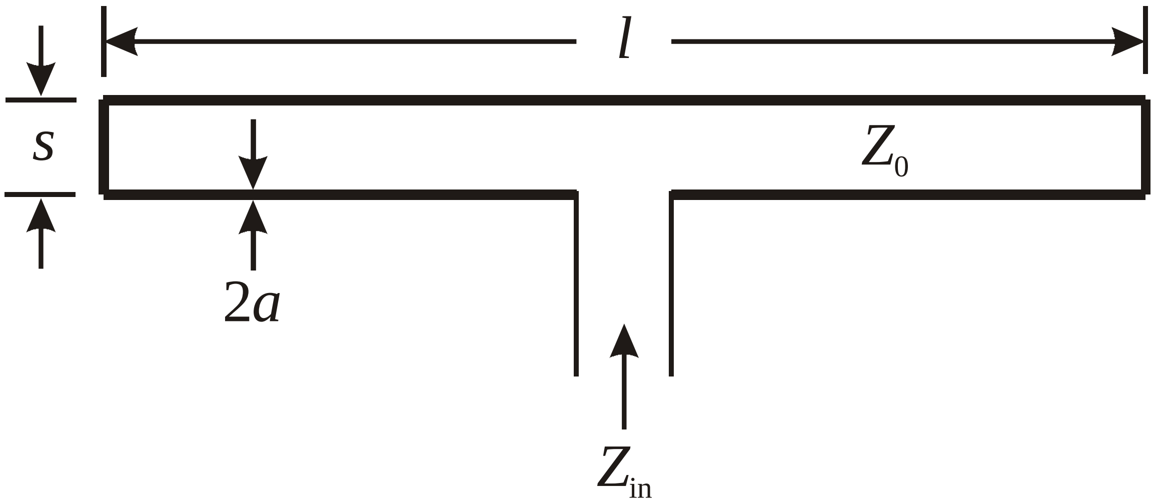
Driving/Matching Techniques For Yagi-Uda Antennas: Folded Dipole

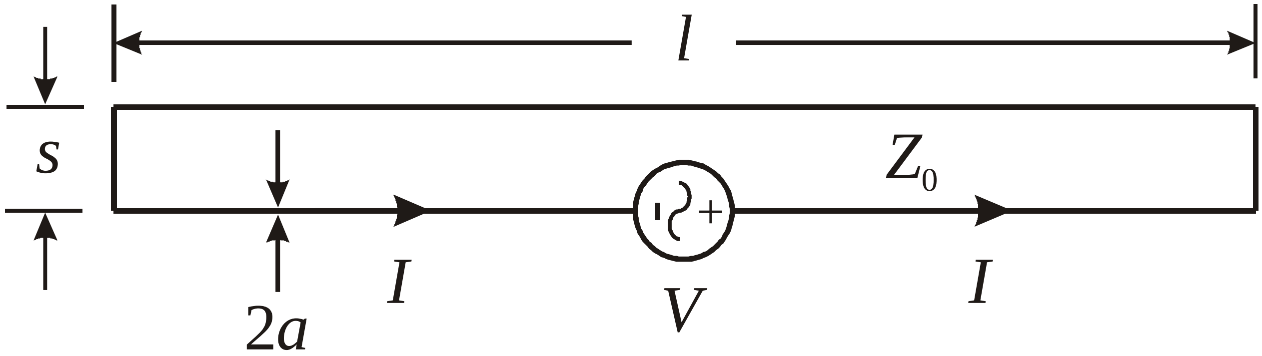
[Sections 9.5 & 9.6 of *Antenna Theory, Analysis and Design* (4e) by Balanis)]

Folded Dipole:



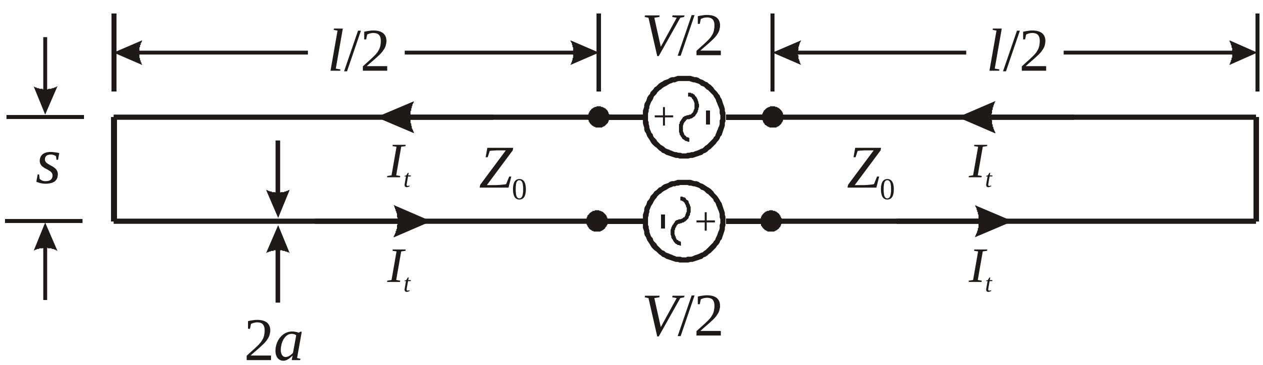
- A good example of a folded dipole application is an FM radio antenna.
- As shown, a folded dipole is a thin ($s << \lambda$) rectangular loop.
- Typically, folded dipoles are used to provide a step-up in impedance by about a factor of 4 when a regular dipole antenna has a lower impedance than the feeding transmission line. [e.g., 4 × 75 Ω = 300 Ω]
- Folded dipoles are useful as driven elements when a twin-lead transmission line is used to feed a Yagi-Uda antenna.
- They can be modeled as a combination of a short-circuited transmission lines and a dipole antenna.
- The characteristic impedance for a uniform twin-lead transmission line is $Z_0 = \frac{\eta}{\pi}\cosh^{-1}\left(\frac{s}{2a}\right)$, where $\eta = \sqrt{\mu / \varepsilon_{\text{eff}}}$ is the characteristic impedance of the material wherein the folded dipole exists.

Model:



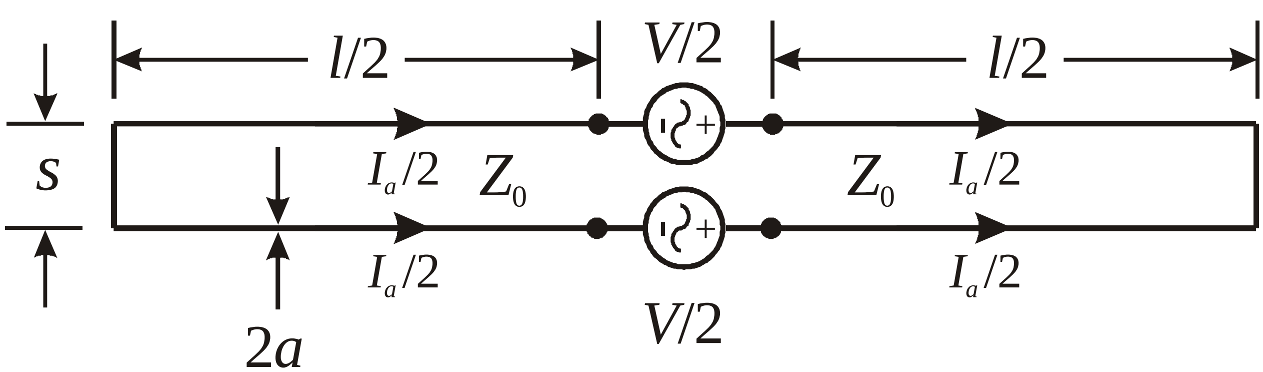
$=$

Transmission Line Mode:

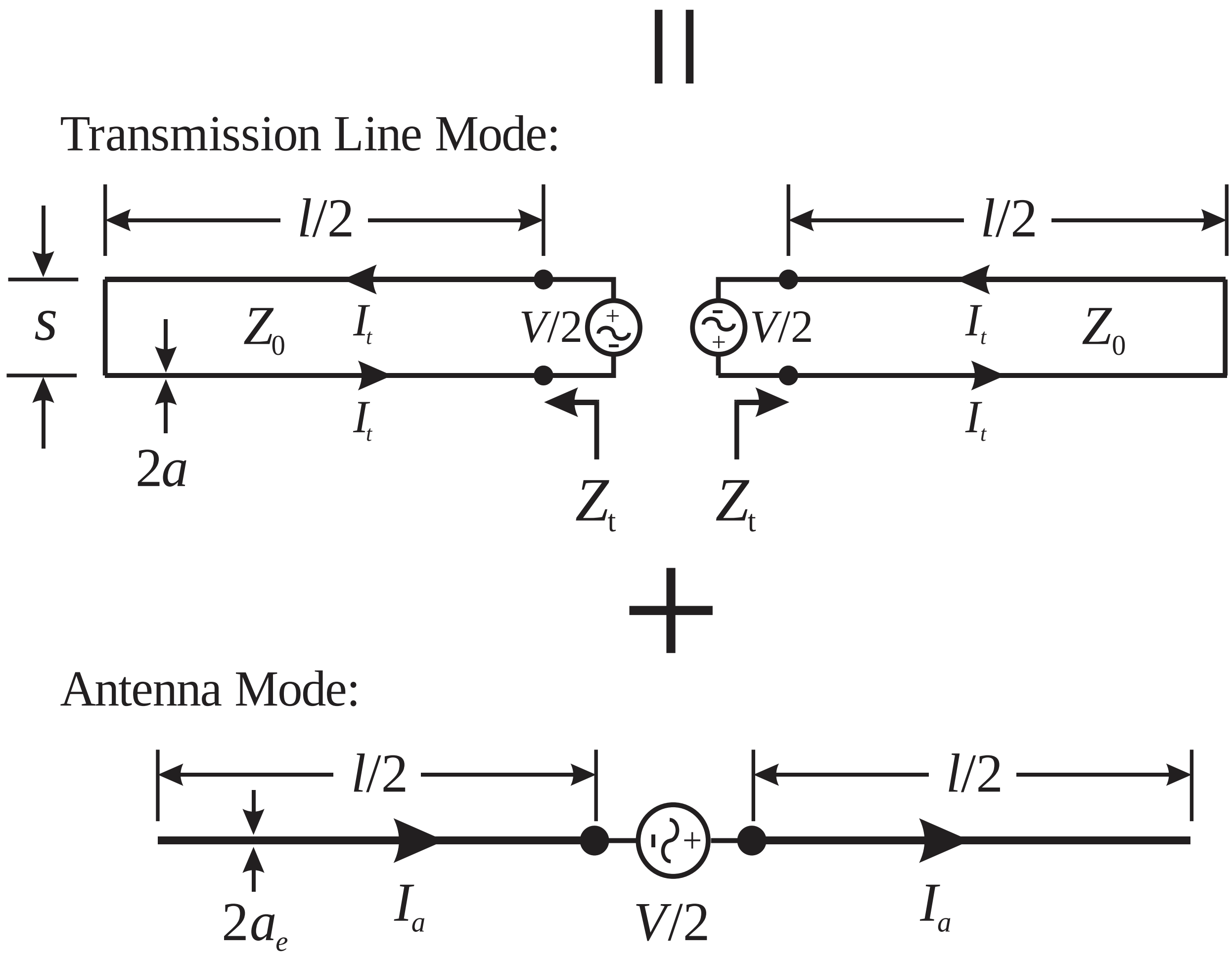


$+$

Antenna Mode:

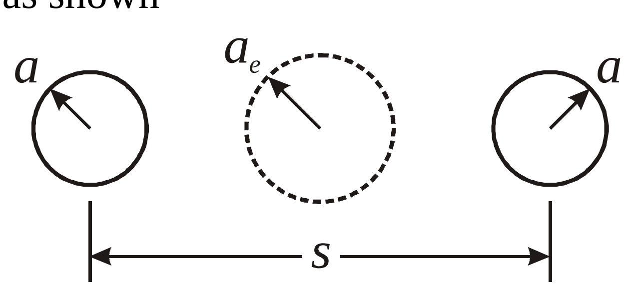


This model can be further refined as:



where a_e is the effective radius of the two wires considered together. The text discusses equivalent radii for several geometries in section 9.5.5 and Table 9.3.

The twin wires of radii a separated by a distance s (where $s << \lambda$) that compose the folded dipole can be modeled as a single wire with an equivalent radius a_e as shown



where $a_e = \sqrt{as}$ (9-24a).

Transmission line mode impedance and current

Transmission line(s) input impedance is $Z_t = \dfrac{V/2}{I_t}$.

The input impedance and admittance, respectively, for a transmission line with a short-circuit load are

$$Z_t = jZ_0 \tan(kl/2) \text{ and } Y_t = \frac{-j}{Z_0 \tan(kl/2)},$$

where $k = \beta = 2\pi/\lambda$.

Notes:

- For $0 < l < 0.5\lambda$, the transmission line mode reactance $Z_0 \tan(kl/2) > 0$, i.e., inductive reactance. This is typically the case encountered when using a folded dipole by itself or in a Yagi-Uda antenna.
- When $l = 0.5\lambda$, $Z_t = jZ_0 \tan(\pi/2) \to \infty$.
- If $0.5\lambda < l < \lambda$, then $Z_0 \tan(kl/2) < 0$, i.e., capacitive reactance.

Solving for the current yields $I_t = \dfrac{V/2}{Z_t} = \dfrac{V}{j2Z_0 \tan(kl/2)}$

Antenna mode impedance and current

- The antenna mode input impedance Z_a (Z_d in text) for a dipole of radius a_e and length l is usually found numerically (e.g., Method of Moments program like NEC) or from figures.
- If a folded dipole is used as the driven element in a Yagi-Uda antenna, insert this equivalent dipole into the antenna when finding Z_a to (partially) account for interaction with other elements.

The antenna mode input impedance is defined as $Z_a = \dfrac{V/2}{I_a}$.

Solving for the antenna mode current, yields $I_a = \dfrac{V/2}{Z_a}$.

Each wire carries half of the current (i.e., $I_a/2$) in the antenna mode.

Total impedance and current for folded dipole

The current at the terminals of the folded dipole is

$$I = I_t + \frac{I_a}{2} = \frac{V/2}{Z_t} + \frac{V/2}{2Z_a} = V\left(\frac{1}{2Z_t} + \frac{1}{4Z_a}\right) = V\left(\frac{2Z_a + Z_t}{4Z_a Z_t}\right).$$

Solving for the input admittance and impedance, yields

$$Y_{\text{in}} = \frac{I}{V} = \frac{1}{2Z_t} + \frac{1}{4Z_a} \quad \Rightarrow \quad \underline{Y_{\text{in}} = \frac{Y_t}{2} + \frac{Y_a}{4}},$$

and

$$Z_{\text{in}} = \frac{V}{I} \quad \Rightarrow \quad \underline{Z_{\text{in}} = \frac{4Z_a Z_t}{2Z_a + Z_t}}.$$

In order for $Y_t/2$ (purely imaginary) to cancel the susceptance of $Y_a/4$, when $0 < l < 0.5\lambda$ (implies susceptance of $Y_t/2$ is negative/inductive), the susceptance of Y_a must be positive/capacitive (i.e., the reactance of Z_a is negative/capacitive). As we saw when studying dipoles, this is typically the case when $l < 0.5\lambda$.

For the case that $l = \lambda/2$ (i.e., a half-wave dipole), the transmission line impedance and admittance become

$$Z_t = jZ_0 \tan(kl/2) = jZ_0 \tan(\pi/2) \rightarrow \infty$$

and

$$Y_t = 0.$$

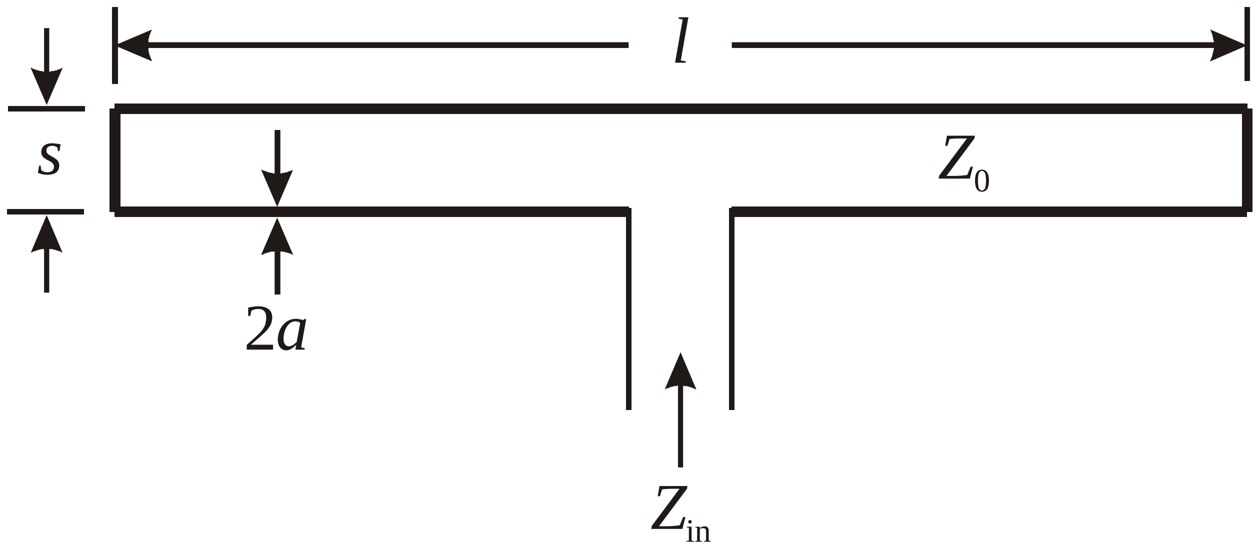
Then, the input impedance and admittance become

$$Z_{\text{in}} = \lim_{Z_t \rightarrow \infty}\left(\frac{4Z_a Z_t}{2Z_a + Z_t}\right) \quad \Rightarrow \quad \underline{Z_{\text{in}} = 4Z_a}$$

and

$$\underline{Y_{\text{in}} = \frac{Y_a}{4}}.$$

Example: Find the input impedance of a folded dipole in free space described by the dimensions: $l = \lambda/2.2 = 0.4545\lambda$, $s = 0.00615\lambda$, and $2a = 0.001\lambda$.



Transmission line characteristic impedance

$$
\begin{aligned}
Z_0 &= \frac{\eta}{\pi}\cosh^{-1}\left(\frac{s}{2a}\right) \\
&= \frac{376.7303}{\pi}\cosh^{-1}\left(\frac{0.00615\lambda}{0.001\lambda}\right) \\
&= 300.143\,\Omega
\end{aligned}
$$

Input impedance of transmission line mode

$$
\frac{kl}{2} = \frac{1}{2}\left(\frac{2\pi}{\lambda}\right)\frac{\lambda}{2.2} = 1.428
$$

$$
\begin{aligned}
Z_t &= jZ_0 \tan(kl/2) \\
&= j300.143\tan(1.428) \\
\Rightarrow \quad & \underline{Z_t = j2087.5404\,\Omega}
\end{aligned}
$$

Input impedance of antenna mode

The equivalent radius of the twin wires forming the folded dipole is

$$a_e = \sqrt{a\,s} = \sqrt{(0.001\lambda / 2)(0.00615\lambda)} = 0.001754\lambda .$$

Use a Method of Moments program (e.g., NEC-2) to find the input impedance of a dipole of length $l = \lambda/2.2$ and radius $a_e = 0.0011754\lambda$.

NEC-2 input file

```
CM EE 483 Folded Dipole Example (folded_example.txt)
CM This file is used to determine the input impdeance of a
CM lossless dipole antenna, center driven at 10MHz (lambda=29.98m).
CM of length lambda/2.2 = 13.6272727m
CM radius = ae = 0.001754(lambda) = 0.052571962 m
CM Used 51 segments (del/a ~ 5.08).  DRIVEN SEGMENT IS #26.
CE
GW 1 51 0.0 0.0 -6.813636365 0.0 0.0 6.813636365 0.052571962
GE 0
EK 0         ! Use extended kernel in simulation
FR 0 1 0 0 10.0 0.0
EX 0 1 26 00 1.0 0.0
XQ 0
EN
```

NEC-2 output file excerpt

```
<snip>
*********************************************
NUMERICAL ELECTROMAGNETICS CODE (NEC-2D)
*********************************************
<snip>
- - - - - - FREQUENCY - - - - - -
   FREQUENCY= 1.0000E+01 MHZ
   WAVELENGTH= 2.9980E+01 METERS
<snip>
 - - - ANTENNA INPUT PARAMETERS - - -
TAG SEG. VOLTAGE (V) CURRENT (A) IMPEDANCE (OHMS) ADMITTANCE (MHOS) POWER
NO. NO.  REAL  IMAG. REAL  IMAG.  REAL   IMAG.   REAL    IMAG.   (Watts)
 1  26 1.0E+00 0.0E+00 1.34318E-02 5.30149E-03 6.44152E+01-2.54245E+01
1.34318E-02 5.30149E-03 6.71590E-03
<snip>
```

From the NEC-2 output file, we see-

$$Z_a = 64.4152 - j\,25.4245\ \Omega.$$

The input impedance of the folded dipole is then calculated

$$Z_{\text{in}} = \frac{4Z_a Z_t}{2Z_a + Z_t} = \frac{4(64.4152 - j25.4245)(j2087.54)}{2(64.4152 - j25.4245) + j2087.54}$$

$$\Rightarrow \quad \underline{Z_{\text{in}} = 269.61 - j87.18\,\Omega}$$

Since $|Z_t| >> |Z_a|$, we could also use the approximation that

$$Z_{\text{in}} \approx 4Z_a = 4(64.4152 - j25.4245) \Rightarrow \underline{Z_{\text{in}} \approx 257.661 - \mathrm{j}101.698\ \Omega},$$

which is fairly close to the preceeding result.

When connected to a 300 **Ω** twin-lead transmission line, this folded dipole would have a reflection coefficient and VSWR of

$$\Gamma_{\text{in}} = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} = \frac{(269.6 - j87.2) - 300}{(269.6 - j87.2) + 300}$$

$$= 0.160225\angle -100.5^\circ$$

and

$$\text{VSWR} = \frac{1 + |\Gamma_{\text{in}}|}{1 - |\Gamma_{\text{in}}|} = \frac{1 + 0.16}{1 - 0.16} \qquad \Rightarrow \quad \underline{\text{VSWR} = 1.38}.$$