Chapter 5 Loop Antennas

- Simple, cheap, and give pretty good performance.

→ another simple, cheap antenna that is the basis for many other antennas

→ usually circular loops, but square, rectangular, elliptical ... shapes work

→ usually broken into two categories < electrically small / electrically large

where small means $C < \lambda/10$ (C: circumference)

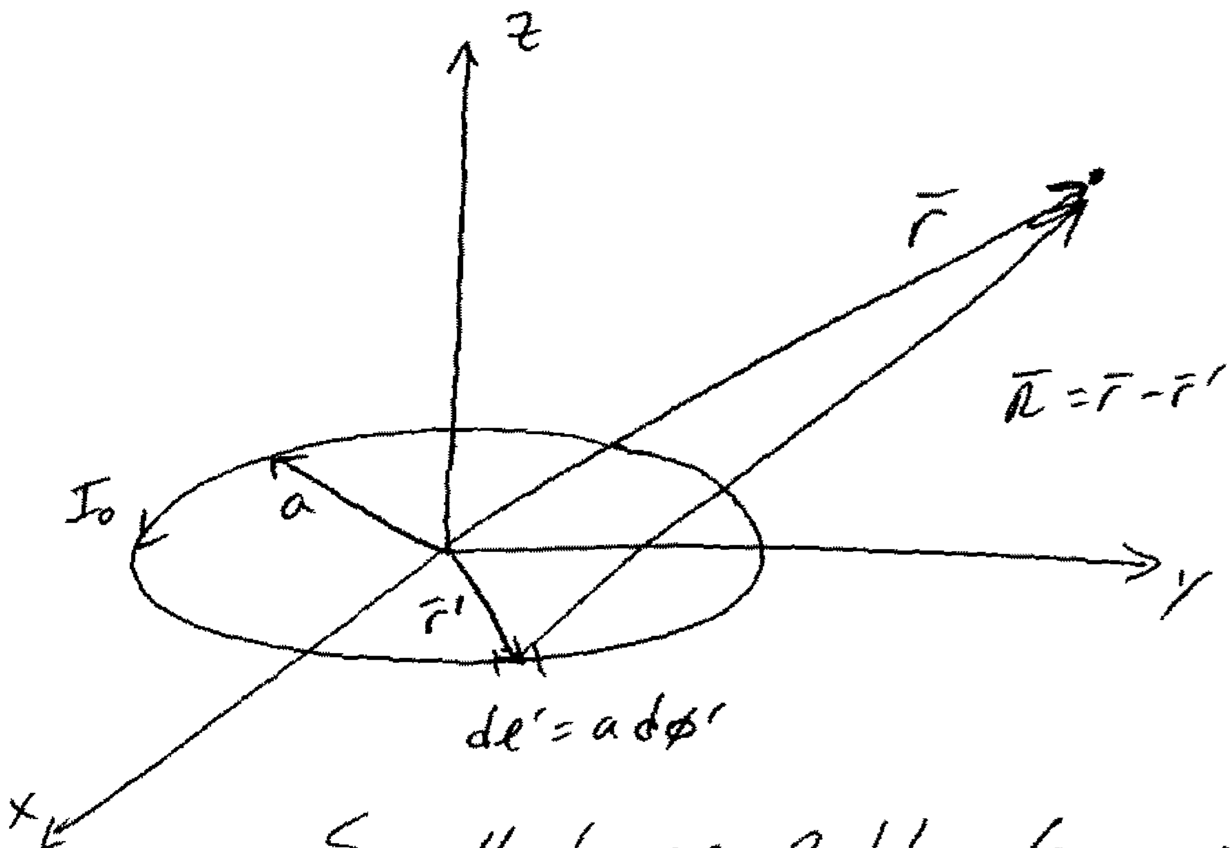
and large means $C \sim \lambda$

→ usually used up to UHF band (~3GHz), but can go higher (tough to make)

→ small loops, like small dipoles, have low radiation resistances (can be smaller than loss resistance). So, they are not used much for transmitting, but do find applications in receive mode. A ferrite core helps increase R_r (boosts $\bar{B} = \mu \bar{H}$) (AM radio a key application)

5.2 Small Circular loop $\left(C < \lambda/10 \text{ or } a < \frac{\lambda}{20\pi}\right)$

→ assume $\bar{I} = \hat{a}_\phi I_o$ ← constant current



Small Loop Problem Geometry

Again

$$\bar{A} = \frac{\mu}{4\pi}\int_C \bar{I}_e \frac{e^{-jkR}}{R}\,dl'$$

⋮ after much math & approximations

$$\bar{A} \simeq \hat{a}_\phi\, j\frac{k\mu a^2 I_o \sin\theta}{4r}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

5.2 cont.

Applying $\bar{H} = \bar{H}_A = \frac{1}{\mu} \bar{\nabla} \times \bar{A}$, yields:

$$H_r = j \frac{ka^2 I_0 \cos\theta}{2r^2} \left[1 + \frac{1}{jkr} \right] e^{-jkr}$$ ← Not radiating

$$H_\theta = -\frac{(ka)^2 I_0 \sin\theta}{4r} \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right] e^{-jkr}$$

$$H_\phi = 0$$

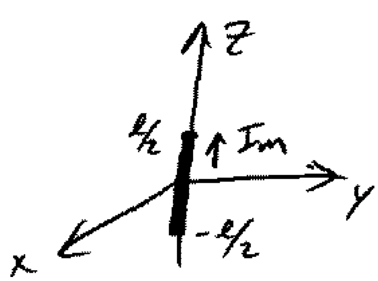
using $\bar{\nabla} \times \bar{H} = \cancel{\bar{J}}^{\,0} + j\omega\epsilon \bar{E}$, yields:

$$E_r = E_\theta = 0$$

$$E_\phi = \eta \frac{(ka)^2 I_0 \sin\theta}{4r} \left[1 + \frac{1}{jkr} \right] e^{-jkr}$$

[Note: $ka = \frac{2\pi}{\lambda} a = \frac{C}{\lambda}$]

These fields very similar to what would obtained for a small magnetic dipole w/ constant magnetic current I_m on a length ℓ along z-axis



5.2 cont.

small magnetic dipole

$$\boxed{\begin{aligned} H_r &= \frac{I_m l \cos\theta}{2\pi\eta r^2}\left[1 + \frac{1}{jkr}\right] e^{-jkr} \\ H_\theta &= j\frac{k I_m l \sin\theta}{4\pi\eta r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right] e^{-jkr} \\ E_\phi &= -j\frac{k I_m l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right] e^{-jkr} \\ E_r &= E_\theta = H_\phi = 0 \end{aligned}}$$

In fact, a small loop + small magnetic dipole are equivalent provided

$$\boxed{I_m l = j S \omega \mu I_o}$$

area of loop

$$\overline{W_{ave}} = \tfrac{1}{2}\,\mathrm{Re}\left\{\bar{E} \times \bar{H}^*\right\}$$

$$= \tfrac{1}{2}\,\mathrm{Re}\left\{-\hat{a}_r E_\phi H_\theta^* + \hat{a}_\theta E_\phi H_r^*\right\}$$

not radiating (falls off as $\frac{1}{r^3}$ or more)

$$\overline{W_{ave}} = \hat{a}_r\,\eta\,\frac{(ka)^4 |I_o|^2 \sin^2\theta}{32 r^2}$$

5.2 cont.

$$P_{rad} = \oiint_{sphere} \overline{W}_{ave} \cdot d\overline{S}_r$$

$$\underline{P_{rad} = \eta \left(\tfrac{\pi}{12}\right) (ka)^4 |I_o|^2} \leftarrow ka = \frac{2\pi a}{\lambda} = \frac{C}{\lambda}$$

letting $\eta \left(\frac{\pi}{12}\right)(ka)^4 |I_o|^2 = \frac{1}{2} |I_o|^2 R_r$

yields

$$\underline{\underline{R_r = \eta \left(\tfrac{\pi}{6}\right)(ka)^4 = \eta \frac{2\pi}{3}\left(\frac{k\underbrace{S}_{\text{area of loop}}}{\lambda}\right)^2 = \eta\left(\tfrac{\pi}{6}\right)\left(\tfrac{C}{\lambda}\right)^4}}$$

What if you have multiple turns of wire?

$$\boxed{R_{r,N} = N^2 R_r}$$

$\leftarrow$ can greatly increase R_r (R_{Loss} goes up however)

ex. Choose $a = \lambda/50$

$$R_r = \eta\left(\tfrac{\pi}{6}\right)(ka)^4 = 376.73\left(\tfrac{\pi}{6}\right)\left(\tfrac{2\pi}{\lambda}\tfrac{\lambda}{50}\right)^4$$

$$\underline{R_r = 0.049189\,\Omega}$$

$N = 30$

$$R_{r,30} = (0.049189)30^2 = \underline{44.27\,\Omega}$$

5.2 cont.

Far-field Region ($kr \gg 1$)

→ only keep field components proportional to $1/r$

$$\boxed{\begin{aligned} H_\theta &\simeq -\frac{(ka)^2 I_0 \sin\theta}{4r} e^{-jkr} = -\frac{\pi S I_0 \sin\theta}{\lambda^2 r} e^{-jkr} \\ E_\phi &\simeq \eta \frac{(ka)^2 I_0 \sin\theta}{4r} e^{-jkr} = \eta \frac{\pi S I_0 \sin\theta}{\lambda^2 r} e^{-jkr} \\ H_r &\simeq H_\phi = E_r = E_\theta = 0 \end{aligned}}$$

Again $Z_{wave} = \frac{-E_\phi}{H_\theta} = \eta$

orthogonal to each other + direction of propagation

Radiation Intensity

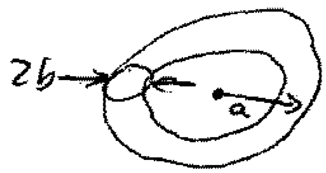
$$\boxed{\begin{aligned} U &= r^2 W_r = \frac{\eta}{2}\left(\frac{k^2 a^2}{4}\right)^2 |I_0|^2 \sin^2\theta = \frac{\eta (ka)^4}{32} |I_0|^2 \sin^2\theta \\ U &= \frac{r^2}{2\eta} |E_\phi|^2 \\ U_{max}\Big|_{\theta=\pi/2} &= \frac{\eta (ka)^4}{32} |I_0|^2 \end{aligned}}$$

5.2 cont.

$$\boxed{\begin{aligned} D_0 &= \frac{4\pi U_{max}}{P_{rad}} = 3/2 = 1.7609\ dB_i \\ A_{em} &= \left(\frac{\lambda^2}{4\pi}\right) D_0 = \frac{3\lambda^2}{8\pi} \end{aligned}}$$

→ all very similar to infinitesimal dipole

Ohmic Losses



→ single turn ($N=1$) $R_L = R_{hf} = \dfrac{\overset{2\pi a}{\ell}}{\underset{2\pi b}{p}} \sqrt{\dfrac{\omega\mu_0}{2\sigma}} = \dfrac{a}{b}\sqrt{\dfrac{\omega\mu_0}{2\sigma}}$

→ multiple turns

* Current not uniformly distributed

$$R_L = R_{ohmic} = \frac{Na}{b}\underbrace{\sqrt{\frac{\omega\mu_0}{2\sigma}}}_{R_S}\left(\frac{R_p}{R_0}+1\right)$$

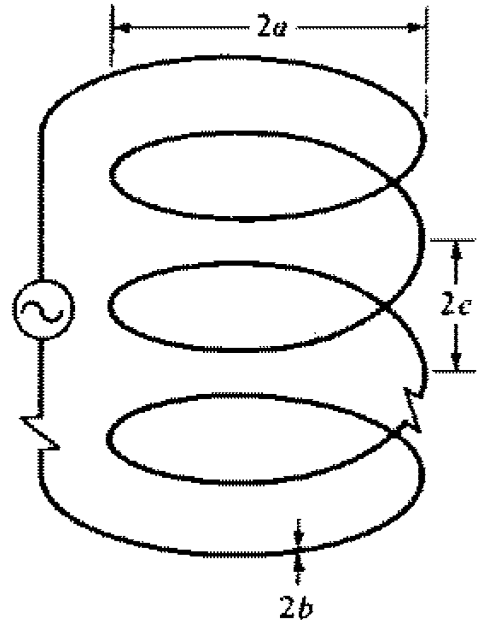
R_S Surface impedance of conductor

$R_p \equiv$ ohmic resistance per unit length due to proximity effect

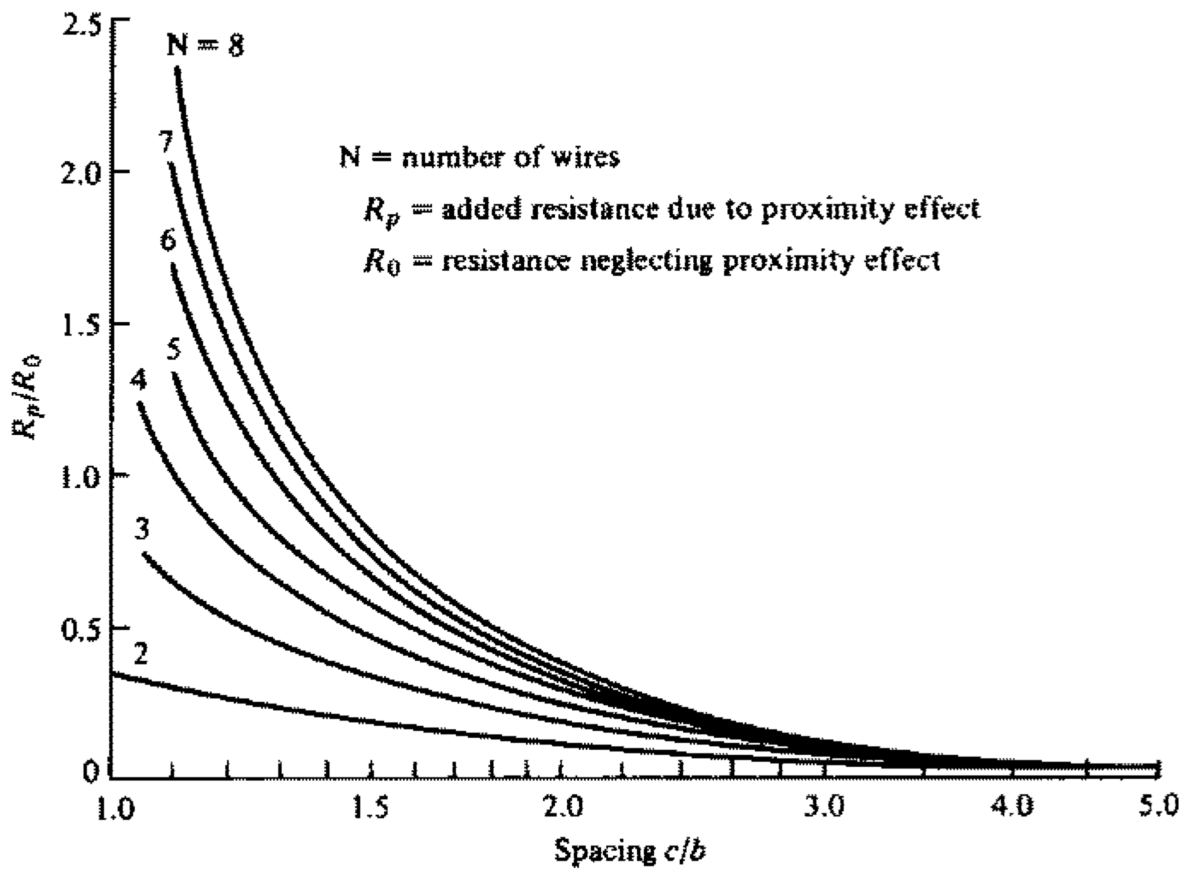
$R_0 \equiv$ ohmic skin effect resistance per-unit length

→ see graph for values of $\frac{R_p}{R_0}$ vs loop spacing

5.2 cont



(a) N–turn circular loop



(b) Ohmic resistance due to proximity (after G. N. Smith)

Figure 5.3 N-turn circular loop and ohmic resistance due to proximity effect. (SOURCE: G. S. Smith, "Radiation Efficiency of Electrically Small Multiturn Loop Antennas," *IEEE Trans. Antennas Propagat.*, Vol. AP-20, No. 5, pp. 656-657, Sept. 1972© (1972) IEEE).

5.3 Circular Loop of Constant Current

→ constant current approx. good up to

$$\frac{\lambda}{10} < C \lesssim \lambda/5, \quad \frac{\lambda}{20\pi} < a \lesssim \frac{\lambda}{10\pi}, \text{ or } \frac{1}{10} < ka \lesssim \frac{1}{5}$$

(analogous to short dipole w/ triangular current distribution)

Here, we assume $R \simeq r - a\sin\theta\cos\phi'$ for phase (i.e., e^{-jkR})

& $R \simeq r$ for amplitude (i.e., $1/R$)

for the far-field.

Then, $\bar{A} \simeq \hat{a}_\phi \, j \dfrac{a\mu I_o e^{-jkr}}{2r} J_1(ka\sin\theta)$

Bessel Function of first Kind, order 1

and

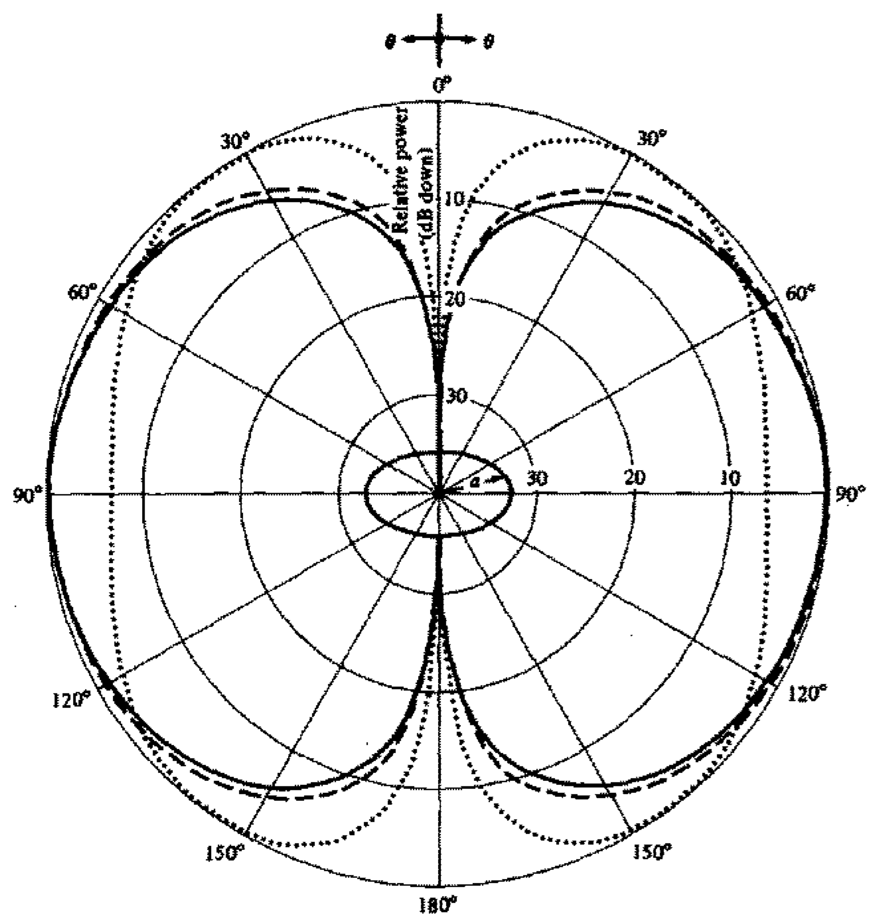
$$\boxed{\begin{aligned} &E_r \simeq E_\theta = 0 \\ &E_\phi \simeq \frac{ak\eta I_o e^{-jkr}}{2r} J_1(ka\sin\theta) \\ &H_r \simeq H_\phi = 0 \\ &H_\theta = -\frac{ak I_o e^{-jkr}}{2r} J_1(ka\sin\theta) \end{aligned}}$$

5.3 cont.

$$\overline{W_{ave}} = \hat{a}_r \frac{1}{2\eta} |E_\phi|^2 = \hat{a}_r \frac{(ka)^2 \eta^2 |I_0|^2}{\eta 8 r^2} J_1^2(ka \sin\theta)$$

$$\overline{W_{ave}} = \hat{a}_r \frac{(a\omega\mu)^2 |I_0|^2}{8 \eta r^2} J_1^2(ka \sin\theta)$$

$$\boxed{\begin{aligned} U = r^2 W_r &= \frac{(ka)^2 \eta |I_0|^2}{8} J_1^2(ka \sin\theta) \\ &= \frac{(a\omega\mu)^2 |I_0|^2}{8\eta} J_1^2(ka \sin\theta) \end{aligned}}$$



a = 0.1λ — $ka = C/\lambda = 0.63$ ← borderline small loop constant I

a = 0.2λ — $ka = C/\lambda = 1.26$

a = 0.5λ — $ka = C/\lambda = 3.1$

} assumption not met

Figure 5.7 Elevation plane amplitude patterns for a circular loop of constant current ($a = 0.1\lambda$, 0.2λ, and 0.5λ). Balanis, Antenna Theory (4th edn)

5.4 Circular Loop w/ Nonuniform Current

What about loops where $C > \lambda/5$ $(a > 0.03\lambda)$?

→ when $C = 2\pi a \simeq \lambda$ $(ka = 1)$, the loop radiates on axis $(\theta = 0, \pi)$ instead of broadside

↳ useful for Yagi-Uda arrays, basis of helical antenna, ...

* Here $a = \lambda/2\pi \simeq \underline{0.159\lambda}$ and the current is no longer constant

↳ Sometimes the current is approximated as

$$I(\phi') = I_0 + 2\sum_{n=1}^{M} I_n \cos(n\phi') \quad \text{(Fourier series)}$$

→ easier/more accurate to do numerical analysis

→ Note: Many texts/articles quantify the relative thickness of the wire as

$$\Omega = 2\ln\left(\frac{2\pi a}{b}\right)$$

$2\pi a$ ← circumference

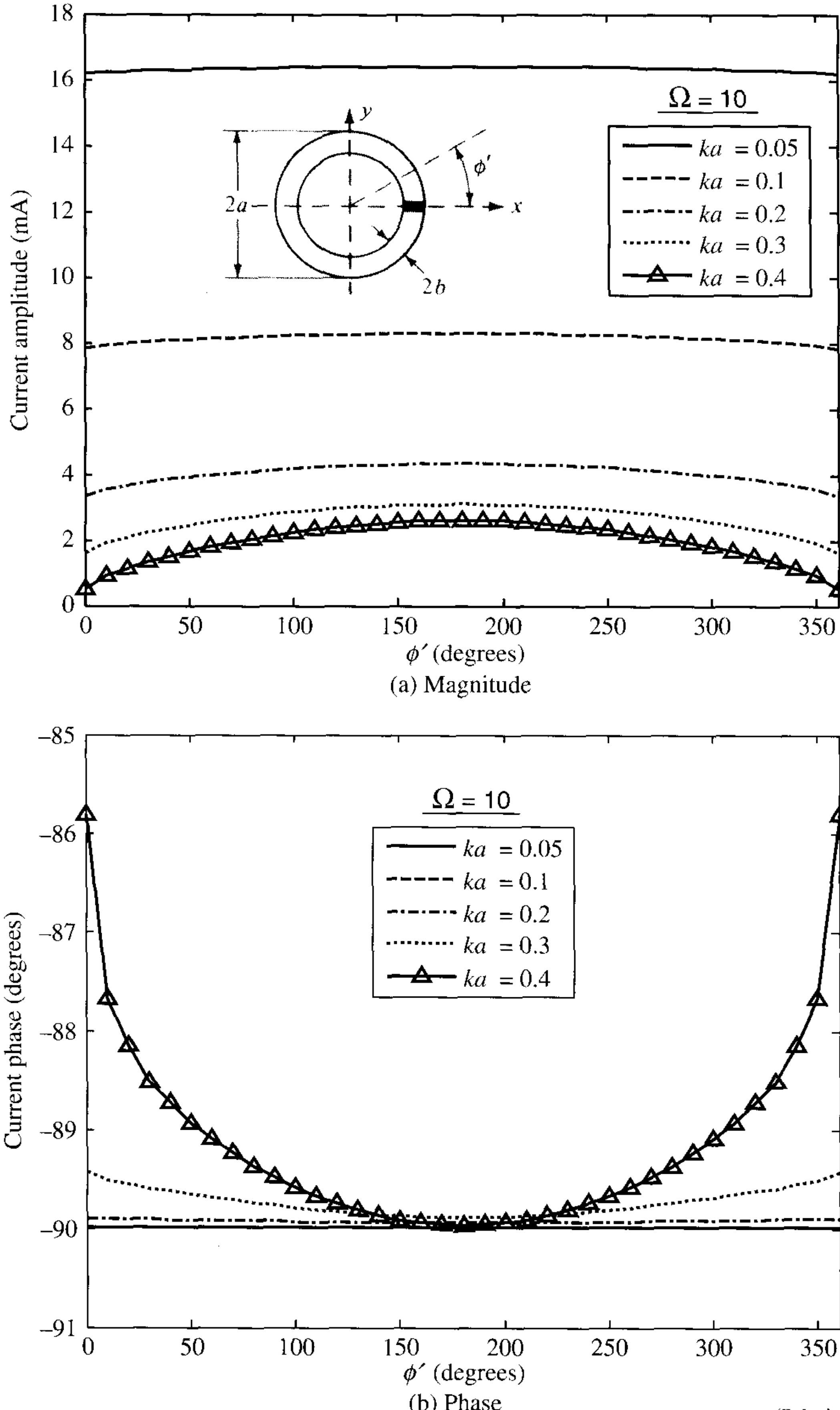
b ← wire radius

Ω big ⇒ thin

Ω small ⇒ thick

ex. $C = \lambda$, $b = 0.01\lambda$ ⇒ $\Omega = 2\ln 100 = \underline{\underline{9.21}}$

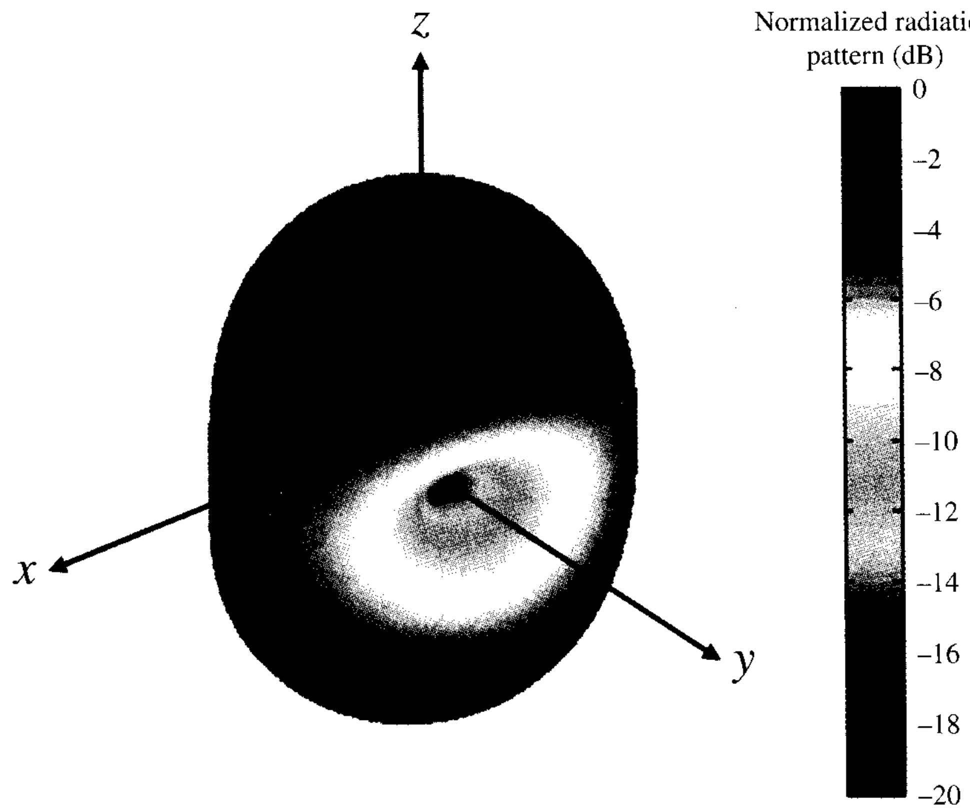
Note: Ω being <u>larger</u> implies the wire is <u>thinner</u>. Typically, both the loop wire radius b and circumference $C = 2\pi a$ are expressed in terms of wavelengths.



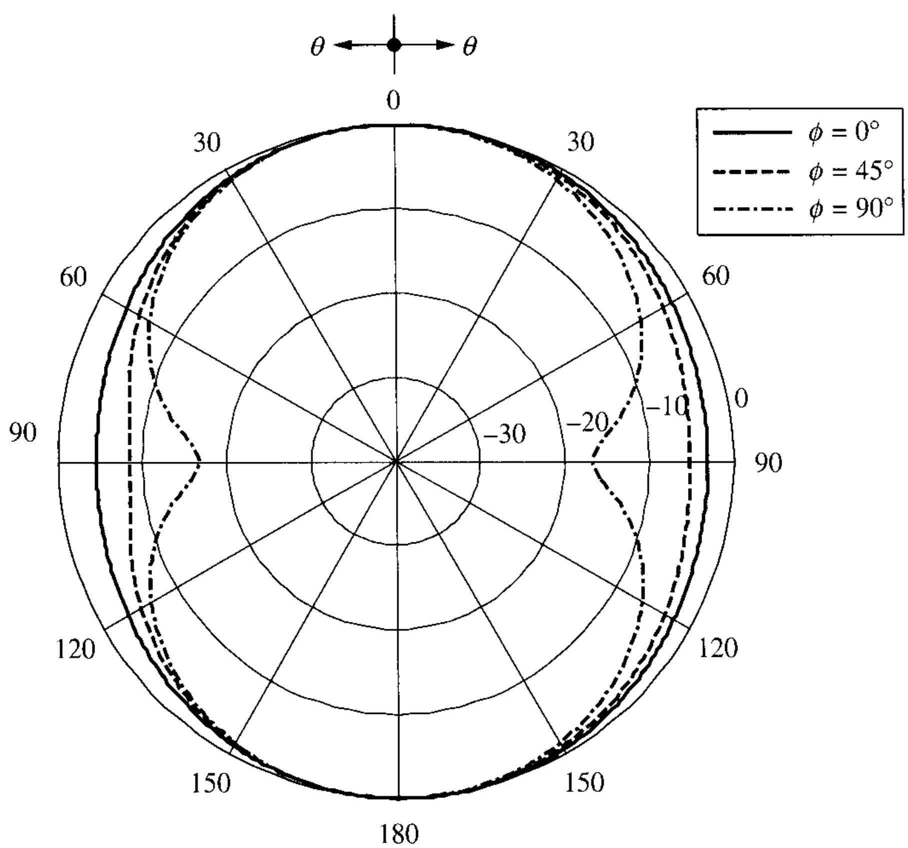
(Balanis, 4th Edn.)

Figure 5.11 Current magnitude and phase distributions on small circular loop antennas.

Note: Current progressively becomes more non-uniform as *ka* increases.



(a) 3-D

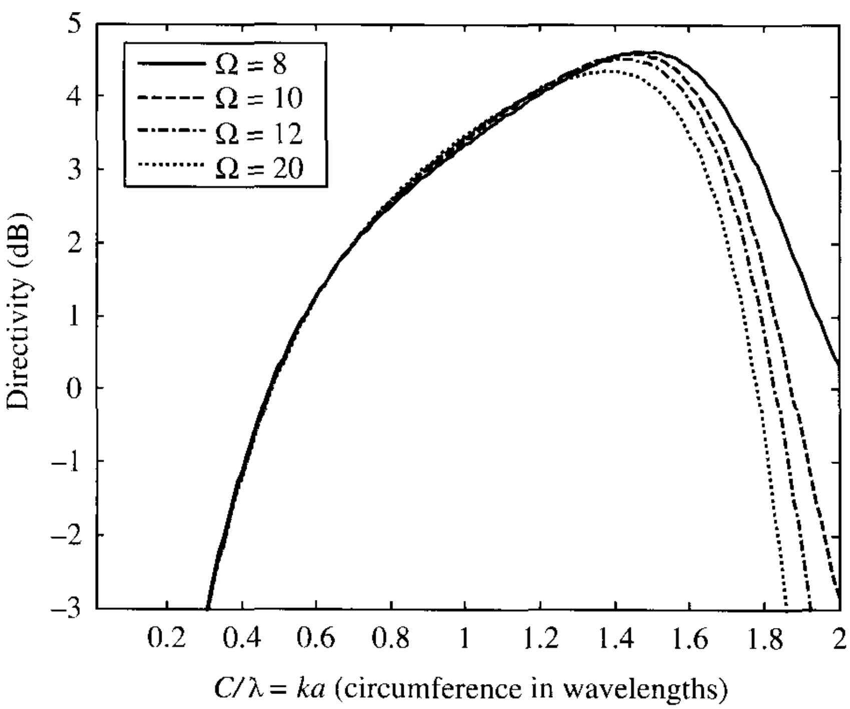


(b) 2-D (elevation)

Figure 5.12 Far-field normalized three- and two-dimensional amplitude patterns for a loop with $C = \lambda$ and $\Omega = 10$. **(Balanis, 4th Edn)**

5.4 cont.

- As shown in Figure 5.13, the directivity of an electrically-large loop is significantly larger than a comparable linear λ/2 dipole (D = 2.15 dBi).
- While the peak directivity occurs near C/λ ~1.4, this size loop is not used much due to impedance matching considerations.
- Near C/λ ~1, the directivity is D ~ 3.4 dBi. D varies with wire thickness described by $\Omega = 2\ln(2\pi a/b)$.



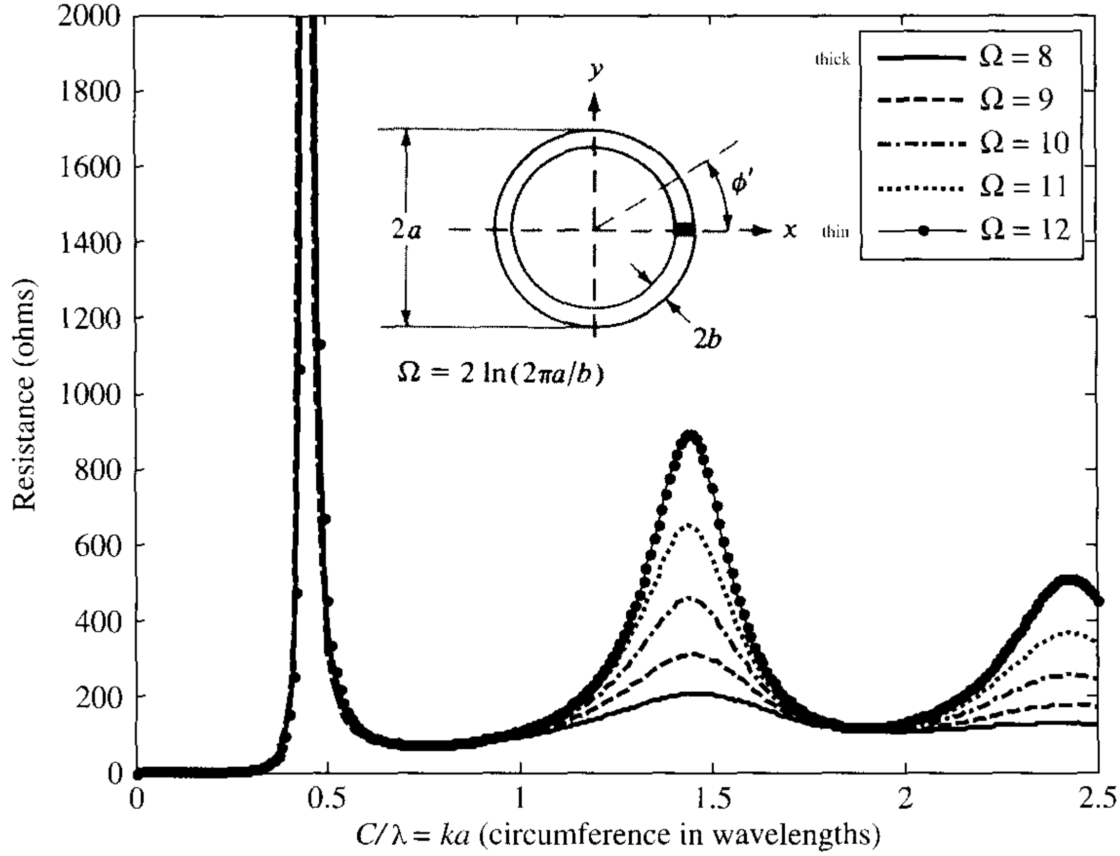
Ω	$C/\lambda = ka$	D (dB)
8	1.48	4.626
10	1.45	4.592
12	1.43	4.523
20	1.39	4.354

Ω	$C/\lambda = ka$	D (dB)
8	1	3.344
10	1	3.412
12	1	3.442
20	1	3.476

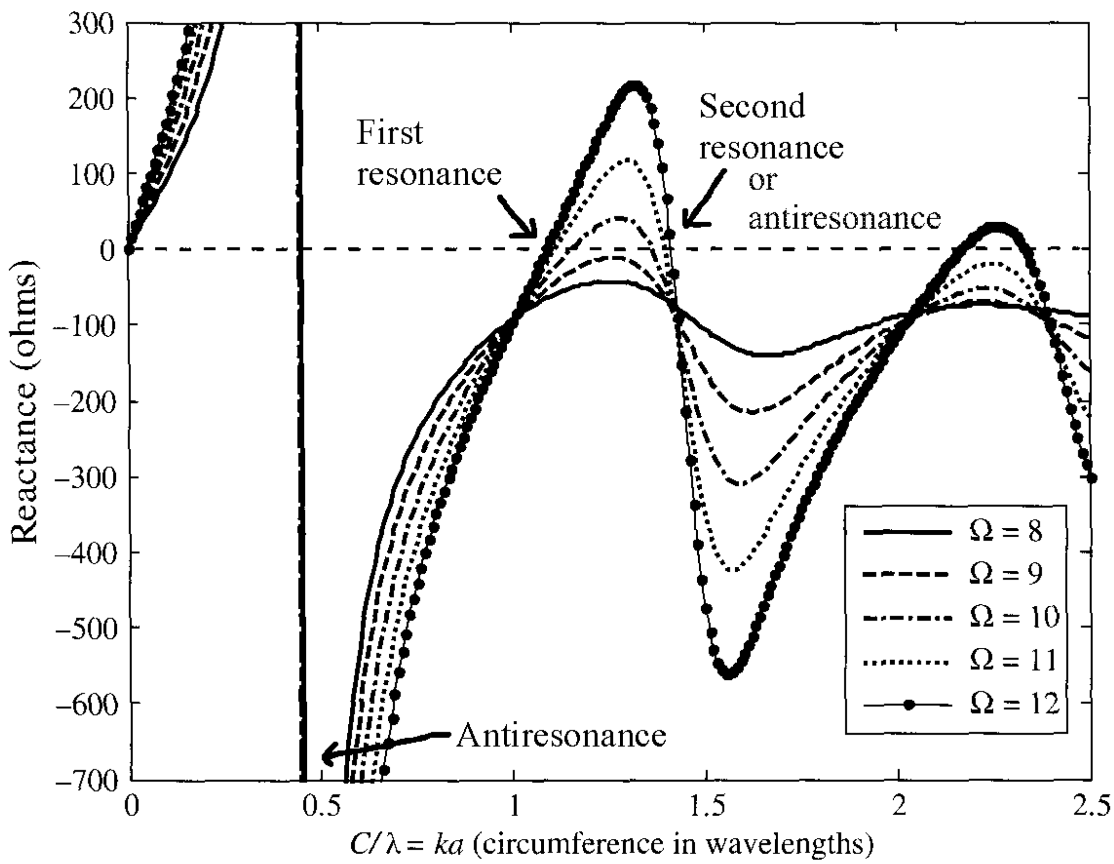
Figure 5.13 Directivity of circular-loop antenna for $\theta = 0, \pi$ versus electrical size (C/λ).
[Balanis, 4th Edn]

Impedance

- As shown in Figure 5.15, the reactance of an electrically-small loop is very <u>inductive</u> with a very small resistance (i.e., hard to match, not efficient).
- There is an anti-resonance near C/λ ~ 0.5 where the resistance gets very large while the reactance crosses zero going from inductive to capacitive.
- Near C/λ ~1, the reactance of a loop goes from capacitive toward resonance (i.e., X_a = 0) when C/λ is a bit bigger than 1. The resistance is ~100 Ω. Loops with thicker wires (i.e, Ω is smaller) may not reach true resonance.



(a) Resistance



(b) Reactance

Figure 5.15 Input impedance of circular-loop antennas. **[Balanis, 4th Edn.]**