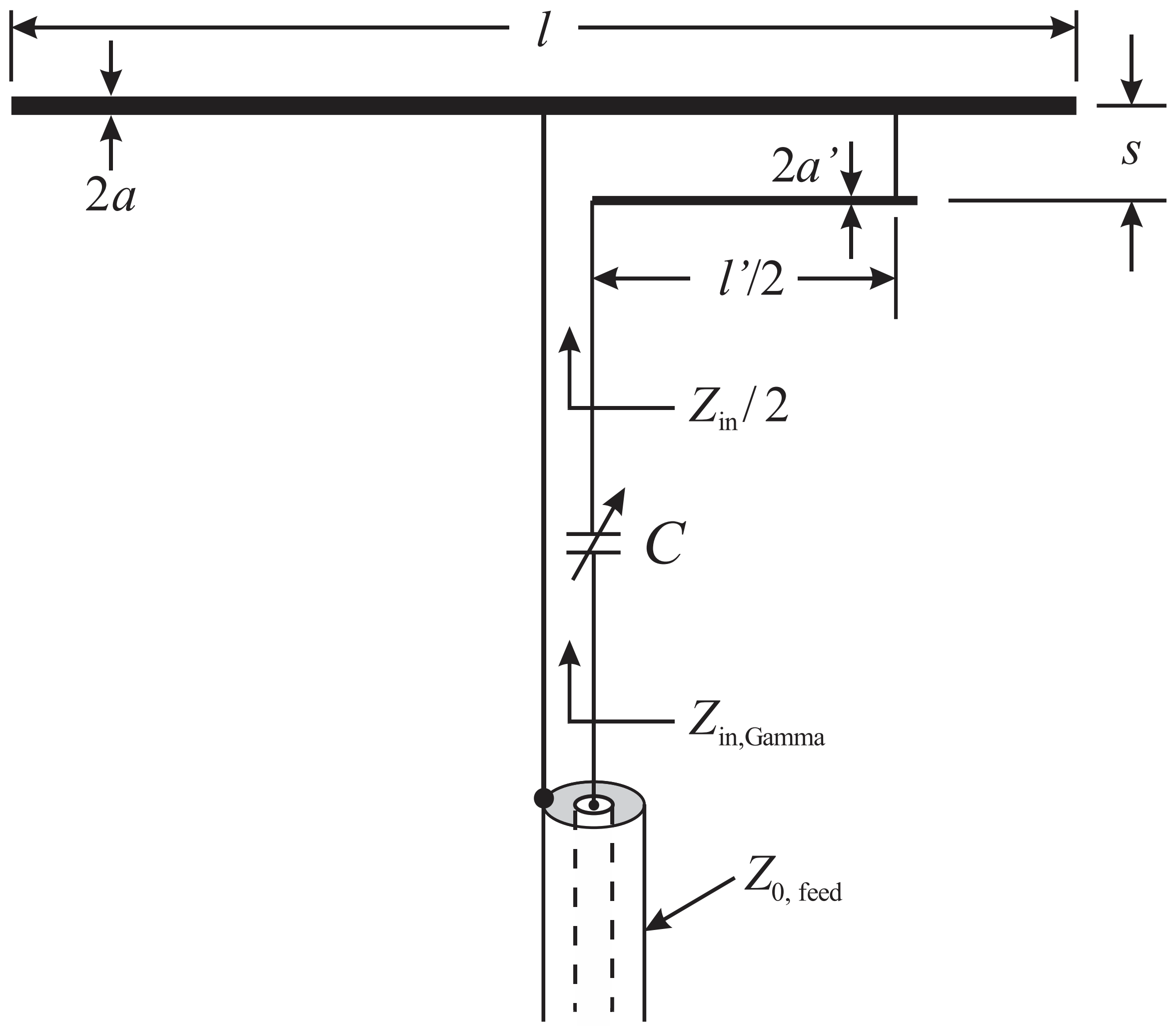
Matching Techniques for Driving Yagi-Uda Antennas: Gamma-Match

[Sections 9.5, 9.6, & 9.8 of *Antenna Theory, Analysis and Design* (4e) by Balanis)]

Gamma-Match:



- The Gamma-Match (Γ-Match) allows dipoles or the driven element of Yagi-Uda antennas to be fed by **coaxial** transmission lines (unbalanced).
- As shown, the shield of the coaxial line is connected to the center of the dipole/driven element (the axis of symmetry) or the metallic boom by "pigtailing" the shield braid. The center conductor of the coaxial line is connected to the Γ-Match.

- Note that the Γ-Match is simply the modified T-Match split down the middle. Therefore, the analysis follows that for the modified T-Match and T-Match.

- The input impedance $Z_{\text{in,Gamma}}$ is simply half that of the modified T-Match $Z_{\text{in,MT}}$ or is equal to half that of the T-Match (Z_{in}) plus the impedance of a single series capacitor-

$$
\begin{aligned}
Z_{\text{in,Gamma}} &= \frac{Z_{\text{in,MT}}}{2} = Z_{\text{cap}} + \frac{Z_{\text{in}}}{2} \\
&= \frac{1}{j\omega C} + \frac{1}{\dfrac{1}{Z_t} + \dfrac{2}{(1+\alpha)^2 Z_a}} \\
&= \frac{1}{j\omega C} + \frac{(1+\alpha)^2 Z_a Z_t}{(1+\alpha)^2 Z_a + 2Z_t}
\end{aligned}
$$

  where Z_a is the input impedance of the antenna mode of the driven element, α is the current divisor factor, and Z_t is the input impedance of the transmission line mode of the Γ-Match, and C is the series capacitance required to counteract the inductive reactance of Z_{in} /2, i.e., we desire $Z_{\text{in,Gamma}} = R_{\text{in}} / 2 \approx Z_{0,\text{feed}}$. $Z_{0,\text{feed}}$ is the characteristic impedance of the feeding transmission line.

- Knowing $Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}}$, to achieve a real input impedance $Z_{\text{in,Gamma}}$ for the Γ-Match, the capacitance is calculated as-

$$
C = \frac{1}{2\pi f X_{\text{in}}}.
$$

  Note the factor of two (2) in the denominator versus the equation for the capacitance required for the modified T-Match.

Design Process:

- We desire to match a given Yagi-Uda antenna to $Z_{0,\text{feed}}$. Usually a specification in terms of the VSWR is given.

1) Select a driven element length l_2 so that $l_1 < l_2 < l_3$ as well as a', s, and $l'/2$ (usually $l'/2 < l_2/4 < \lambda/4$). These values may be changed later.

- Diameter of feed $2a'$- usually you will want this to be less than the Yagi-Uda element diameters $2a$ to make $\alpha > 1$.
- Spacing s- make less than $s_{12}/4$, more than 1 cm (practical construction), and less than 5 cm (don't want the characteristic impedance Z_0 of the Γ-Match section to be too large).
- Length $l'/2$- make less than fourth of the initial driven element length l_2, i.e., $l'/2 \le l_2/4$, to avoid overly disturbing the current distribution on l_2.

2) Calculate the characteristic impedance Z_0 of the transmission line mode of the Γ-Match-

$$Z_0 = \frac{\eta}{2\pi}\cosh^{-1}\left(\frac{s^2 - a^2 - a'^2}{2aa'}\right)$$

where $\eta = \sqrt{\mu/\varepsilon_{\text{eff}}}$ is the intrinsic impedance of the material wherein the Γ-Match exists.

3) Calculate the transmission line mode input impedance Z_t -

$$Z_t = jZ_0 \tan\left(kl'/2\right) \text{ where } k = \beta = 2\pi/\lambda.$$

4) Calculate the parameters u, v, and α-

$$u = \frac{a}{a'},\quad v = \frac{s}{a'},\quad \text{and}\quad \alpha = \frac{\cosh^{-1}\left(\dfrac{v^2 - u^2 + 1}{2v}\right)}{\cosh^{-1}\left(\dfrac{v^2 + u^2 - 1}{2vu}\right)} \quad \begin{array}{l} \alpha > 1 \text{ when } a > a' \\ \alpha = 1 \text{ when } a = a'. \\ \alpha < 1 \text{ when } a < a' \end{array}$$

5) Calculate the equivalent radius a_e of the Γ-Match section -

$$a_e \approx a' e^{\frac{u^2 \ln u + 2u \ln v}{(1+u)^2}}.$$

6) Find input impedance of antenna mode Z_a (i.e., use NEC-2) of the driven element as part of the overall Yagi-Uda antenna. Remember to modify the driven element to have an equivalent radius of a_e over the length $l'/2$ corresponding to the Γ-Match and radius a for the remainder. [Note: Put feed on either the first or second (more realistic) segment of the a_e part.]

7) Find overall T-Match input impedance Z_{in} of the driven element as part of the overall Yagi-Uda antenna-

$$Z_{\text{in}} = \frac{2(1+\alpha)^2 Z_a Z_t}{(1+\alpha)^2 Z_a + 2Z_t}$$

8) Determine if $Z_{\text{in}} / 2$ meets your matching specification. If so, stop the design process (the capacitor is not needed). If not, check if $X_{\text{in}} > 0$ (inductive reactance). If so, go to next steps, else go back to step 1) and adjust l_1, $\boldsymbol{l_2}$, l_3, a', s, and/or l'/2 so $X_{\text{in}} > 0$. [Hint: Making l_2 (or l_1) longer can increase inductive reactance.]

9) If $X_{\text{in}} > 0$, calculate C –

$$C = \frac{1}{2\pi f X_{\text{in}}}$$

and

$$Z_{\text{in,Gamma}} = R_{\text{in}} / 2 = \frac{1}{j\omega C} + \frac{(1+\alpha)^2 Z_a Z_t}{(1+\alpha)^2 Z_a + 2Z_t}.$$

10) Determine if $Z_{\text{in,Gamma}}$ meets your matching specification. If not, back to step 1) and adjust l_2, l'/2, s, l_1, l_3, & a' can be varied (try in that order), and repeat steps 2) - 8).

- Remember, the magnitude of the input impedance $|Z_{\text{in,Gamma}}|$ is greatly affected by α (i.e., when α increases, $|Z_{\text{in,Gamma}}|$ increases, and vice versa). In turn, α is inversely related to s (i.e., if s decreases, α increases, and vice versa).
- Try changing l'/2 *toward* the length suggested by –

$$l'/2 = \frac{1}{k}\tan^{-1}\left[\frac{1}{Z_0 \operatorname{Im}\left(\frac{2Y_a}{(1+\alpha)^2}\right)}\right]$$

  to better offset the antenna mode reactance.
- See http://montoya.sdsmt.edu/ee483_583/notes/matching_tips.pdf for further tips.