Omega$
- From NEC: $Z_a = 18.356 + j\,25.3879\ \Omega$ & Gain = 11.6122 dBi
- Using MathCad: $Z_{\text{in}}/2 = 27.746 + j\ 58.205\ \Omega$. After adding series capacitor $C = 4.226$ pF, $Z_{\text{in,Gamma}} = 27.746\ \Omega$. With this input impedance, $|\Gamma| = 0.286$, and VSWR = 1.802 > 1.1 (high).

NEC input file for first Γ-Match attempt

```
CM Yagi-Uda Antenna for UHF channel 43 (NO BOOM)
CM
CM THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN
CM ELEMENT OF A 5 ELEMENT YAGI-UDA ANTENNA. CENTER FREQUENCY IS 647 MHz
CM W/ WAVELENGTH OF 0.46335 m.
CM
CM THE DIMENSIONS ARE:
CM element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in,
CM equivalent radius of Gamma-Match portion of driven element
CM is ae=0.00666 m which has a length of l'/2=0.05 m  < l2
CM
CM l1=0.48 l=0.2224m, l3=l5=0.419 l=0.1941m, l4=0.412 l=0.1909m,
CM driven element l2=0.47 l= 0.218m
CM ELEMENT SPACINGS Sij=0.2 l=0.09267m
CM SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
CE THE DRIVEN SEGMENT IS #1 on l2 Tag 3/GW 3.
GW 1 17 -0.1112  0.0 0.0     0.1112  0.0 0.0     0.003175 !Reflector
GW 2 9  -0.109   0.0 0.09267 0.0     0.0 0.09267 0.003175 !Driven tip
GW 3 4   0.0     0.0 0.09267 0.05    0.0 0.09267 0.00666  !Driven middle
GW 4 5   0.05    0.0 0.09267 0.109   0.0 0.09267 0.003175 !Driven tip
GW 5 15 -0.09705 0.0 0.18534 0.09705 0.0 0.18534 0.003175 !Director 1
GW 6 15 -0.09545 0.0 0.27801 0.09545 0.0 0.27801 0.003175 !Director 2
GW 7 15 -0.09705 0.0 0.37068 0.09705 0.0 0.37068 0.003175 !Director 3
GE 0 0
FR 0 1 0 0 647 0
EX 0 3 1 0 1.0 0.0
RP 0 2 3 0000 0.0 0.0 180.0 90.0
PT -1
XQ 0
EN
```

NEC output file for first Γ-Match attempt

```
Yagi-Uda Antenna for UHF channel 43 (NO BOOM)

THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN ELEMENT OF
A 5 ELEMENT ANTENNA. CENTER FREQUENCY IS 647 MHz W/ WAVELENGTH OF 0.46335 m.

    THE DIMENSIONS ARE:
    element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in,
    equivalent radius of Gamma-Match portion of driven element
    is ae=0.00666 m which has a length of l'/2=0.05 m  < l2

    l1=0.48 l=0.2224m, l3=l5=0.419 l=0.1941m, l4=0.412 l=0.1909m,
    driven element l2=0.47 l= 0.218m
    ELEMENT SPACINGS Sij=0.2 l=0.09267m
    SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
    THE DRIVEN SEGMENT IS #9 on l2.
   ***********************************************************
      - - - STRUCTURE SPECIFICATION - - -
       COORDINATES MUST BE INPUT IN METERS OR BE SCALED TO METERS
       BEFORE STRUCTURE INPUT IS ENDED

WIRE                                                    NO. OF  FIRST  LAST  TAG
NO.    X1     Y1      Z1    X2      Y2    Z2      RADIUS   SEG. SEG.   SEG.  NO.
1  -0.11120 0.0 0.00000  0.11120 0.0 0.00000 0.00317  17     1    17    1
2  -0.10900 0.0 0.09267  0.00000 0.0 0.09267 0.00317   9    18    26    2
3   0.00000 0.0 0.09267  0.05000 0.0 0.09267 0.00666   4    27    30    3
4   0.05000 0.0 0.09267  0.10900 0.0 0.09267 0.00317   5    31    35    4
5  -0.09705 0.0 0.18534  0.09705 0.0 0.18534 0.00317  15    36    50    5
6  -0.09545 0.0 0.27801  0.09545 0.0 0.27801 0.00317  15    51    65    6
7  -0.09705 0.0 0.37068  0.09705 0.0 0.37068 0.00317  15    66    80    7

TOTAL SEGMENTS USED=80  NO. SEG. IN A SYMMETRIC CELL=80  SYMMETRY FLAG=  0

***** INPUT LINE  1  FR 0  1  0  0  6.470E+02  0.0  0.0  0.0  0.0  0.0
***** INPUT LINE  2  EX 0  3  1  0  1.00000E+00  0.0  0.0  0.0  0.0  0.0
***** INPUT LINE  3  RP 0  2  3  0  0.0 0.0  1.800E+02 9.000E+01  0.0  0.0

      FREQUENCY= 6.4700E+02 MHZ    WAVELENGTH= 4.6337E-01 METERS

    - - - ANTENNA INPUT PARAMETERS - - -

TAG SEG.    VOLTAGE     IMPEDANCE (OHMS)        ADMITTANCE (MHOS)
NO. NO.  REAL IMAG.     REAL     IMAG.          REAL     IMAG.
3  27    1.0   0.0   1.83560E+01 2.53879E+01 1.87022E-02-2.58667E-023

       - - - RADIATION PATTERNS - - -
  THETA   PHI       Gain
 DEGREES  DEGREES    DB
    0.00  0.00     11.61220
  180.00  0.00     -0.14719

 ***** INPUT LINE  4  PT  -1    0    0    0  0.0  0.0  0.0  0.0  0.0  0.0
 ***** INPUT LINE  5  XQ   0    0    0    0  0.0  0.0  0.0  0.0  0.0  0.0
 ***** INPUT LINE  6  EN   0    0    0    0  0.0  0.0  0.0  0.0  0.0  0.0
```

MathCad file for first Γ-Match attempt

Gamma-Match equations

$c := 2.9979 \cdot 10^{8}$ m/s $fc := 647 \cdot 10^{6}$ Hz $\lambda := \frac{c}{fc}$ $\lambda = 0.46335$ m $k := \frac{2 \cdot \pi}{\lambda}$ $k = 13.56023$ rad/m

$Zdesired := 50$ Ω $d := 0.635 \cdot 10^{-2}$ m $a := d \cdot 0.5$ $a = 0.003175$ m

$dprime := 0.3175 \cdot 10^{-2}$ m $aprime := dprime \cdot 0.5$ $aprime = 0.0015875$ m $s := 2. \cdot 10^{-2}$ m

$$Z0 := \frac{376.73}{2 \cdot \pi} \cdot acosh\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right] \qquad Z0 = 260.329 \ \Omega$$

$u := \frac{a}{aprime}$ $v := \frac{s}{aprime}$

$u = 2$ $v = 12.5984$

$$\alpha := \frac{acosh\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{acosh\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]} \qquad \alpha = 1.36748$$

$$ae := aprime \cdot e^{\frac{1}{(1+u)^2} \cdot \left(u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)\right)}$$

$ae = 0.00666$ $ae \cdot 100 = 0.66609$ cm

$lprime := 10. \cdot 10^{-2}$ m $lprime \cdot 0.5 = 0.05$ m

$$Zt := j \cdot Z0 \cdot \tan\left(\frac{k \cdot lprime}{2}\right) \qquad Zt = 209.66289i \ \Omega$$

$Yt := \frac{1}{Zt}$ $Yt = -4.77i \times 10^{-3}$

Za from NEC (a MoM program)

$Za := 18.356 + j \cdot 25.3879$ Ω $Ya := \frac{1}{Za}$ $Ya = 0.0187 - 0.02587i$ Mhos

$$Yin2 := Yt + \frac{Ya \cdot 2}{(1 + \alpha)^2} \qquad Yin2 = 6.673 \times 10^{-3} - 0.014i$$

$\frac{1}{Zdesired} = 0.02$ Mhos

$Zin2 := \frac{1}{Yin2}$ $Zin2 = 27.746 + 58.205i$ Ω $Zdesired = 50$ Ω

$$C := if\left(Im(Zin2) > 0, \frac{1}{2 \cdot \pi \cdot fc \cdot Im(Zin2)}, 0\right) \qquad C = 4.226 \times 10^{-12}$$

$C \cdot 10^{12} = 4.226$ pF

$$Zcap := if\left(C > 0, \frac{1}{j \cdot 2 \cdot \pi \cdot fc \cdot C}, 0\right) \qquad Zcap = -58.205i \ \Omega$$

$Zing := Zcap + Zin2$ $\boxed{Zing = 27.746}$ Ω

$\Gamma := \frac{(Zing - Zdesired)}{Zing + Zdesired}$ $\Gamma = -0.286$ $|\Gamma| = 0.286$

$VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$ $VSWR = 1.802$

First try

Second attempt at Γ-Match

Keep Γ-Match diameter $2a' = 0.125" = 0.3175$ cm $\Rightarrow$ $a' = 0.15875$ cm

Keep Γ-Match spacing $s = 2$ cm and length $l'/2 = 5$ cm

Change driven element length to **$l_2' = 20.8$ cm** (shorten from 21.8 cm)

Using NEC-2 and MathCad (see attached pages)-

- Using MathCad for Γ-match: $Z_0 = 260.329\ \Omega$, $\alpha = 1.36748$, $a_e = 0.666$ cm, & $Z_t = j209.663\ \Omega$ (no changes)
- From NEC: $Z_a = 14.9299 + j\,1.27235\ \Omega$ & Gain = 11.56149 dBi
- Using MathCad: $Z_{in}/2 = 38.953 + j\,11.15\ \Omega$. After adding series capacitor $C = 22.06$ pF, $Z_{in,Gamma} = 38.953\ \Omega$. With this input impedance, $|\Gamma| = 0.124$, and VSWR = 1.284 > 1.1 (high).

NEC input file for second Γ-Match attempt

```
CM Yagi-Uda Antenna for UHF channel 43 (NO BOOM)
CM
CM THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN
CM ELEMENT OF A 5 ELEMENT YAGI-UDA ANTENNA. CENTER FREQUENCY IS 647 MHz
CM W/ WAVELENGTH OF 0.46335 m.
CM
CM THE DIMENSIONS ARE:
CM element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in,
CM equivalent radius of Gamma-Match portion of driven element
CM is ae=0.00666 m which has a length of l'/2=0.05 m  < l2
CM
CM l1=0.48 l=0.2224m, l3=l5=0.419 l=0.1941m, l4=0.412 l=0.1909m,
CM driven element l2=0.4489 l= 0.208m
CM ELEMENT SPACINGS Sij=0.2 l=0.09267m
CM SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
CE THE DRIVEN SEGMENT IS #1 on l2 Tag 3/GW 3.
GW 1 17 -0.1112  0.0 0.0     0.1112  0.0 0.0     0.003175 !Reflector
GW 2 9  -0.104   0.0 0.09267 0.0     0.0 0.09267 0.003175 !Driven tip
GW 3 4   0.0     0.0 0.09267 0.05    0.0 0.09267 0.00666  !Driven middle
GW 4 5   0.05    0.0 0.09267 0.104   0.0 0.09267 0.003175 !Driven tip
GW 5 15 -0.09705 0.0 0.18534 0.09705 0.0 0.18534 0.003175 !Director 1
GW 6 15 -0.09545 0.0 0.27801 0.09545 0.0 0.27801 0.003175 !Director 2
GW 7 15 -0.09705 0.0 0.37068 0.09705 0.0 0.37068 0.003175 !Director 3
GE 0 0
FR 0 1 0 0 647 0
EX 0 3 1 0 1.0 0.0
RP 0 2 3 0000 0.0 0.0 180.0 90.0
PT -1
XQ 0
EN
```

NEC output file for second Γ-Match attempt

```
Yagi-Uda Antenna for UHF channel 43 (NO BOOM)

THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN ELEMENT OF
A 5 ELEMENT ANTENNA. CENTER FREQUENCY IS 647 MHz W/ WAVELENGTH OF 0.46335 m.

  THE DIMENSIONS ARE:
  element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in,
  equivalent radius of Gamma-Match portion of driven element
  is ae=0.00666 m which has a length of l'/2=0.05 m  < l2

  l1=0.48 l=0.2224m, l3=l5=0.419 l=0.1941m, l4=0.412 l=0.1909m,
  driven element l2=0.4489 l= 0.208m
  ELEMENT SPACINGS Sij=0.2 l=0.09267m
  SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
  THE DRIVEN SEGMENT IS #9 on l2.
  ******************************************************
- - - STRUCTURE SPECIFICATION - - -
  COORDINATES MUST BE INPUT IN METERS OR BE SCALED TO METERS
  BEFORE STRUCTURE INPUT IS ENDED

  WIRE                                              NO. OF  FIRST  LAST  TAG
  NO.  X1      Y1  Z1       X2      Y2  Z2      RADIUS  SEG. SEG. SEG.  NO.
  1   -0.11120 0.0 0.0      0.11120 0.0 0.0     0.00317  17   1   17    1
  2   -0.10400 0.0 0.09267  0.0     0.0 0.09267 0.00317   9  18   26    2
  3    0.0     0.0 0.09267  0.05000 0.0 0.09267 0.00666   4  27   30    3
  4    0.05000 0.0 0.09267  0.10400 0.0 0.09267 0.00317   5  31   35    4
  5   -0.09705 0.0 0.18534  0.09705 0.0 0.18534 0.00317  15  36   50    5
  6   -0.09545 0.0 0.27801  0.09545 0.0 0.27801 0.00317  15  51   65    6
  7   -0.09705 0.0 0.37068  0.09705 0.0 0.37068 0.00317  15  66   80    7

TOTAL SEGMENTS USED= 80  NO. SEG. IN A SYMMETRIC CELL= 80  SYMMETRY FLAG=  0

***** INPUT LINE  1  FR  0  1  0  0  6.47000E+02  0.0  0.0  0.0  0.0  0.0
***** INPUT LINE  2  EX  0  3  1  0  1.00000E+00  0.0  0.0  0.0  0.0  0.0
***** INPUT LINE  3  RP  0  2  3  0  0.0  0.0  1.800E+02  9.00E+01  0.0  0.0

  FREQUENCY= 6.4700E+02 MHZ   WAVELENGTH= 4.6337E-01 METERS

  - - - ANTENNA INPUT PARAMETERS - - -
TAG SEG.   VOLTAGE   IMPEDANCE (OHMS)       ADMITTANCE (MHOS)
NO. NO.  REAL IMAG.  REAL       IMAG.       REAL        IMAG.
3   27   1.0  0.0   1.49299E+01 1.27235E+00 6.64966E-02 -5.66692E-03

  - - - RADIATION PATTERNS - - -

  THETA   PHI     MAJOR
 DEGREES  DEGREES   DB
  0.00    0.00    11.56149
  180.00  0.00    -0.12704

 ***** INPUT LINE  4  PT  -1  0  0  0  0.0  0.0  0.0  0.0  0.0  0.0
 ***** INPUT LINE  5  XQ   0  0  0  0  0.0  0.0  0.0  0.0  0.0  0.0
 ***** INPUT LINE  6  EN   0  0  0  0  0.0  0.0  0.0  0.0  0.0  0.0
```

MathCad file for second Γ-Match attempt

Gamma-Match

$c := 2.9979 \cdot 10^{8}$ $fc := 647 \cdot 10^{6}$ $\lambda := \frac{c}{fc}$ $\lambda = 0.46335$ $k := \frac{2 \cdot \pi}{\lambda}$ $k = 13.56023$

$Zdesired := 50\ \Omega$ $d := 0.635 \cdot 10^{-2}\ m$ $a := d \cdot 0.5$ $a = 0.003175\ m$

$dprime := 0.3175 \cdot 10^{-2}\ m$ $aprime := dprime \cdot 0.5$ $aprime = 0.0015875\ m$ $s := 2. \cdot 10^{-2}\ m$

$$Z0 := \frac{376.73}{2 \cdot \pi} \cdot acosh\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right] \qquad Z0 = 260.329\ \Omega$$

$u := \frac{a}{aprime}$ $v := \frac{s}{aprime}$

$u = 2$ $v = 12.5984$

$$\alpha := \frac{acosh\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{acosh\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]} \qquad \alpha = 1.36748$$

$$ae := aprime \cdot e^{\frac{1}{(1+u)^2} \cdot \left(u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)\right)}$$

$ae = 0.00666$ $ae \cdot 100 = 0.66609$ cm

$lprime := 10. \cdot 10^{-2}\ m$ $lprime \cdot 0.5 = 0.05\ m$

$$Zt := j \cdot Z0 \cdot \tan\left(\frac{k \cdot lprime}{2}\right) \qquad Zt = 209.66289i\ \Omega$$

$Yt := \frac{1}{Zt}$ $Yt = -4.77i \times 10^{-3}$

Za from NEC (a MoM program)

$Za := 14.9299 + j \cdot 1.27253\ \Omega$ $Ya := \frac{1}{Za}$ $Ya = 0.0665 - 0.00567i$ Mhos

$$Yin2 := Yt + \frac{Ya \cdot 2}{(1 + \alpha)^2} \qquad Yin2 = 0.024 - 6.792i \times 10^{-3}$$

$\frac{1}{Zdesired} = 0.02$ Mhos

$Zin2 := \frac{1}{Yin2}$ $Zin2 = 38.953 + 11.15i\ \Omega$ $Zdesired = 50\ \Omega$

$$C := if\left(Im(Zin2) > 0, \frac{1}{2 \cdot \pi \cdot fc \cdot Im(Zin2)}, 0\right) \qquad C = 2.206 \times 10^{-11}$$

$C \cdot 10^{12} = 22.061$ pF

$$Zcap := if\left(C > 0, \frac{1}{j \cdot 2 \cdot \pi \cdot fc \cdot C}, 0\right) \qquad Zcap = -11.15i\ \Omega$$

$Zing := Zcap + Zin2$ $\boxed{Zing = 38.953}\ \Omega$

$$\Gamma := \frac{(Zing - Zdesired)}{Zing + Zdesired} \qquad \Gamma = -0.124$$

$|\Gamma| = 0.124$

$$VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|} \qquad VSWR = 1.284$$

Second try

Third attempt at Γ-Match

Keep Γ-Match diameter $2a' = 0.125" = 0.3175\,\text{cm} \Rightarrow a' = 0.15875\,\text{cm}$

Keep Γ-Match spacing $s = 2$ cm and length $l'/2 = 5$ cm

Keep driven element length $l_2' = 20.8\,\text{cm}$

Increase reflector length **$l_1' = 22.4\,\text{cm}$** & decrease director length **$l_3' = 19.1\,\text{cm}$**

Using NEC-2 and MathCad (see attached pages)-

- Using MathCad for Γ-match: $Z_0 = 260.329\ \Omega$, $\alpha = 1.36748$, $a_e = 0.666$ cm, & $Z_t = j209.663\ \Omega$ (no changes)
- From NEC: $Z_a = 18.2632 - j\,0.2054\ \Omega$ & Gain = 11.45567 dBi
- Using MathCad: $Z_{\text{in}}/2 = 48.555 + j\ 11.309\ \Omega$. After adding series capacitor $C = 21.752$ pF, $Z_{\text{in,Gamma}} = 48.555\ \Omega$. With this input impedance, $|\Gamma| = 0.015$, and **VSWR = 1.03 < 1.1** (DONE!).

NEC input file for third Γ-Match attempt

```
CM Yagi-Uda Antenna for UHF channel 43 (NO BOOM)
CM
CM THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN
CM ELEMENT OF A 5 ELEMENT ANTENNA. CENTER FREQUENCY IS 647 MHz
CM W/ WAVELENGTH OF 0.46335 m.
CM
CM THE DIMENSIONS ARE:
CM element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in,
CM equivalent radius of Gamma-Match portion of driven element
CM is ae=0.00666 m which has a length of l'/2=0.05 m  < l2
CM l1=0.483 l=0.224m, l3=0.4122 l=0.191 m, l4=0.412 l=0.1909m,
CM l5=0.419 l=0.1941m
CM driven element l2=0.4489 l= 0.208m
CM ELEMENT SPACINGS Sij=0.2 l=0.09267m
CM SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
CE THE DRIVEN SEGMENT IS #1 on l2 Tag 3.
GW 1 17 -0.112  0.0 0.0     0.112  0.0 0.0     0.003175   !Reflector
GW 2 9  -0.104   0.0 0.09267 0.0     0.0 0.09267 0.003175 !Driven tip
GW 3 4   0.0     0.0 0.09267 0.05    0.0 0.09267 0.00666  !Driven middle
GW 4 5   0.05    0.0 0.09267 0.104   0.0 0.09267 0.003175 !Driven tip
GW 5 15 -0.09505 0.0 0.18534 0.09505 0.0 0.18534 0.003175 !Director 1
GW 6 15 -0.09545 0.0 0.27801 0.09545 0.0 0.27801 0.003175 !Director 2
GW 7 15 -0.09705 0.0 0.37068 0.09705 0.0 0.37068 0.003175 !Director 3
GE 0 0
FR 0 1 0 0 647 0
EX 0 3 1 0 1.0 0.0
RP 0 2 3 0000 0.0 0.0 180.0 90.0
PT -1
XQ 0
EN
```

NEC output file for third Γ-Match attempt

```
Yagi-Uda Antenna for UHF channel 43 (NO BOOM)

THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN ELEMENT OF
A 5 ELEMENT ANTENNA. CENTER FREQUENCY IS 647 MHz W/ WAVELENGTH OF 0.46335 m.

   THE DIMENSIONS ARE:
   element diameter d=0.635cm=0.25in, radius a=d/2=0.3175cm=0.125in,
   equivalent radius of Gamma-Match portion of driven element
   is ae=0.00666 m which has a length of l'/2=0.05 m  < l2

   l1=0.483 l=0.224m, l3=0.4122 l=0.191 m, l4=0.412 l=0.1909m,
   l5=0.419 l=0.1941m
   driven element l2=0.4489 l= 0.208m
   ELEMENT SPACINGS Sij=0.2 l=0.09267m
   SELECT SEGMENT LENGTH OF APPROX. 1.25cm=0.025 l
   THE DRIVEN SEGMENT IS #9 on l2.
      ***********************************************************
     - - - STRUCTURE SPECIFICATION - - -
    COORDINATES MUST BE INPUT IN METERS OR BE SCALED TO METERS
    BEFORE STRUCTURE INPUT IS ENDED

WIRE                                                      NO. OF FIRST LAST  TAG
 NO.  X1       Y1     Z1      X2       Y2     Z2     RADIUS   SEG.  SEG. SEG. NO.
 1   -0.11200 0.0 0.00000  0.11200 0.0 0.00000  0.00317  17     1    17   1
 2   -0.10400 0.0 0.09267  0.00000 0.0 0.09267  0.00317   9    18    26   2
 3    0.00000 0.0 0.09267  0.05000 0.0 0.09267  0.00666   4    27    30   3
 4    0.05000 0.0 0.09267  0.10400 0.0 0.09267  0.00317   5    31    35   4
 5   -0.09505 0.0 0.18534  0.09505 0.0 0.18534  0.00317  15    36    50   5
 6   -0.09545 0.0 0.27801  0.09545 0.0 0.27801  0.00317  15    51    65   6
 7   -0.09705 0.0 0.37068  0.09705 0.0 0.37068  0.00317  15    66    80   7

TOTAL SEGMENTS USED= 80  NO. SEG. IN A SYMMETRIC CELL= 80  SYMMETRY FLAG=  0

***** INPUT LINE  1  FR  0   1   0   0  6.47000E+02  0.0  0.0  0.0  0.0  0.0
***** INPUT LINE  2  EX  0   3   1   0  1.00000E+00  0.0  0.0  0.0  0.0  0.0
***** INPUT LINE  3  RP  0   2   3   0  0.0 0.0 1.80E+02  9.00E+01  0.0  0.0

FREQUENCY= 6.4700E+02 MHZ  WAVELENGTH= 4.6337E-01 METERS

      - - - ANTENNA INPUT PARAMETERS - - -
TAG  SEG. VOLTAGE(V) CURRENT (AMPS)     IMPEDANCE (OHMS)     ADMITTANCE (MHOS)
NO. NO. REAL IMAG. REAL      IMAG.      REAL         IMAG.     REAL IMAG.
3   27  1.0  0.0 5.48E-02 6.16E-04 1.82632E+01-2.05397E-01 5.48E-02 6.16E-04

     - - - RADIATION PATTERNS - - -
   THETA  PHI      Gain
 DEGREES  DEGREES    DB
    0.00  0.00    11.45567
  180.00  0.00    -1.78089

 ***** INPUT LINE  4  PT  -1    0    0    0  0.0  0.0  0.0  0.0  0.0  0.0
 ***** INPUT LINE  5  XQ   0    0    0    0  0.0  0.0  0.0  0.0  0.0  0.0
 ***** INPUT LINE  6  EN   0    0    0    0  0.0  0.0  0.0  0.0  0.0  0.0
```

MathCad file for third Γ-Match attempt

Gamma-Match equations

$c := 2.9979 \cdot 10^{8}$ $fc := 647 \cdot 10^{6}$ $\lambda := \frac{c}{fc}$ $\lambda = 0.46335$ $k := \frac{2 \cdot \pi}{\lambda}$ $k = 13.56023$

$Zdesired := 50\ \Omega$ $d := 0.635 \cdot 10^{-2}\ m$ $a := d \cdot 0.5$ $a = 0.003175\ m$

$dprime := 0.3175 \cdot 10^{-2}\ m$ $aprime := dprime \cdot 0.5$ $aprime = 0.0015875\ m$ $s := 2. \cdot 10^{-2}\ m$

$$Z0 := \frac{376.73}{2 \cdot \pi} \cdot acosh\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right] \qquad Z0 = 260.329\ \Omega$$

$u := \frac{a}{aprime}$ $v := \frac{s}{aprime}$

$u = 2$ $v = 12.5984$

$$\alpha := \frac{acosh\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{acosh\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]} \qquad \alpha = 1.36748$$

$$ae := aprime \cdot e^{\frac{1}{(1+u)^2} \cdot \left(u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)\right)}$$

$ae = 0.00666$ $ae \cdot 100 = 0.66609\ cm$

$lprime := 10. \cdot 10^{-2}\ m$ $lprime \cdot 0.5 = 0.05\ m$

$$Zt := j \cdot Z0 \cdot \tan\left(\frac{k \cdot lprime}{2}\right) \qquad Zt = 209.66289i\ \Omega$$

$Yt := \frac{1}{Zt}$ $Yt = -4.77i \times 10^{-3}$

Za from NEC (a MoM program)

$Za := 18.2632 - j \cdot 0.205397\ \Omega$ $Ya := \frac{1}{Za}$ $Ya = 0.05475 + 0.00062i$ Mhos

$$Yin2 := Yt + \frac{Ya \cdot 2}{(1 + \alpha)^2} \qquad Yin2 = 0.02 - 4.55i \times 10^{-3}$$

$\frac{1}{Zdesired} = 0.02$ Mhos

$Zin2 := \frac{1}{Yin2}$ $Zin2 = 48.555 + 11.309i\ \Omega$ $Zdesired = 50\ \Omega$

$$C := if\left(Im(Zin2) > 0, \frac{1}{2 \cdot \pi \cdot fc \cdot Im(Zin2)}, 0\right) \qquad C = 2.175 \times 10^{-11}$$

$C \cdot 10^{12} = 21.752$ pF

$$Zcap := if\left(C > 0, \frac{1}{j \cdot 2 \cdot \pi \cdot fc \cdot C}, 0\right) \qquad Zcap = -11.309i\ \Omega$$

$Zing := Zcap + Zin2$ $\boxed{Zing = 48.555}\ \Omega$

$$\Gamma := \frac{(Zing - Zdesired)}{Zing + Zdesired} \qquad \Gamma = -0.015$$

$|\Gamma| = 0.015$

$VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$ $\boxed{VSWR = 1.03}$

Third try, DONE!

Yagi-Uda antenna CH 13 final design (f_c = 647 MHz & λ = 46.3 cm)

element diameters $d = 2a = 0.25" = 0.635$ cm $\Rightarrow$ $a = 0.125" = 0.3175$ cm

element spacings $s_{ij} = 0.2\lambda = 9.274$ cm

Γ-Match diameter $2a' = 0.3175$ cm, spacing s = 2 cm, & length $l'/2$ = 5 cm

reflector length $l_1' = 22.4$ cm & driven element length $l_2' = 20.8$ cm

director lengths $l_3' = 19.1$ cm, $l_4' = 19.09$ cm, & $l_5' = 19.41$ cm

$C = 21.752$ pF $\Rightarrow$ $Z_{in,Gamma} = 48.555\ \Omega$, and Gain = 11.456 dBi