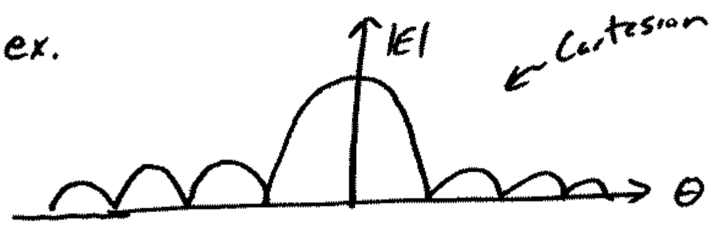
Chapter 2 Fundamental Parameters of Antennas

2.1 Introduction

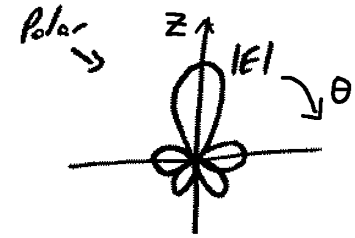
- Definitions of various parameters are needed to describe performance of antennas.
- Many based on *IEEE Standard Definitions of Terms for Antennas* 145-1993, reaffirmed 2004, and a revision of 145-1983.

2.2 Radiation Pattern - spatial distribution of a quantity that characterizes the EM field generated by an antenna (e.g. power, $|E|$) aka antenna pattern.

ex.



or



2.2.1 Radiation Pattern Lobes - part of radiation pattern bounded by areas of relatively weak radiation intensity. Some classifications are the main/major, minor, side, + back lobes.

ex.

major/main lobe (main beam) - radiation lobe w/ or containing direction of maximum radiation

minor lobe - any lobe but major/main lobe

side lobe - lobe in direction other than intended

back lobe - lobe whose axis is ~180° from main lobe

→ usually want to minimize minor lobe(s) + express strength relative to the main lobe (e.g. want minor lobes -20dB or less than main lobe)

2.2.2 Isotropic radiator - hypothetical, lossless antenna having equal radiation in all directions (spherical radiation pattern). It's used as a reference for actual antennas w/ directive properties

Directional antenna(s) - antenna(s) having the property of radiating or receiving EM waves more effectively in some directions than others. Usually applied to antennas whose max. directivity is greater than a $\lambda/2$ dipole

Omnidirectional antenna(s) - antenna having an essentially non-directional pattern in a given plane of the antenna and a directional pattern in any orthogonal plane. ex. dipole ($\lambda/2$)

2.2.3 Principal Patterns - applies to linearly polarized antennas and is usually described in terms of the E-plane + H-plane patterns.

E-plane - plane (of radiation pattern) containing the electric field vector + direction of maximum radiation.

H-plane - same only w/ magnetic field.

ex. horn, dipole

Plotting antenna radiation patterns:

polar.m from MATLAB:

>> help polar

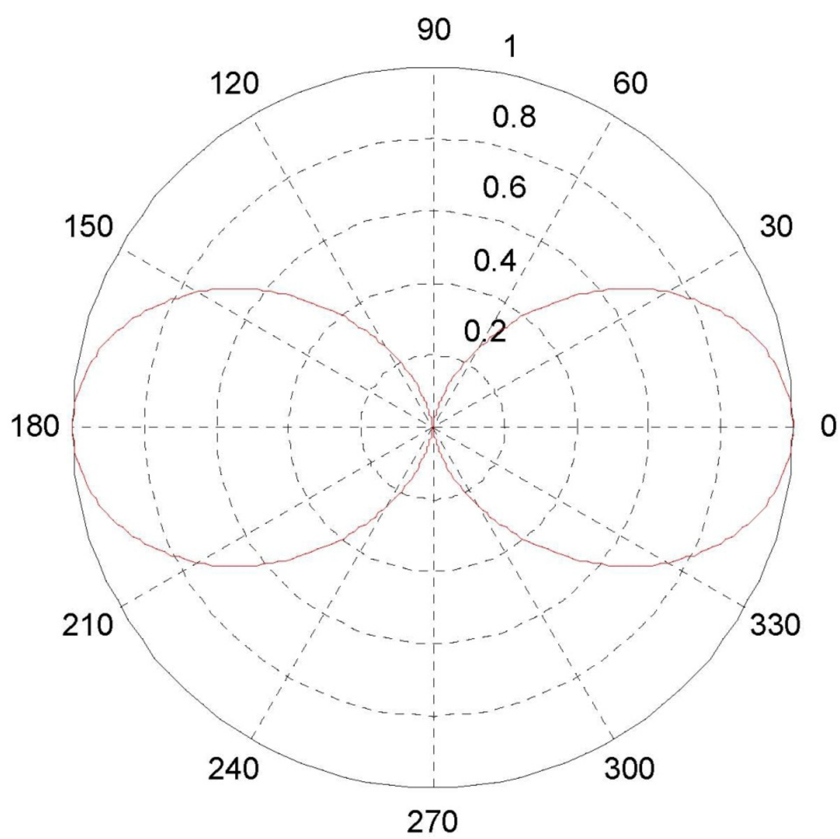
POLAR Polar coordinate plot.
POLAR(THETA, RHO) makes a plot using polar coordinates of
the angle THETA, in radians, versus the radius RHO.
POLAR(THETA,RHO,S) uses the linestyle specified in string S.
See PLOT for a description of legal linestyles.

See also PLOT, LOGLOG, SEMILOGX, SEMILOGY.

Example:

(From MATLAB Command Window)

```
>> ang1 = 0:1:359;                                    % angles in degrees
>> rho1 = cos(ang1*pi/180).*cos(ang1*pi/180);  % radial values
>> polar(ang1*pi/180,rho1,'r-')                     % plot (converted angles to radians)
```



Notes: These plots are strictly linear and radial values must be positive.

radpat.m found on course webpage:

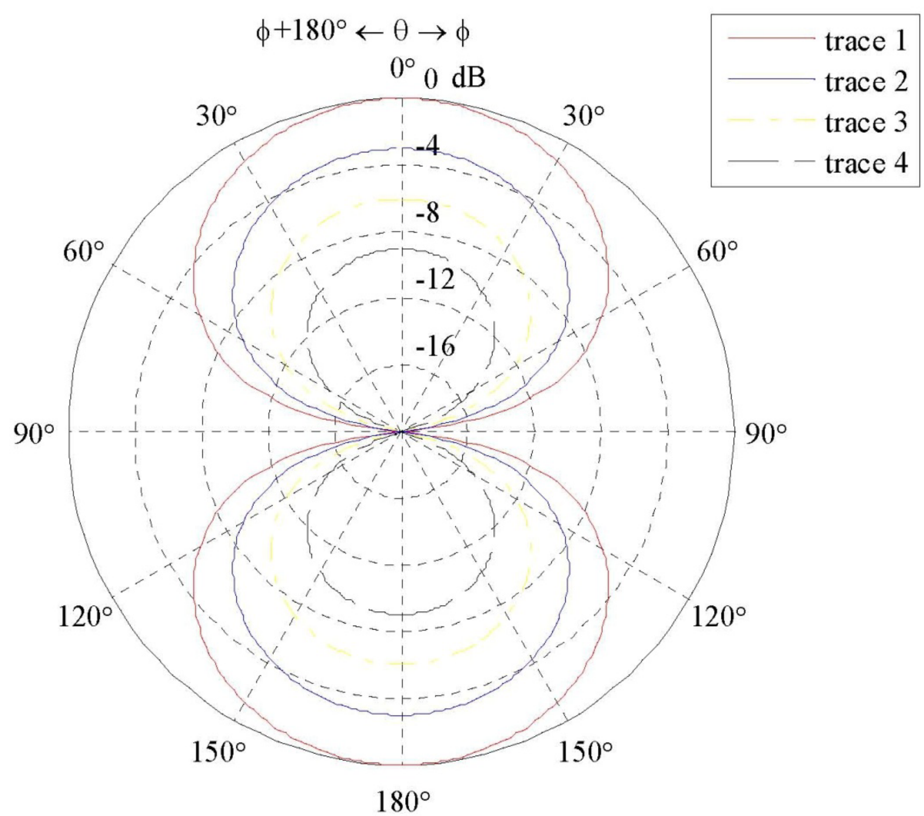
```
function radpat(ang1,R1,st1,ang2,R2,st2,ang3,R3,st3,ang4,R4,st4)
%RADPAT  Polar coordinate plot used for antenna radiation patterns.
%        RADPAT(ANG1,R1,ST1,ANG2,R2,ST2,ANG3,R3,ST3,ANG4,R4,ST4)
%          plots up to four curves in dB format.
%
%        ANGi are angles in degrees,
%        Ri are radiation pattern values (radii for plot traces), &
%        STi are the linestyles.
%        See PLOT for a description of legal linestyles.
%
%        Ri can be in dB or not in dB (resulting plot is in dB).
%        Axis labels can be placed on horizontal or vertical axis.
%        Choice of normalized or unnormalized (show gains) patterns.
%        Minimum dB level at plot center can be specified.
%        Maximum dB level at outermost plot circle can be specified for unnormalized patterns.
%        Line width of radiation patterns can be specified.
%        Legend can be placed. To move the legend, press left mouse button on the legend and
%          drag to the desired location.
%        Grid linetype can be specified.
%        Default values are inside [], press Enter to chose default.
%        0 degrees can be at North/Top or East/Right side of plot.
%
%        Example:  radpat(a1,r1,'r-',a2,r2,'y--')
%
%        Based on polarpat.m by D. Liu, 9/13/1996.
%        T.J. Watson Research center, IBM
%        P.O.Box 218
%        Yorktown Heights, NY 10598
%        Email: dliu@watson.ibm.com
%
%        Updated by Thomas P. Montoya, SDSM&T, 1/23/2006
%        * allow up to four traces
%        * added degree symbols to plot spoke labels
%        * for plots vs. theta keep spoke labels in 0 to +180 deg
%          range and indicate that negative theta angles are for
%          phi+180 deg and
%        * orient plot so that 0 degrees at the top (North)
```

Note: The resulting radiation pattern plot is in dB regardless of whether the input variable(s) (e.g., rho1) is originally in dB or not.

Example:

(From MATLAB Command Window)

```
>> ang = 0:1:359;                                   % Define angles in degrees
>> rho1 = cos(ang1*pi/180).*cos(ang1*pi/180);   % Define radiation patterns
>> rho2 = 0.5*rho1;
>> rho3 = 0.5*rho2;
>> rho4 = 0.5*rho3;
>> radpat(ang,rho1,'r-',ang,rho2,'b-',ang,rho3,'y-.',ang,rho4,'k--')
Are input values in dB (Y/N)[Y]? n
Normalize to the Maximum Gain Value (Y/N)[Y]? y
Minimum dB value at plot center [-40]? -20
Are the angles theta values? (Y/N)[Y]? y
Labels on Vertical or Horizontal axis (V/H)[V]? v
Pattern line width [1.25]:
Legend for traces on graph (Y/N)[N]? y
Enter label for trace 1: trace 1
Enter label for trace 2: trace 2
Enter label for trace 3: trace 3
Enter label for trace 4: trace 4
Put a box around the legend (Y/N)[Y]?
Line type of grid(-, --, -., :)[:]?
```



Notes: You may need to move labels around on the MatLab figure window using the mouse (click arrow icon, then left click and drag with mouse).

polarpat.m found on internet & course webpage:

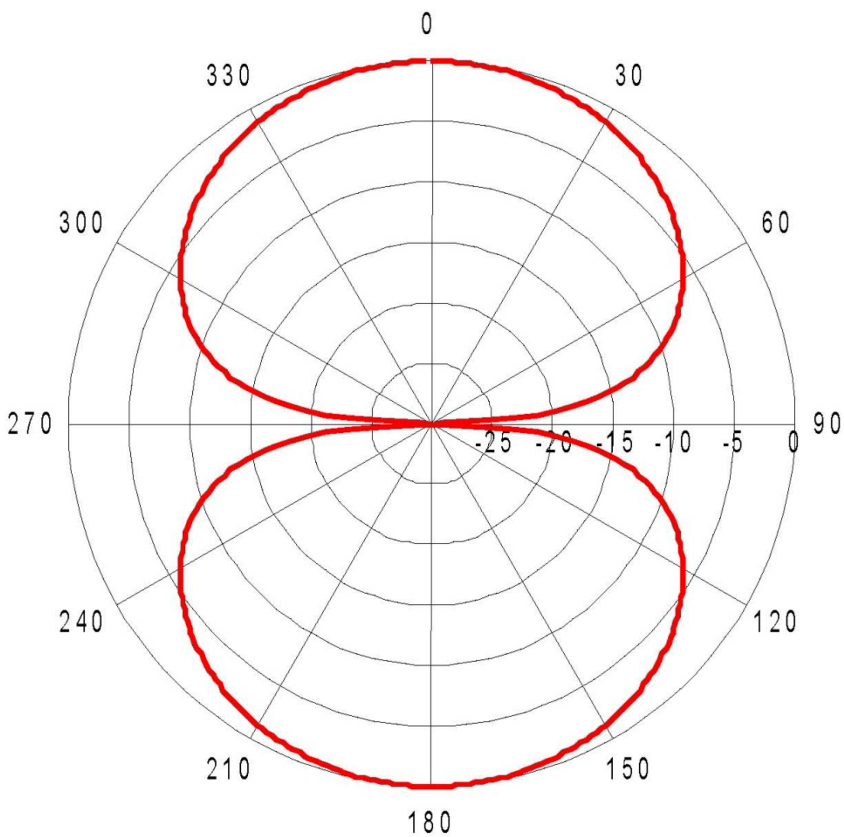
```
function polarpat(ang1,rho1,st1,ang2,rho2,st2,ang3,rho3,st3)
%POLARPAT  Polar coordinate plot used for antenna radiation patterns.
%       POLARPAT(ANG1,RHO1,ST1,ANG2,RHO2,ST2,ANG3,RHO3,ST3) plots up to
%       three curves. ANGi is angles in degress, RHOi is radius, and
%       STi is linestyle.
%       RHOi can be in dB or not in dB.
%       Axis labels can be placed horzontally or vertically.
%       Choice of normalized or unnormalized (showing gains) patterns.
%       Minimum level at the polar center can be specified.
%       Maximum level at the polar outmost circle can be specified for
%         unnormalized patterns.
%       Line width of radiation patternns can be specified.
%       Legend can be placed. To move the legend, press the left mouse
%         button on the legend and drag to the desired location.
%       Grid linetypes can be specified.
%       Default value is inside [], press Enter to chose default.
%       See PLOT for a description of legal linestyles.
%       0 degree can be in the East or North direction.
%       Example:  polarpat(a1,r1,'r-',a2,r2,'y--')
%       Written by Duixian Liu, on September 13, 1996.
%       T.J. Watson Research center
%       IBM
%       P.O.Box 218
%       Yorktown Heights, NY 10598
%       Email: dliu@watson.ibm.com
…
```

Note: The resulting radiation pattern plot is in dB regardless of whether the input variable(s) (e.g., rho1) is originally in dB or not.

Example:

(From MATLAB Command Window)

```
>> ang1 = 0:1:359;          % define angles in degrees
>> rho1 = cos(ang1*pi/180).*cos(ang1*pi/180);
>> polarpat(ang1,rho1,'r-')
Are input values in dB (Y/N)[Y]? N
Normalize to the Maximum Gain Value (Y/N)[Y]? Y
The minimum dB value at polar center [-50]? -30
Put axis label Vertically or Horizontally (V/H)[H]?
Pattern line width [1.0]: 1
Is 0 degree in the North or East (N/E)[E]? N
Line type of grid(-, --, -., :)[-]? -
>>
```



Notes: You may need to move labels around on the MatLab figure window using the mouse (click arrow icon, then left click and drag with mouse)

2.2.4 Field Regions

– space surrounding an antenna is usually divided into 3 regions: a) reactive near-field, b) radiating near-field (Fresnel), and c) far-field (Fraunhofer).

→ boundaries between regions somewhat arbitrary (can be determined by phase error, size of reactive components vs radiating components of fields)

Reactive near-field – part of near-field where the reactive field components dominate

$(kr \lesssim 1)$

Rule of Thumb $R < 0.62\sqrt{D^3/\lambda}$ ← D is largest dimension of antenna

(valid for $D \gtrsim \lambda$)

Fresnel/radiating near-field – part of near-field where the radiating components of the fields dominate <u>and</u> the angular field distribution is dependent on distance from antenna

$$0.62\sqrt{D^3/\lambda} < R < 2D^2/\lambda \quad \left(\pi/8 \text{ phase error}\right)$$

Far-field/Fraunhofer – region of space where field distribution wrt angle unchanged by distance.

$$R > 2D^2/\lambda$$

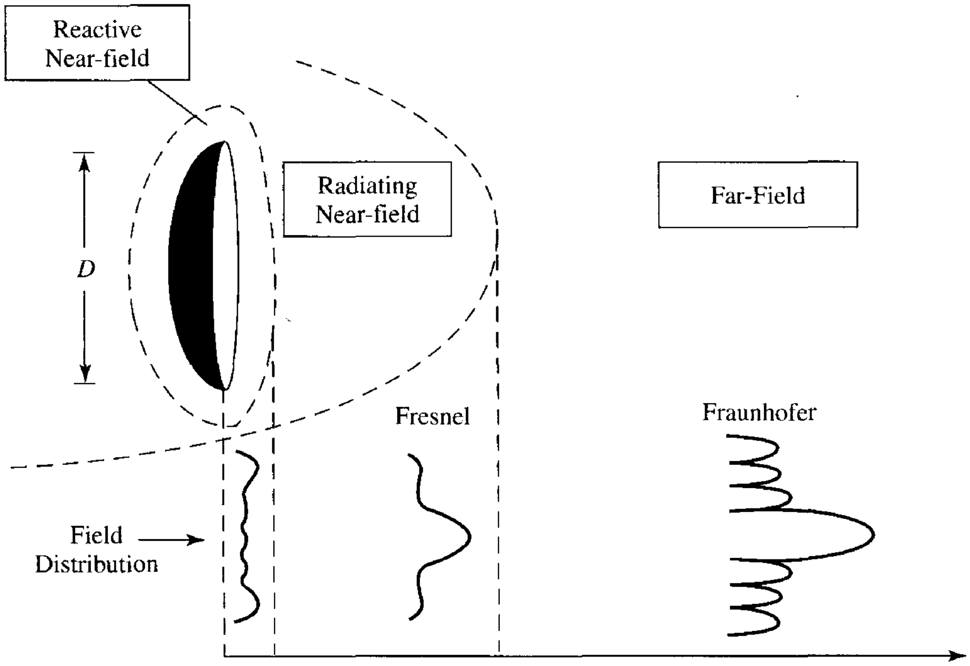


Figure 2.8 Typical changes of antenna amplitude pattern shape from reactive near field toward the far field. (SOURCE: Y. Rahmat-Samii, L. I. Williams, and R. G. Yoccarino, The UCLA Bi-polar Planar-Near-Field Antenna Measurement and Diagnostics Range," *IEEE Antennas & Propagation Magazine*, Vol. 37, No. 6, December 1995 © 1995 IEEE).

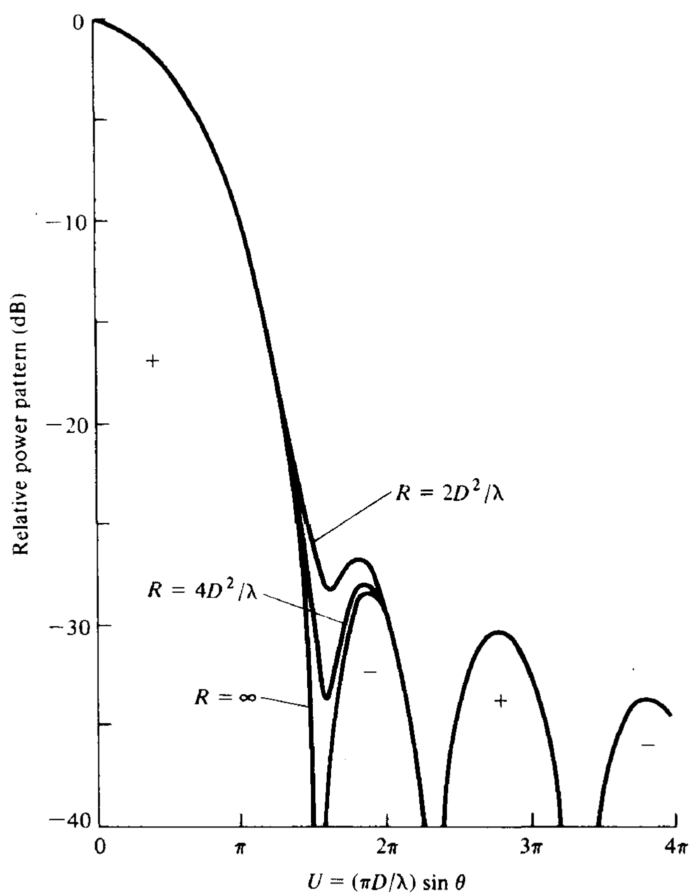
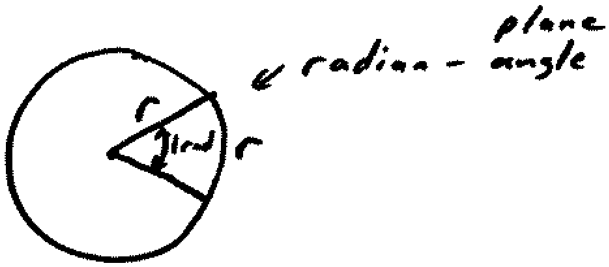
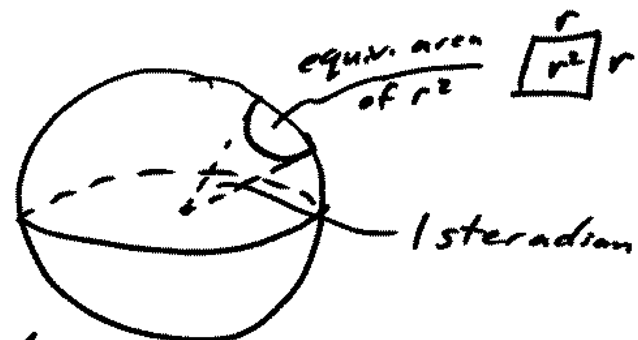


Figure 2.9 Calculated radiation patterns of a paraboloid antenna for different distances from the antenna. (SOURCE: J. S. Hollis, T. J. Lyon, and L. Clayton, Jr. (eds.), *Microwave Antenna Measurements*, Scientific-Atlanta, Inc., July 1970).

2.2.5 Radian + Steradian –



steradian (sr) – solid angle



a closed sphere includes 4π steradians $\left(4\pi r^2/r^2 = 4\pi\right)$

2.3 Radiation Power Density

Instantaneous Poynting Vector $\quad \bar{\mathscr{W}} = \bar{\mathscr{E}} \times \bar{\mathscr{H}} \quad (W/m^2)$

Instantaneous Power $\quad \mathscr{P} = \oiint_S \bar{\mathscr{W}} \cdot d\bar{s} = \oiint_S \bar{\mathscr{W}} \cdot \hat{n}\, da$

$\hat{n}$ unit vector normal to surface

For time-harmonic signals (e.g., sinusoids), we can calculate the average Poynting vector + power as

$$\bar{W}_{ave} = \left[\bar{\mathscr{W}}\right]_{ave} = \tfrac{1}{2}\,\mathrm{Re}\left[\bar{E} \times \bar{H}^*\right] = \bar{W}_{rad}$$

where $\bar{\mathscr{E}} = \mathrm{RE}\left[\bar{E}\, e^{j\omega t}\right]$

$$P_{rad} = P_{ave} = \oiint_S \bar{W}_{rad} \cdot d\bar{s} \quad (W)$$

Side discussion on Poynting Vector under certain conditions (see section 10.10 of EE 381/382 text "Elements of Electromagnetics (6th Edn)" by Sadiku)

→ $\bar{\mathcal{E}}$ & $\bar{\mathcal{H}}$ are time-harmonic <u>and</u> orthogonal

→ lossless medium (μ, ϵ) where intrinsic impedance $\equiv \eta = \sqrt{\mu/\epsilon}$
& <u>source-free</u>

$$\eta_0 = 376.7303\ \Omega$$

→ propagation vector $\equiv \bar{k} = \omega\sqrt{\mu\epsilon}\ \hat{a}_k = \frac{\omega}{u}\hat{a}_k$ ← direction of wave propagation

→ $\bar{\mathcal{E}} = \mathrm{Re}\{\bar{E}e^{j\omega t}\}$ and $\bar{\mathcal{H}} = \mathrm{Re}\{\bar{H}e^{j\omega t}\}$

Using Maxwell's equations, it can be shown:

$$\boxed{\bar{H} = \frac{\hat{a}_k \times \bar{E}}{\eta}} \quad \text{and} \quad \boxed{\bar{E} = \eta(\bar{H} \times \hat{a}_k)}$$

Also, using $\bar{W}_{ave} = \frac{1}{2}\mathrm{Re}\{\bar{E} \times \bar{H}^*\}$, the above info, and vector identities, it can be shown:

$$\boxed{\bar{W}_{ave} = \hat{a}_k \frac{|\bar{E}|^2}{2\eta}}$$ ← Most commonly used

OR

$$\boxed{\bar{W}_{ave} = \hat{a}_k \frac{\eta|\bar{H}|^2}{2}}$$

2.3 cont.

For an isotropic radiator ← No θ or ϕ

$$\bar{W}_{rad} = \bar{W}_0 = \hat{a}_r W_0 = \hat{a}_r \left(\frac{P_{rad}}{4\pi r^2} \right) \quad (W/m^2)$$

$$P_{rad} = \oiint_S \bar{W}_0 \cdot d\bar{s} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \hat{a}_r W_0 \cdot \hat{a}_r r^2 \sin\theta \, d\theta \, d\phi$$

$$= 4\pi r^2 W_0 \quad (W)$$

2.4 Radiation Intensity

Radiation Intensity – power radiated from an antenna per unit solid angle in a given direction

$$U(\theta,\phi) = r^2 W_{rad} \quad \left(\frac{W}{\text{solid} \measuredangle} \text{ or } \frac{W}{Sr} \right)$$

far-field

$$= \frac{r^2}{2\eta} |\bar{E}(r,\theta,\phi)|^2 \simeq \frac{r^2}{2\eta} \left[|E_\theta|^2 + |E_\phi|^2 \right]$$

$\eta \equiv$ intrinsic impedance of material

isotropic radiator $U_0 = \frac{P_{rad}}{4\pi}$

To find the magnitude squared of a complex vector: $|\bar{C}|^2 = \bar{C} \cdot \bar{C}^*$ ← complex conjugate

$$P_{rad} = \oiint_\Omega U \, d\Omega = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U \sin\theta \, d\theta \, d\phi$$

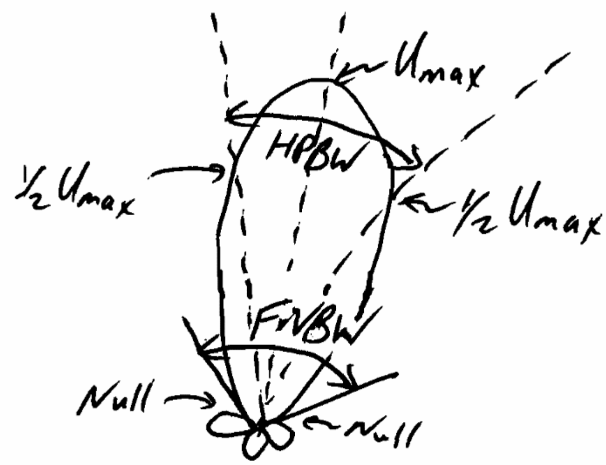
↖ diff. solid angle

2.5 Beamwidth

<u>Beamwidth</u> is an antenna parameter defined as the angular separation between two like points on opposite sides of the maximum of the radiation pattern.

Most common are the <u>Half-power Beamwidth (HPBW)</u> and the <u>First-Null Beamwidth (FNBW)</u>, but can have -10dB BW ...

IEEE def'n of HPBW "In a plane containing the direction of the maximum of a beam, the angle between the two directions in which the radiation intensity is one-half value of the beam."



Can use to describe resolution capabilities of antennas used for RADAR targets or to distinguish adjacent sources. $\sim \frac{FNBW}{2} \sim HPBW$

2.6 Directivity

Directivity – the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} \quad \text{(dimensionless)}$$

$$D_{max} = D_0 = \frac{U|_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}}$$

→ isotropic source $D = D_{max} = 1$

For antennas w/ orthogonal polarization components, define <u>partial directivity</u> as "that part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity averaged over all directions."

ex. $D_0 = D_\theta + D_\phi$

where $$D_\theta = \frac{4\pi U_\theta}{(P_{rad})_\theta + (P_{rad})_\phi}$$

$$D_\phi = \frac{4\pi U_\phi}{(P_{rad})_\theta + (P_{rad})_\phi}$$

2.6 cont.

ex. For an infinitesimal dipole, the far-zone fields can be represented as:

$$\bar{E} = \hat{a}_\theta \, j E_0 \frac{e^{-jkr}}{r} \sin\theta$$

$$\bar{H} = \hat{a}_\phi \frac{jE_0}{\eta} \frac{e^{-jkr}}{r} \sin\theta \qquad \eta = \sqrt{\mu/\epsilon}$$

$$\bar{W}_{ave} = \bar{W}_{rad} = \tfrac{1}{2} \operatorname{Re}\{\bar{E} \times \bar{H}^*\}$$

$$= \tfrac{1}{2} \operatorname{Re}\left\{\hat{a}_\theta \, j E_0 \frac{e^{-jkr}}{r} \sin\theta \times \hat{a}_\phi \frac{-jE_0}{\eta} \frac{e^{+jkr}}{r} \sin\theta\right\}$$

$$\underline{\bar{W}_{rad} = \tfrac{1}{2} \operatorname{Re}\left\{\hat{a}_r \frac{E_0^2}{\eta} \frac{\sin^2\theta}{r^2}\right\} = \hat{a}_r \frac{E_0^2}{2\eta} \frac{\sin^2\theta}{r^2} \quad (W/m^2)}$$

$$P_{rad} = \oint\!\!\oint_S \bar{W}_{rad} \cdot \hat{n}\, da = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \hat{a}_r \frac{E_0^2}{2\eta} \frac{\sin^2\theta}{r^2} \cdot \hat{a}_r r^2 \sin\theta \, d\theta \, d\phi$$

S ← sphere of radius r

$$= \frac{E_0^2}{2\eta} \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta = \frac{E_0^2}{2\eta} (2\pi) \left(\frac{4}{3}\right)$$

$$\underline{P_{rad} = \frac{4\pi E_0^2}{3\eta} \quad (W)}$$

$$U(\theta) = r^2 W_{rad} = r^2 |\bar{W}_{rad}| = \frac{r^2}{2\eta} |\bar{E}_\theta|^2 = \underline{\frac{E_0^2}{2\eta} \sin^2\theta \quad (W/sr)}$$

$$D(\theta) = \frac{U(\theta)}{U_0} = \frac{4\pi U}{P_{rad}} = \frac{4\pi \frac{E_0^2}{2\eta} \sin^2\theta}{4\pi \, E_0^2 / 3\eta} = \underline{3/2 \sin^2\theta}$$

$$\underline{D_0 = D_{max} = 3/2 = 10 \log_{10}(1.5) = 1.76 \text{ dBi}} \quad (\theta = \pi/2)$$

2.6 cont.

Plot 2D + 3D directivity plots for isotropic and $\lambda/2$ dipole antennas (Fig. 2.13)

$$\left.\begin{array}{ll} \text{isotropic} & D_i = 1 \\ \lambda/2 \text{ dipole} & D_{\lambda/2} \simeq 1.67 \sin^3\theta \end{array}\right\} \begin{array}{l} \text{No } \phi \text{ dependence} \\ \text{(or } r \text{ dependance)} \end{array}$$

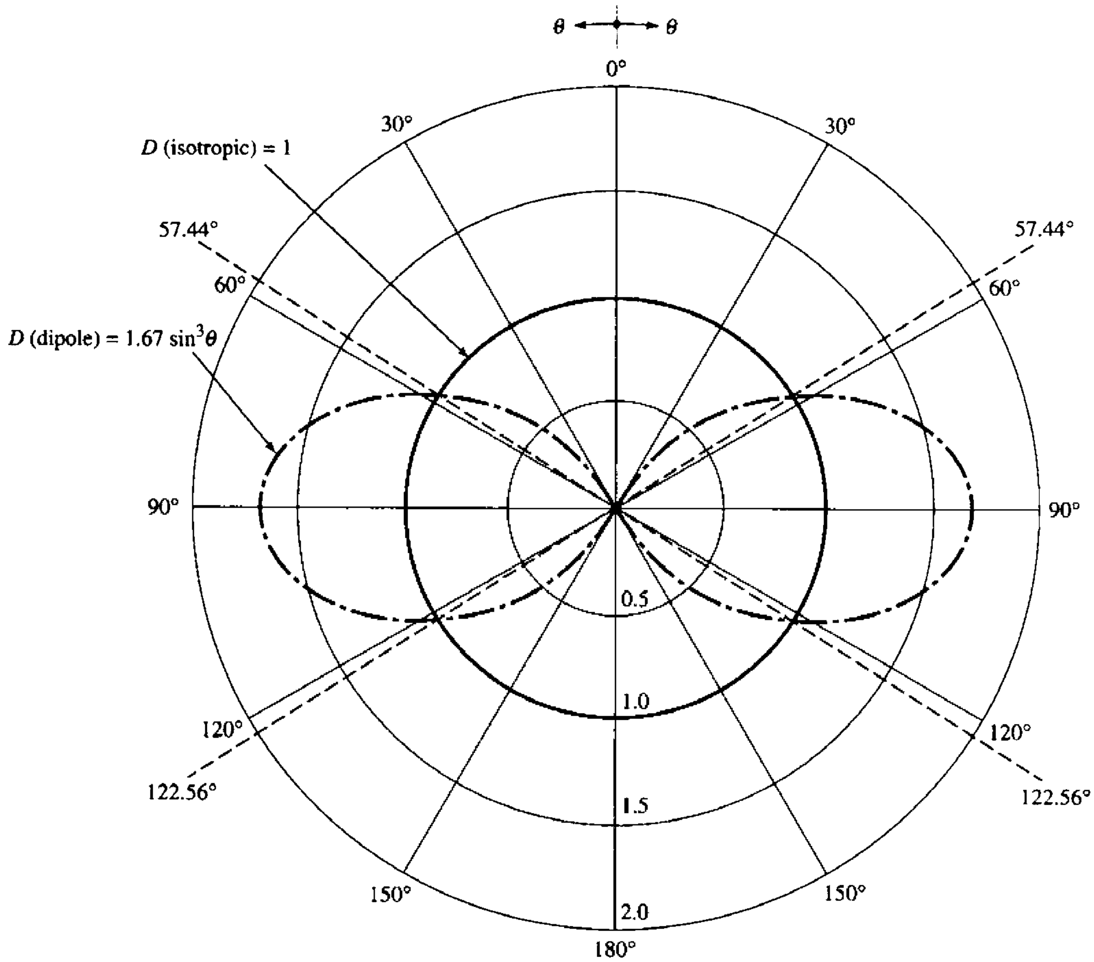
Note that $D_{\lambda/2} > D_i$ when $57.4° < \theta < 122.6°$

$$(D_{\lambda/2})_{max} = 1.67 \text{ @ } \theta = \pi/2$$

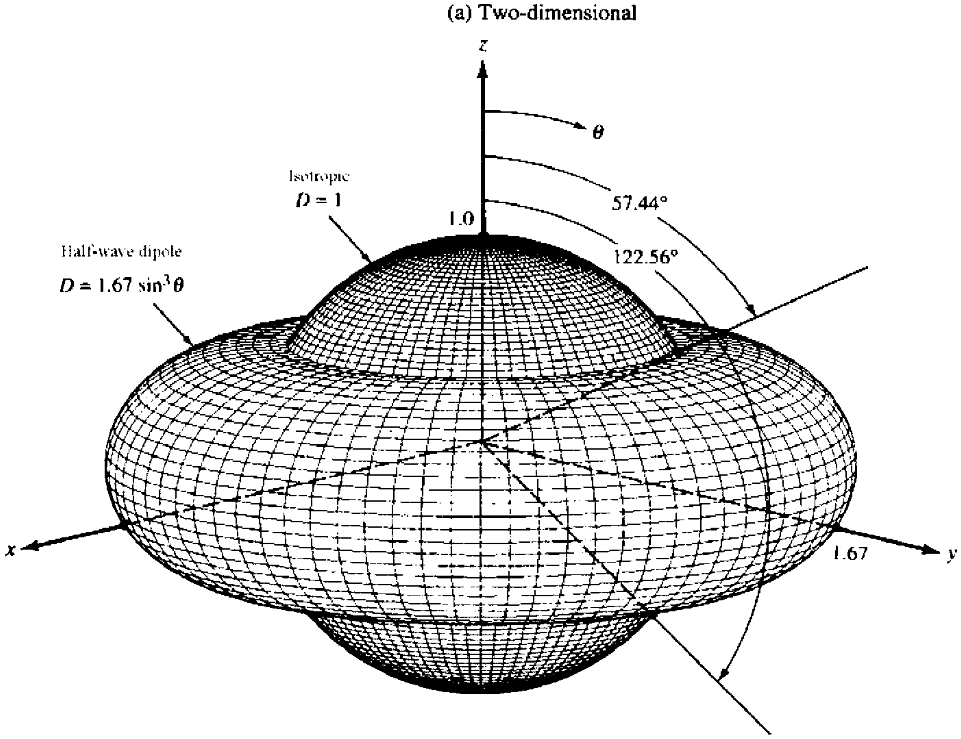
So, the $\lambda/2$ dipole is $\frac{1.67}{1} \cong 1.67 \cong 10\log 1.67 \cong 2.23\,dB_i$ more directive than an isotropic antenna

dB_i = decibels above isotropic antenna

Note: $(D_{\lambda/2})_{max} = 2.15\,dB_i$ when an exact expression is used



(a) Two-dimensional



(b) Three-dimensional

Figure 2.13 Two- and three-dimensional directivity patterns of a $\lambda/2$ dipole. (SOURCE: C. A. Balanis, "Antenna Theory: A Review." *Proc. IEEE,* Vol. 80, No. 1. January 1992. © 1992 IEEE.)

2.6 cont.

Let's now look at the more general case where

$$U = B_0 F(\theta,\phi) \approx \frac{1}{2\eta}\left[|E_\theta^{FF}(\theta,\phi)|^2 + |E_\phi^{FF}(\theta,\phi)|^2 \right]$$

B_0 ← constant

$$U_{max} = B_0 F(\theta,\phi)\big|_{max}$$

$$P_{rad} = \oint\!\!\oint_{\Omega} U(\theta,\phi)\, d\Omega = B_0 \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

Now, the directivity and max. directivity can be written as:

$$D(\theta,\phi) = 4\pi \frac{F(\theta,\phi)}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi}$$

$$D_{max} = D_0 = 4\pi \frac{F(\theta,\phi)\big|_{max}}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi}$$

$$= \frac{4\pi}{\left[\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi\right] \Big/ F(\theta,\phi)\big|_{max}}$$

$$= \frac{4\pi}{\Omega_A}$$

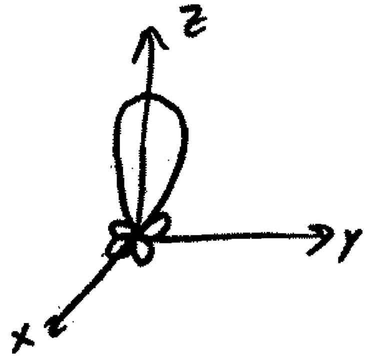
$\Omega_A \equiv$ Beam Solid Angle - solid angle thru which all the power of an antenna would flow if its $U = U_{max}$ for all ∡s in Ω_A.

2.6.1 Directional Patterns

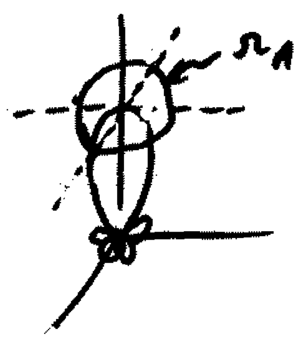
Approximations (useful for design purposes)

Suppose the antenna in question has one narrow major lobe + very small (negligible) minor lobes

ex.



then $\Omega_A \simeq \theta_{1r}\,\theta_{2r}$ ← in radians, where θ_{1r} + θ_{2r} are the half-power beamwidths in 2 ⊥ planes (i.e., E + H)



Kraus

then $D_{max} = D_0 = \dfrac{4\pi}{\Omega_A} \simeq \dfrac{4\pi}{\theta_{1r}\,\theta_{2r}}$ ← in radians

For θ_1 + θ_2 in degrees

$$D_{max} = D_0 = \frac{4\pi\,(180/\pi)^2}{\theta_{1d}\,\theta_{2d}} = \frac{41{,}253}{\theta_{1d}\,\theta_{2d}}$$

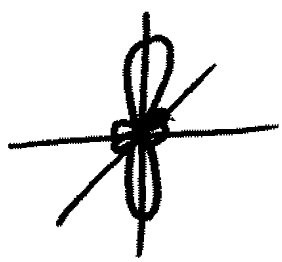
2.6.1 cont.

For planar arrays, a better approximation is

$$D_{max} = D_0 = \frac{32{,}400}{\Theta_{1d}\,\Theta_{2d}} \quad \text{(Scott)}$$

* We can use these approximations for an antenna w/ 2 identical major lobes by simply dividing by 2.

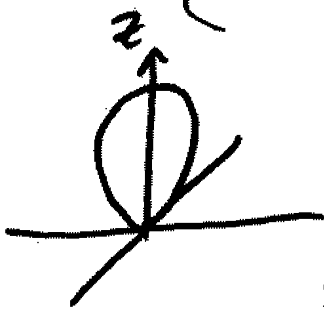
ex.



$$D_{max} = \frac{2\pi}{\Omega_A} = \frac{2\pi}{\Theta_{1r}\,\Theta_{2r}}$$

ex. For an antenna,

$$U = \begin{cases} B_0 \cos^2\theta & 0 \le \theta \le \pi/2,\ 0 \le \phi \le 2\pi \\ 0 & \text{elsewhere} \end{cases}$$



Θ_{1r} in x-z plane

Θ_{2r} in y-z plane

Since there is no ϕ dependence

$\Theta_{1r} = \Theta_{2r}$ $\quad \cos^2\theta = 0.5 \Rightarrow \theta = 45^\circ = \pi/4$

Hence $\Theta_{1r} = \Theta_{2r} = \pi/2$ $\quad D_0 = \dfrac{4\pi}{(\frac{\pi}{2})(\frac{\pi}{2})} = \underline{\underline{5.09}}$

2.6.1 cont.

ex. cont. Find exact D_0

$$U_{max} = B_0 \cos^2\theta\big|_{max} = B_0$$

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} B_0 \cos^2\theta \sin\theta \, d\theta \, d\phi$$

$$= B_0 (2\pi - 0)(1/3) = \frac{2\pi B_0}{3}$$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi B_0}{2\pi B_0/3} = \underline{\underline{6}}$$

* Often express quantities in decibels

$$D(dB) = 10\log_{10}[D] \quad \leftarrow \text{dimensionless}$$

$$D_{max}(dB) = 10\log_{10}[D]$$

$$D_0 = 6 = 10\log_{10} 6 = \underline{7.78\text{ dBi}}$$ relative to isotropic antenna

$$D_{0,approx} = 5.09 = 10\log_{10} 5.09 = \underline{7.07\text{ dBi}}$$

⇒ Not too good of a result, but remember the approx. is meant for <u>narrow</u> main lobes ($\Theta_{1d} = \Theta_{2d} = 90°$ is NOT narrow)

<u>Tai + Pereira</u> have also come up w/ an approx. formula for maximum directivity using:

$$\frac{1}{D_0} = \frac{1}{2}\left(\frac{1}{D_1} + \frac{1}{D_2}\right)$$

where $D_1 \simeq \dfrac{16\ln 2}{\Theta_{1r}^2}$ and $D_2 \simeq \dfrac{16\ln 2}{\Theta_{2r}^2}$

Again, Θ_{1r} + Θ_{2r} are the HPBWs in the E+H planes

2.6.1 cont.

So $\frac{1}{D_0} \simeq \frac{1}{32 \ln 2}\left(\theta_{1r}^2 + \theta_{2r}^2\right)$

or $\boxed{D_0 = \frac{32 \ln 2}{\theta_{1r}^2 + \theta_{2r}^2} = \frac{(32 \ln 2)\left(\frac{180}{\pi}\right)^2}{\theta_{1d}^2 + \theta_{2d}^2}}$ Tai & Pereira

ex. From previous example for

$$U = \begin{cases} B_0 \cos^2\theta & 0 \le \theta \le \pi/2 \quad 0 \le \phi \le \pi/2 \\ 0 & \text{elsewhere} \end{cases}$$

$\theta_{1r} = \theta_{2r} = \pi/2$ or $90°$

$$(D_0)_{approx} = \frac{32 \ln 2}{(\pi/2)^2 + (\pi/2)^2} = \underline{4.495}$$

Not too good

Kraus $(D_0)_{approx} = \underline{5.09}$

$(D_0)_{exact} = \underline{6}$

2.6.2 Omnidirectional Patterns

Let $U \simeq |\sin^n\theta| \quad 0 \le \theta \le \pi,\ 0 \le \phi < 2\pi$

$$D_0 \simeq \frac{101}{HPBW(deg) - 0.0027(HPBW(deg))^2}$$ McDonald

or

$$D_0 \simeq -172.4 + 191\sqrt{0.818 + 1/HPBW(deg)}$$ Pozar

2.7 Numerical Techniques

→ read thru this section

→ won't cover in class since many popular math packages exist (MathCad, Mathematica, Matlab, ...) to do numerical integrations.

2.8 Antenna Efficiency

Total antenna efficiency, e_o, takes into account:

- reflections due to mismatch between t-line and antenna
- Ohmic losses in dielectric & conductors

$$e_o = e_r e_c e_d = e_r e_{cd} = e_{cd}(1-|\Gamma|^2)$$

e_r = reflection (mismatch) efficiency $= 1-|\Gamma|^2$

e_c = conduction effiency
e_d = dielectric efficiency
} usually measured as a combined quantity

Γ = reflection coeff. $= \dfrac{Z_{in,ant} - Z_o}{Z_{in,ant} + Z_o}$ ← function of frequency

2.9 Gain

→ Takes into account the efficiency of an antenna (unlike directivity)

Absolute Gain – the ratio of the radiation intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically.

$$G = \text{gain} = 4\pi \frac{U(\theta,\phi)}{P_{in}}, \quad P_{in} \geq P_{rad} \text{ in general}$$

2.9 cont.

Notes – Does NOT include losses due to impendance and polarization mismatches

– If antenna is lossless $G = D$

– If a direction is not given, assume the gain is in the direction of maximum radiation

P_{rad} = ant. rad. eff. × total input power $\Rightarrow e_{cd} = \frac{P_{rad}}{P_{in}}$
$= e_{cd} P_{in}$

$$G(\theta,\phi) = e_{cd}\left[4\pi \frac{U(\theta,\phi)}{P_{rad}}\right] = e_{cd} D(\theta,\phi)$$

$$G_{max} = G_o = G(\theta,\phi)\big|_{max} = e_{cd} D_o$$

Realized Gain – The gain of an antenna reduced by the losses due to the mismatch of the antenna input impedance to a specified impedance (e.g. t-line)

$$G_{re} = e_r e_{cd} D = (1 - |\Gamma|^2) G$$

Relative Gain – The ratio of the gain of an antenna in a given direction to the gain of a reference antenna (e.g. dipole, horn)

ex. $10\,dBd = 10 + 2.15 = 12.15\,dBi$ since $D_{\lambda/2} = 2.15\,dBi$

2.9 cont.

Partial Gain - In a given direction, that part of the radiation intensity corresponding to a given polarization divided by the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically

ex. $G_\theta = \dfrac{4\pi U_\theta}{P_{in}}$ $\quad G_\phi = \dfrac{4\pi U_\phi}{P_{in}}$

$$G = G_\theta + G_\phi$$

approximation $G_0 = G_{max} \approx \dfrac{30{,}000}{\Theta_{1d}\,\Theta_{2d}}$ ← slightly smaller than directivity approximation

decibels $G(dB) = 10\log_{10}[G] = 10\log_{10}[e_{cd} D]$

2.10 Beam Efficiency

$$BE = \frac{\text{Power transmitted (received) w/in cone angle } \Theta_1}{\text{Power transmitted (received) by antenna}}$$

→ assume major lobe aligned w/ z-axis

$$BE = \frac{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Theta_1} U(\theta,\phi)\sin\theta\, d\theta\, d\phi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi}$$

→ Θ_1 could be angle to first null or minimum

BE in 90%s considered very good

2.11 Bandwidth

Bandwidth – The range of frequencies w/in which the performance of the antenna, wrt some characteristic, conforms to a specified standard

e.x. input impedance, pattern, beamwidth, gain ...
(VSWR)

Narrowband antennas – bandwidth expressed as fraction of center freq.

$$\frac{f_{up} - f_{low}}{f_c} \times 100\%$$

Broadband antennas – bandwidth given as ratio of upper frequency to lower frequency (e.g. 10:1 means $f_{up} = 10 f_{low}$)

→ can have a pattern bandwidth that is different than the impedance bandwidth

ex. Yagi-Uda → Narrowband BW ≈ 2%

ex. LPDA → Broadband BW up to 30:1 possible

↳ limiting factor is practicality, can't make smallest elements littler than T.L. dimensions @ f_{high} & eventually T.L. losses / size restrict f_{low}

2.12 Polarization

Polarization (of an antenna)

In a given direction from the antenna, the polarization of the EM wave transmitted by the antenna determines its polarization. Unless otherwise specified, assume direction is that of maximum directivity/gain.

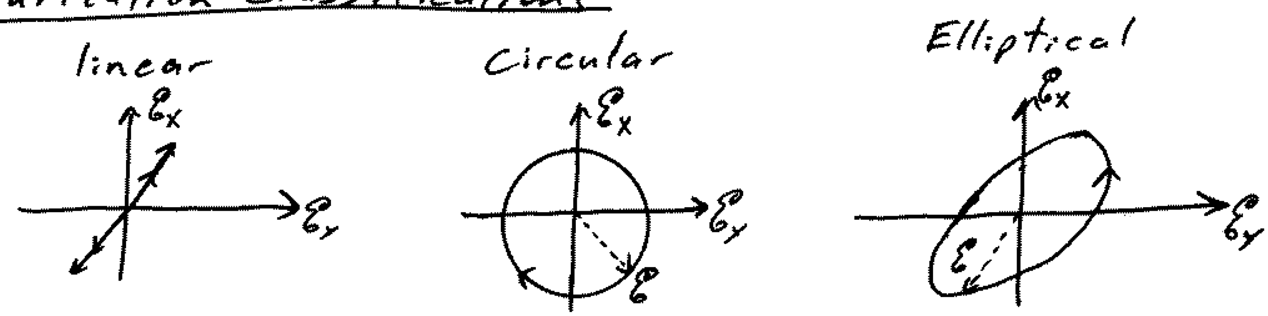
Polarization (of radiated wave)

In a given direction from an antenna and at a point in the far-field, the polarization of the plane wave (at least locally) that represents the radiated EM wave at that point. Type found by following tip of the electric field vector at a fixed point in space wrt time and the sense in which it is traced as oberved in the direction of propagation. The trace/curve traced out is called the polarization ellipse.

Polarization (received by antenna)

Polarization of a plane wave, incident on the antenna from a given direction w/ a given power density, which results in the maximum power available at the antenna terminals.

Polarization Classifications

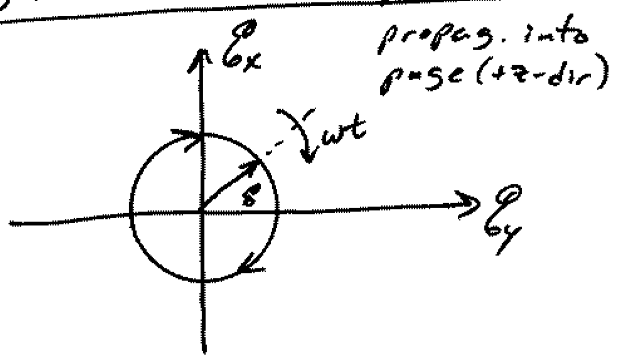


2.12 cont.

Handedness / Sense

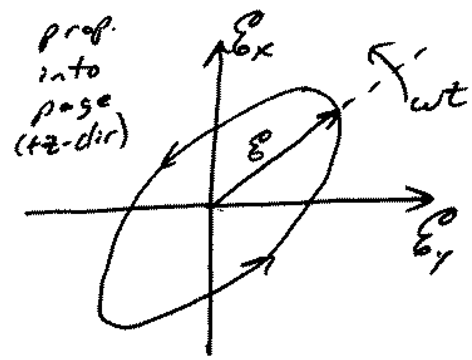
The handedness or sense of the polarization (applicable for circular + elliptical polarizations) is determined by putting thumb in direction of propagation and curling fingers in the direction traced by the electric field vector wrt time → the hand whose fingers curl in the correct direction determines handedness / sense.

ex. Right-hand / CW polarization



RH Circular

Left-hand / CCW



LH elliptical

Linear Polarization

The electric (or magnetic) field vector traces out a straight line wrt time at a given point in space.

This occurs when the electric field vector has:

a) only one component (e.g. $\bar{\mathcal{E}} = \hat{a}_x \mathcal{E}_x$)

b) two orthogonal components that are in phase or 180° (π) out of phase. E.g., $\bar{\mathcal{E}} = \hat{a}_x \mathcal{E}_x + \hat{a}_y \mathcal{E}_x e^{j180°}$

2.12 cont.

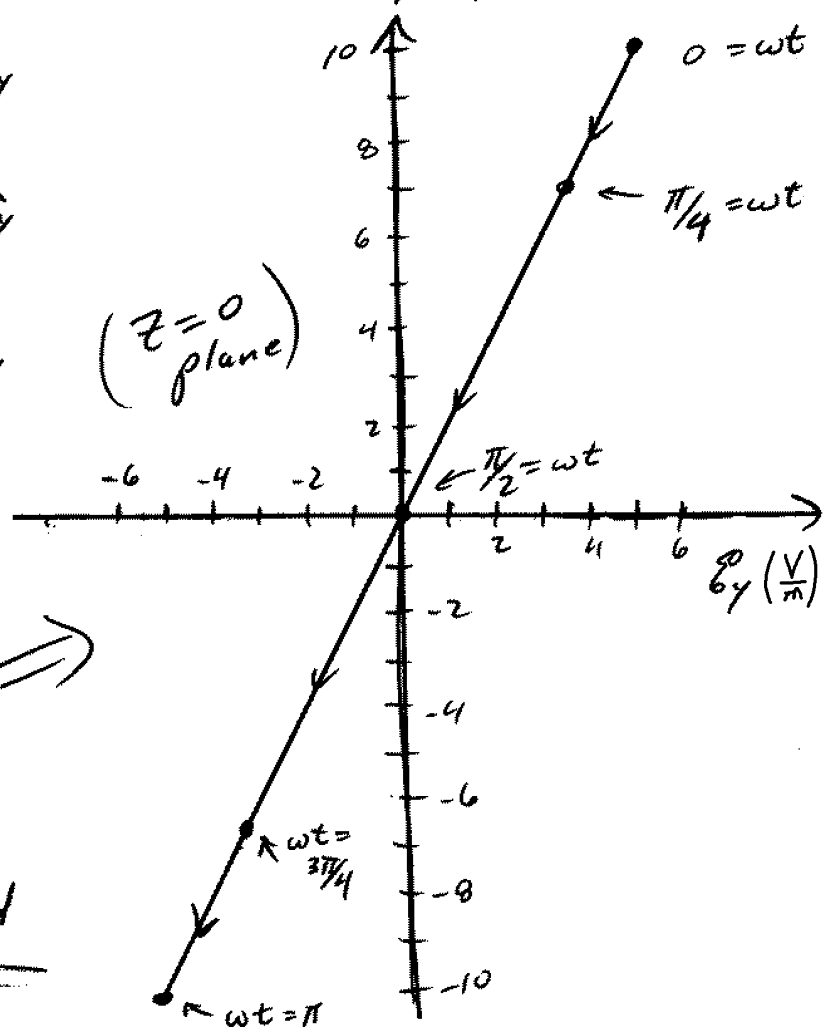
ex. Say $\bar{\mathcal{E}} = 10\cos(\omega t - \beta z)\,\hat{a}_x + 5\cos(\omega t - \beta z)\,\hat{a}_y$ V/m

→ from "$-\beta z$", we know propagation is in $+z$ direction

→ plot location of tip of $\bar{\mathcal{E}}$ as function of time; choose $z=0$ plane

$$\bar{\mathcal{E}} = 10\cos(\omega t)\,\hat{a}_x + 5\cos(\omega t)\,\hat{a}_y \ \text{V/m}$$

ωt	$\bar{\mathcal{E}}$ (V/m)
0	$10\hat{a}_x + 5\hat{a}_y$
$\pi/4$	$7.07\hat{a}_x + 3.536\hat{a}_y$
$\pi/2$	0
$3\pi/4$	$-7.07\hat{a}_x - 3.536\hat{a}_y$
π	$-10\hat{a}_x - 5\hat{a}_y$
$\frac{5\pi}{4}$	$-7.07\hat{a}_x - 3.536\hat{a}_y$
$\frac{3\pi}{2}$	0
$7\pi/4$	$7.07\hat{a}_x + 3.536\hat{a}_y$



plotted first half-cycle

Linearly Polarized

2.12 cont.

Circular Polarization

The electric (or magnetic) field vector traces out a circle wrt time at a given point in space.

The necessary + sufficient conditions for circular polarization are:

a) field has <u>two</u> orthogonal components,

b) the two components must have <u>same</u> magnitude, and

c) the two components must have a time-phase difference which is an <u>odd</u> multiple of 90° ($\pi/2$).

ex. $\bar{\mathscr{E}} = 10\cos(\omega t - \beta z)\,\hat{a}_x + 10\cos(\omega t - \beta z + 270°)\,\hat{a}_y$ V/m

(see plot on next page)

Elliptical Polarization

The electrical (or magnetic) field vector traces out an elliptical locus wrt time at a given point in space.

The necessary + sufficient conditions for elliptical polarization are:

a) field has two orthogonal components,

b) components can have equal or unequal magnitudes,

c) (1) The time-phase difference $\neq 0$ or $\neq n180°$

(2) If components equal, time-phase difference $\neq$ odd mult. of 90°

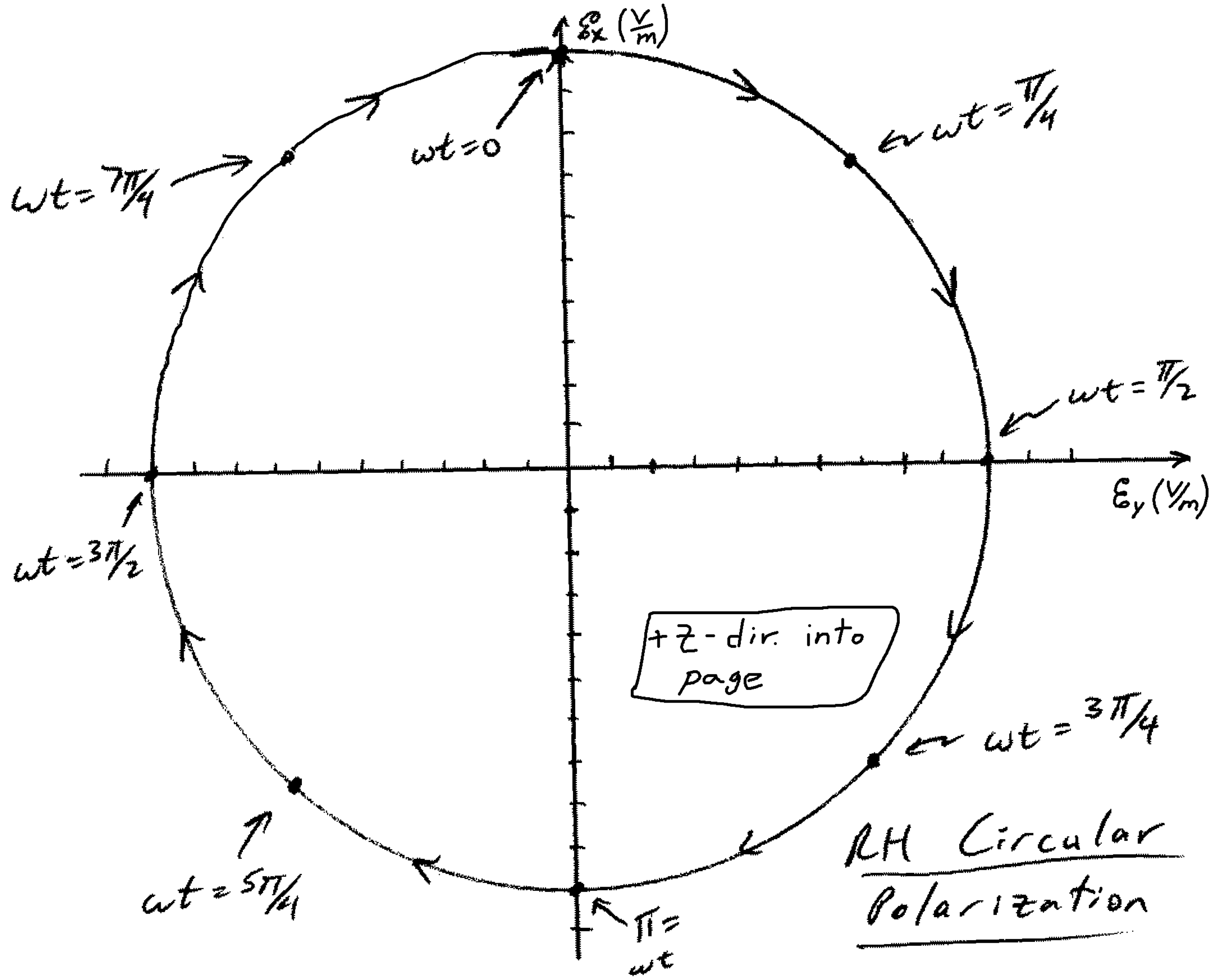
$\Rightarrow$ If EM wave <u>not</u> linear or circular $\Rightarrow$ <u>elliptical</u>

(see example on following pages)

ex. Plot out $\bar{\mathcal{E}} = 10\cos(\omega t - \beta z)\,\hat{a}_x + 10\cos(\omega t - \beta z + 270^\circ)\,\hat{a}_y$ V/m

→ choose $z = 0$ plane $\bar{\mathcal{E}} = 10\cos(\omega t)\,\hat{a}_x + 10\cos(\omega t + 270^\circ)\,\hat{a}_y$ V/m

ωt (rad)	ωt (deg)	$\bar{\mathcal{E}}$ (V/m)
0	0	$10\,\hat{a}_x + 0\,\hat{a}_y$
$\pi/4$	45°	$7.07\,\hat{a}_x + 7.07\,\hat{a}_y$
$\pi/2$	90°	$0\,\hat{a}_x + 10\,\hat{a}_y$
$3\pi/4$	135°	$-7.07\,\hat{a}_x + 7.07\,\hat{a}_y$
π	180°	$-10\,\hat{a}_x + 0\,\hat{a}_y$
$5\pi/4$	225°	$-7.07\,\hat{a}_x - 7.07\,\hat{a}_y$
$3\pi/2$	270°	$0\,\hat{a}_x - 10\,\hat{a}_y$
$7\pi/4$	315°	$7.07\,\hat{a}_x - 7.07\,\hat{a}_y$



RH Circular Polarization

2.12 cont.

Elliptical Polarization Example

Let $\tilde{\mathcal{E}} = 10\cos(\omega t+\beta x)\hat{a}_y + 6\cos(\omega t+\beta x+35^o)\hat{a}_z$ V/m

Note: Wave propagates in the *-x*-direction. On the $x = 0$ plane, let time progress from $t = 0$ to $t = T$ (one period).

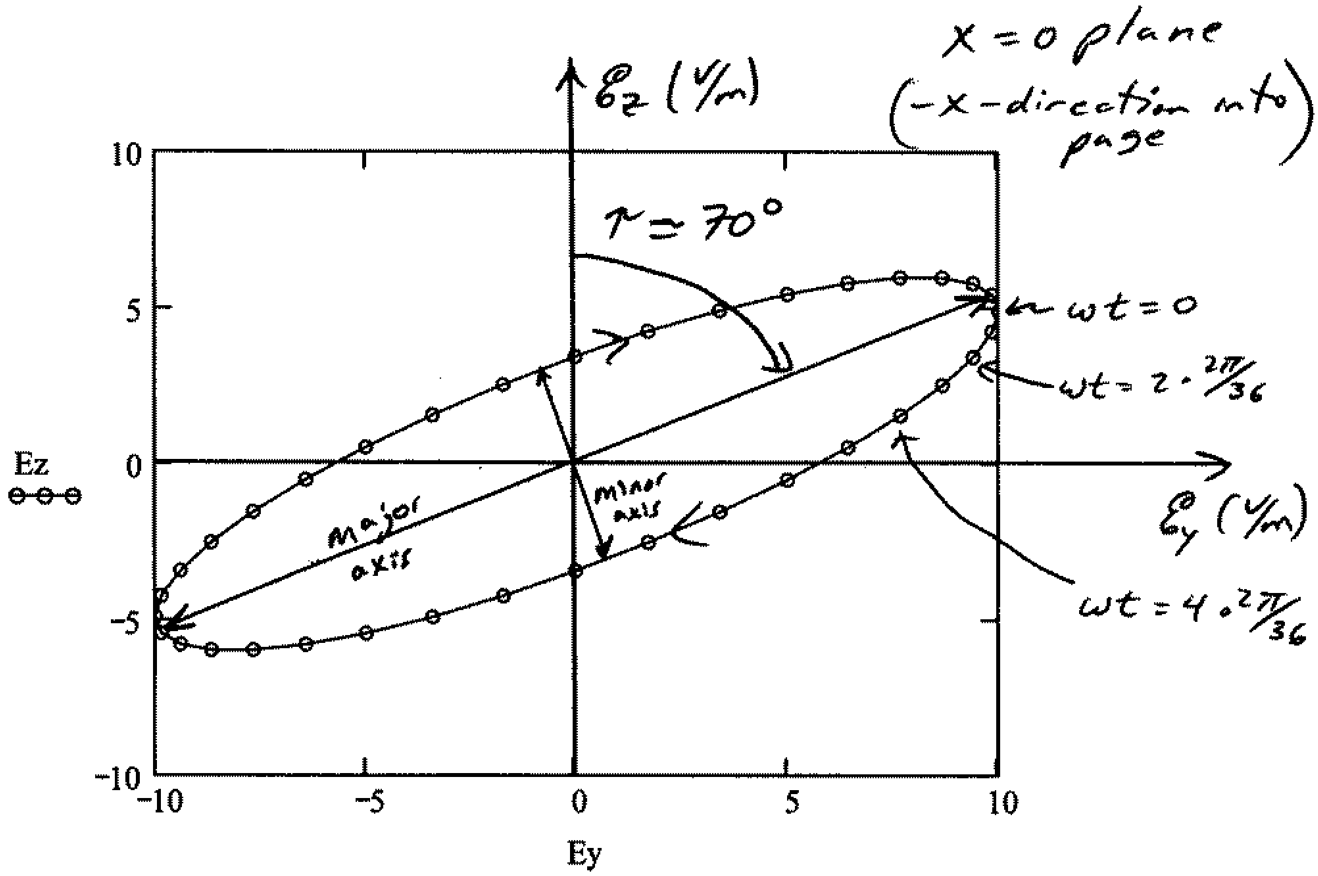
$n := 0..36 \qquad wt_n := n\cdot\frac{2\pi}{36}$

$Ey_n := 10\cdot\cos(wt_n) \qquad Ez_n := 6\cdot\cos\left(wt_n + 35\cdot\frac{\pi}{180}\right)$

$Ey_0 = 10 \qquad Ez_0 = 4.915$

$Ey_2 = 9.397 \qquad Ez_2 = 3.441$

$Ey_4 = 7.66 \qquad Ez_4 = 1.553$



RH Elliptical Polarization

$$AR = \frac{\text{Major}}{\text{minor}} \approx \frac{11.7\text{cm}}{2.6\text{ cm}} = 4.5$$

2.12 cont.

Some parameters used to quantize or describe elliptical polarization include:

a) Axial ratio $\equiv AR = \dfrac{\text{Magnitude of major axis}}{\text{Magnitude of minor axis}}$

$$1 \le AR < \infty$$

circular ↑ ↑ linear

b) Tilt angle $\equiv \tau$ is the angle the major axis of the ellipse makes wrt a specified axis in a specified direction

ex. Text specifies τ as the angle wrt the positive $\hat{e}_y$ axis in the CW direction for -z-dir. propagation.

2.12.2 Polarization loss factor + efficiency

Polarization loss factor $\equiv PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\cos\psi_p|^2$

where ψ_p is the angle between $\hat{\rho}_w$ and $\hat{\rho}_a$

$\hat{\rho}_w$ unit vector of incident wave $\bar{E}^{inc} = \hat{\rho}_w E^{inc}$

$\hat{\rho}_a$ unit vector of receiving antenna (in transmit mode) where $\bar{E}_a = \hat{\rho}_a E_a$ is the far-zone electric field in the direction of interest.

Polarization Efficiency $\equiv p_e = \dfrac{|\bar{\ell}_e \cdot \bar{E}^{inc}|^2}{|\bar{\ell}_e|^2 |\bar{E}^{inc}|^2}$

where $\bar{\ell}_e$ is the vector effective length of antenna

$\bar{E}^{inc}$ is the incident electric field

2.12 cont.

ex. $\bar{E}^{inc} = \hat{a}_y 5 e^{-j\beta z}$ (V/m) $\rightarrow \hat{\rho}_w = \hat{a}_y$

$\bar{E}_a = (\hat{a}_x + \hat{a}_y) E(r,\theta,\phi) \rightarrow \hat{\rho}_a = \dfrac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}$

$$PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\hat{a}_y \cdot \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)\right|^2 = \left|\tfrac{1}{\sqrt{2}}\right|^2 = \underline{\underline{\tfrac{1}{2}}}$$

$$= 10 \log \tfrac{1}{2} = \underline{\underline{-3\,dB}}$$

ex. $\bar{E}_a = (\hat{a}_\theta - j\hat{a}_\phi) E(r,\theta,\phi) \rightarrow \hat{\rho}_a = \dfrac{\bar{E}_a}{|\bar{E}_a|} = \dfrac{(\hat{a}_\theta - j\hat{a}_\phi)E(r,\theta,\phi)}{\sqrt{\bar{E}_a \cdot \bar{E}_a^*}}$

$$= \frac{(\hat{a}_\theta - j\hat{a}_\phi)E(r,\theta,\phi)}{\sqrt{(\hat{a}_\theta - j\hat{a}_\phi)E(r,\theta,\phi) \cdot (\hat{a}_\theta + j\hat{a}_\phi)E^*(r,\theta,\phi)}}$$

$$= \frac{(\hat{a}_\theta - j\hat{a}_\phi)\,\cancel{E(r,\theta,\phi)}}{\sqrt{1^2+1^2}\;\cancel{E(r,\theta,\phi)}}$$

$$= \frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}}$$

If $\hat{\rho}_w = \dfrac{\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{2}}$

$$PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = (\tfrac{1}{2})(\tfrac{1}{2})|1^2 + 1^2|^2 = 1 = \underline{0\,dB\ (\text{No loss})}$$

2.12 cont.

Antenna Receiving mode vs. Transmit mode polarization

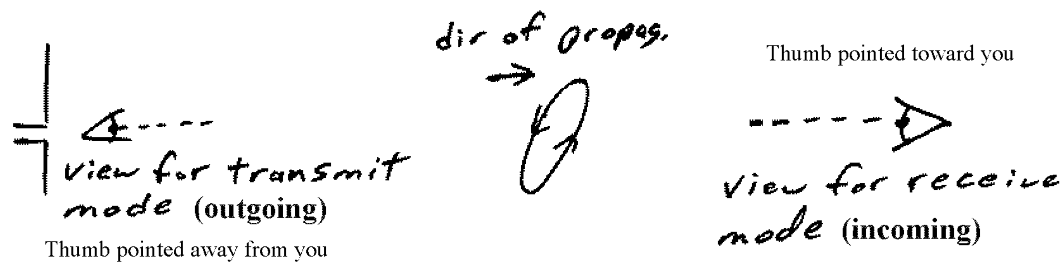
→ Transmit mode almost always used to describe antennas

1) In the same plane of polarization, the polarization ellipses have the same AR, sense of polarization, and spatial orientation.

2) The senses of polarization and the spatial orientation are determined by viewing the polarization ellipses in the respective directions of wave propagation. E.g. Transmit mode – "look" in direction wave leaves antenna or <u>outward</u> from antenna. Receive mode – "look" in direction of wave as it propagates <u>toward</u> antenna.

→ Although the transmitted and received waves have the <u>same</u> sense of polarization, they would <u>appear</u> opposite if both viewed from same direction

→ Their tilt angles are the negative of one another when measured wrt a common reference.



2.13 Input Impedance

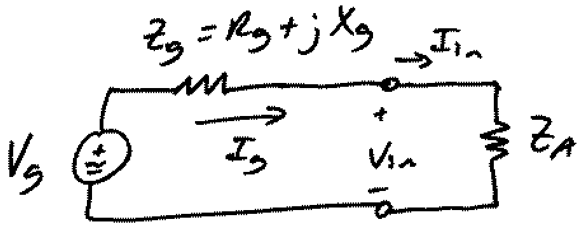
Input Impedance – impedance measured at antenna input terminals as defined by ratio of voltage to current or ratio of appropriate electric field and magnetic field components.

$$Z_A = \frac{V_{in}}{I_{in}} = R_A + jX_A \quad (\Omega)$$

$$R_A = R_r + R_L$$

R_r ← Rad. resistance; R_L ← loss resistance

Simple model (omits any connecting trans. line)



$$I_g = \frac{V_g}{Z_g + Z_A}$$

$$|I_g| = \frac{|V_g|}{\left[(R_g + R_L + R_r)^2 + (X_g + X_A)^2\right]^{1/2}}$$

$$\text{Power radiated} \equiv P_r = \tfrac{1}{2}|I_g|^2 R_r = \frac{|V_g|^2}{2}\,\frac{R_r}{(R_g+R_L+R_r)^2+(X_g+X_A)^2} \quad (W)$$

Power lost (Ohmic + dielectric losses in antenna)

$$\equiv P_L = \tfrac{1}{2}|I_g|^2 R_L = \frac{|V_g|^2}{2}\,\frac{R_L}{(R_g+R_L+R_r)^2+(X_g+X_A)^2} \quad (W)$$

Power dissipated in generator

$$\equiv P_g = \tfrac{1}{2}|I_g|^2 R_g = \frac{|V_g|^2}{2}\,\frac{R_g}{(R_g+R_L+R_r)^2+(X_g+X_A)^2} \quad (W)$$

2.13 cont.

Maximum Power Delivered to antenna when it is conjugate-matched to the source.

i.e. $R_L + R_r = R_g$ and $X_A = -X_g$

Then
$$P_r = \frac{|V_g|^2}{8}\left(\frac{R_r}{(R_r+R_L)^2}\right) \quad (W)$$

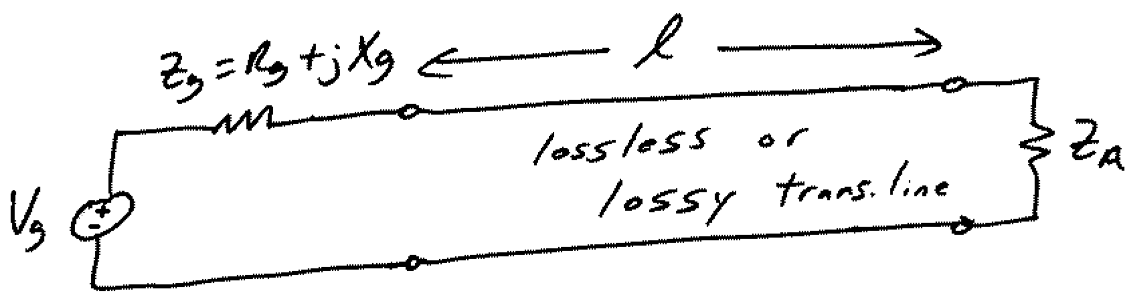
$$P_L = \frac{|V_g|^2}{8}\left(\frac{R_L}{(R_r+R_L)^2}\right) \quad (W)$$

$$P_g = \frac{|V_g|^2}{8}\left(\frac{R_g}{(R_r+R_L)^2}\right) = \frac{|V_g|^2}{8R_g} \quad (W)$$

Note: $P_g = P_r + P_L$ and the power supplied by the generator is:

$$P_s = P_g + P_r + P_L = \frac{|V_g|^2}{4R_g} = \frac{|V_g|^2}{4(R_r+R_L)} \quad (W)$$

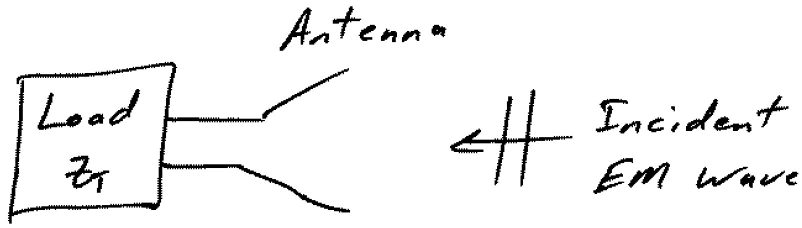
Better Model

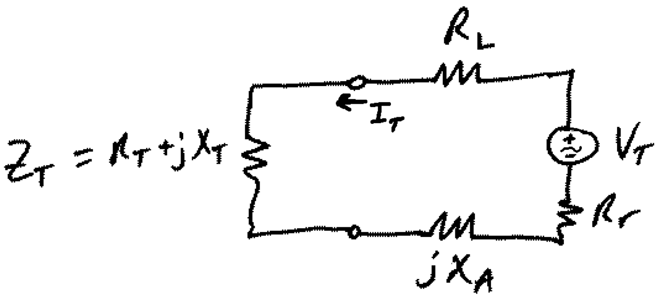


→ obviously transmission line must be considered for power calculations.

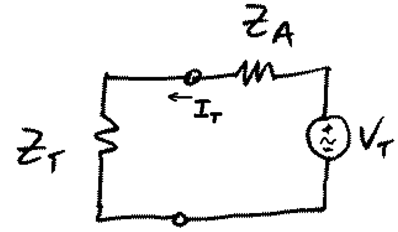
2.13 cont.

Receive mode simple model (omit trans. line)





or



where V_T is the open ckt (i.e. Thevenin) voltage induced at the antenna terminals by the incident EM wave

Max. power delivered for <u>conjugate-match</u>

$$R_T = R_L + R_r \quad \text{and} \quad -X_T = X_A$$

Power to load $\equiv P_T = \dfrac{|V_T|^2}{8R_T}$

Scattered or re-radiated power $\equiv P_r = \dfrac{|V_T|^2}{8}\left[\dfrac{R_r}{(R_r+R_L)^2}\right] = \dfrac{|V_T|^2}{8}\left(\dfrac{R_r}{R_T^2}\right)$

Power lost $\equiv P_L = \dfrac{|V_T|^2}{8}\left[\dfrac{R_L}{(R_r+R_L)^2}\right] = \dfrac{|V_T|^2}{8}\left(\dfrac{R_L}{R_T^2}\right)$

Power collected $\equiv P_c = \frac{1}{2} V_T I_T^* = \dfrac{|V_T|^2}{4}\left(\dfrac{1}{R_r+R_L}\right)$

☆ w/ conjugate matching → ½ power to load
→ ½ power re-radiated &/or lost

2.14 Antenna Radiation Efficiency

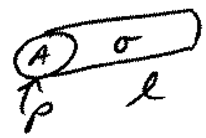
→ conduction (ohmic) + dielectric losses are difficult to calculate so they are usually measured (together)

→ e_c + e_d difficult to separate ⇒ e_{cd}

$$\boxed{e_{cd} = \frac{\text{Power to } R_r}{\text{Power to } R_r + R_L} = \frac{R_r}{R_r + R_L}}$$

How can we estimate R_L due to conduction losses?

$$R_{dc} = \frac{\ell}{\sigma A} \quad (\Omega)$$



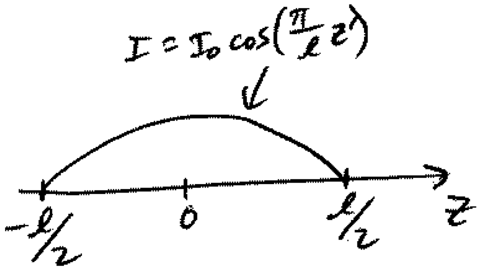
$$R_{ac} = R_{hf} = \frac{\ell}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} \quad (\Omega) \rightarrow r_{hf} = \frac{R_{hf}}{\ell} = \frac{1}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} \left(\frac{\Omega}{m}\right)$$

↳ P ≡ perimeter of conductor (e.g. circumference)

To get R_L, we need to know the current distribution on/in the conductor (due to standing waves ... it may not be constant) to calculate P_L, then relate it to power lost $P_L = \frac{1}{2}|I_T|^2 R_L$.

ex. $\lambda/2$ dipole

$$P_L = \int dP = \int_{-\ell/2}^{\ell/2} \tfrac{1}{2}|I|^2 r_{hf}\, dz' \leftarrow \text{integrate power loss per unit length}$$



$$= \tfrac{1}{2} I_0^2 r_{hf} \int_{-\ell/2}^{\ell/2} \cos^2\left(\tfrac{\pi}{\ell} z'\right) dz'$$

$$= \tfrac{1}{2} I_0^2 r_{hf} \left[\frac{z'}{2} + \frac{\sin\left(\frac{2\pi}{\ell} z'\right)}{4\pi/\ell}\right]\Bigg|_{-\ell/2}^{\ell/2}$$

$$= \tfrac{1}{2} I_0^2 r_{hf} \frac{\ell}{2} = \tfrac{1}{2} I_0^2 \frac{R_{hf}}{2}$$

Set $P_L = \frac{1}{2}|I_T|^2 R_L = \frac{1}{2} I_0^2 \frac{R_{hf}}{2}$ (assume equal) $\Rightarrow \boxed{R_L = R_{hf}/2}$

2.15 Antenna Vector Effective Length and Equivalent Areas

→ Antennas, in receiving mode, capture power from EM waves

→ we use effective lengths & equivalent areas to describe/characterize how antennas do this

Vector Effective Length $\bar{l}_e$

→ AKA effective height

→ quantity used to find voltage induced at open-ckt terminals of antenna

→ far-field quantity

→ complex vector $\bar{l}_e(\theta,\phi) = \hat{a}_\theta l_\theta + \hat{a}_\phi l_\phi$

We define $\bar{l}_e$ using

$$\boxed{\bar{E}_a = \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi = -j\eta \frac{k I_{in}}{4\pi r} \bar{l}_e e^{-jkr}}$$

where $\bar{E}_a \equiv$ far-field radiated electric field

$\eta \equiv$ intrinsic impedance of medium

$k = 2\pi/\lambda \equiv$ wave number

$I_{in} \equiv$ current into antenna terminals

2.15 cont.

The open circuit voltage is found for the antenna in receive mode as:

$$\boxed{V_{oc} = \bar{E}^{i} \cdot \bar{l}_e}$$

incident field

ex. $\lambda/2$ dipole ($l = \lambda/2$)

input current

From chap. 4, (4.84) $\bar{E} = \hat{a}_\theta\, j\eta \dfrac{I_0 e^{-jkr}}{2\pi r}\left[\dfrac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]$

far-field

$$\hat{a}_\theta\, j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right] = -j\eta \frac{k I_{in}}{4\pi r}\, \bar{l}_e\, e^{-jkr}$$

$$\bar{l}_e = \hat{a}_\theta\, \cancel{j}\cancel{\eta} \frac{\cancel{I_0}\, \cancel{e^{-jkr}}}{2\cancel{\pi}\cancel{r}}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right] \frac{4\cancel{\pi}\cancel{r}}{-\cancel{j}\cancel{\eta} k \cancel{I_{in}}}\, \cancel{e^{+jkr}}$$

$$= -\hat{a}_\theta \frac{2}{k}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right] \quad \text{where } k = \frac{2\pi}{\lambda}$$

$$\boxed{\begin{aligned} \bar{l}_e &= -\hat{a}_\theta \frac{\lambda}{\pi}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right] \quad \text{or} \\ \bar{l}_e &= -\hat{a}_\theta \frac{2l}{\pi}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right] \end{aligned}}$$

since $l = \lambda/2$

2.15 cont.

Several equivalent areas have been defined for antennas

Effective Area (AKA Aperture Area) A_e

In a given direction, the ratio of the power available at the antenna terminals of a receiving antenna to the power flux density of an EM plane wave incident on the antenna from that direction w/ the wave being polarization matched to the antenna. Usually, we only find A_e in the direction of maximum directivity / gain.

$$A_e = \frac{P_T}{W_i} = \frac{\frac{1}{2} |I_T|^2 R_T}{W_i} \quad (m^2)$$

$P_T \rightarrow$ Power delivered to load (W)

$W_i \rightarrow$ Power density of incident wave (W/m^2)
(Mag. of Poynting vector)

From the simple receive mode equiv. ckt

$$A_e = \frac{|V_T|^2}{2W_i} \left[\frac{R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2} \right]$$ ← any load

w/ conjugate matching ($R_T = R_r + R_L$ and $X_A = -X_T$)

$$\left[A_e\right]_{max} = \frac{|V_T|^2}{8 W_i} \left[\frac{R_T}{(R_r + R_L)^2} \right] = \frac{|V_T|^2}{8 W_i} \left(\frac{1}{R_r + R_L} \right) = A_{em}$$

2.15 cont.

Notes: * $\boxed{P_T = A_e W_i}$ ← gives available power at terminals of antenna

* Remember, w/ conj. matching, only half captured power makes it to load (other half lost or re-radiated)

Other equivalent areas are the scattering, loss, and capture areas. For a conjugate-matched load + antenna, they are

Scattering Equiv. Area $\equiv A_S = \dfrac{|V_T|^2}{8W_i}\left[\dfrac{R_r}{(R_L+R_r)^2}\right] \Leftrightarrow P_r = A_S W_i$ (always true)

Loss Equiv. Area $\equiv A_L = \dfrac{|V_T|^2}{8W_i}\left[\dfrac{R_L}{(R_L+R_r)^2}\right] \Leftrightarrow P_L = A_L W_i$ (always true)

Capture Equiv. Area $\equiv A_c = \dfrac{|V_T|^2}{8W_i}\left[\dfrac{R_T + R_r + R_L}{(R_L+R_r)^2}\right] \Leftrightarrow P_c = A_c W_i$ (always true)

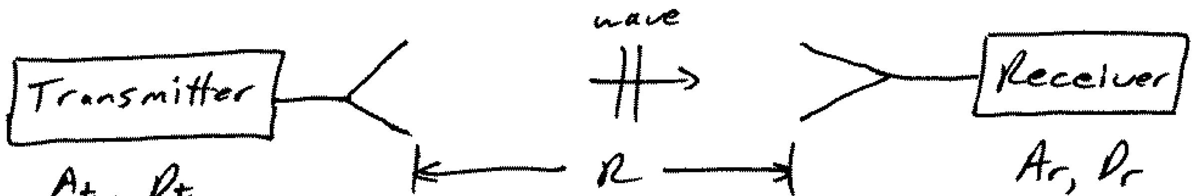
Aperture Efficiency $\equiv \epsilon_{ap} = \dfrac{A_{em}}{A_p} = \dfrac{\text{Max. Eff. Area}}{\text{physical Area}}$

→ For aperture antennas, $A_{em} \leq A_p$ or $\epsilon_{ap} \leq 1$

→ Wire antennas can easily have $\epsilon_{ap} > 1$

Note: $\boxed{A_C = A_e + A_S + A_L}$ ← always true

2.16 Maximum Directivity & Effective Area



A_t, D_t — effective area + directivity of transmitter

A_r, D_r — eff. area + directivity of receiving antenna

$$W_t = W_o D_t = \frac{P_t}{4\pi R^2} D_t$$

isotropic pwr density (W_o)

← Power density trans. and available to Receiver

Rec. Power → $P_r = W_t A_r = \dfrac{P_t D_t A_r}{4\pi R^2} \Rightarrow D_t A_r = \dfrac{P_r}{P_t}(4\pi R^2)$

If we reverse directions, we get

$$D_r A_t = \frac{P_r}{P_t}(4\pi R^2) \Longrightarrow \underline{D_t A_r = D_r A_t}$$

So $\boxed{\dfrac{D_t}{A_t} = \dfrac{D_r}{A_r}}$

In dir. of max. radiation $\boxed{\dfrac{D_{ot}}{A_{tm}} = \dfrac{D_{or}}{A_{rm}}}$

Directivities + effective areas increase/decrease in direct proportion

↓ w/ some derivation

$$\boxed{A_{em} = \frac{\lambda^2}{4\pi} D_o}$$

← relate max. eff. area + directivity

OR

$$\boxed{A_{em} = \frac{\lambda^2}{4\pi} G_o = \frac{\lambda^2}{4\pi} e_{cd} D_o}$$

← if antenna has losses

2.16 cont.

If we also consider T.L mismatch + polarization mismatch

$$\boxed{A_{em} = e_{cd}\left(1-|\Gamma|^2\right)\left(\frac{\lambda^2}{4\pi}\right)|\hat{\rho}_w \cdot \hat{\rho}_a|^2 D_0} \quad (m^2)$$

e_{cd}: conduction-dielectric losses

$(1-|\Gamma|^2)$: e_t (mismatch w/ T.L.)

$|\hat{\rho}_w \cdot \hat{\rho}_a|^2$: PLF (polarization losses)

ex. We have an antenna with a gain of 9.8 dBi and an input impedance of 60Ω connected to a 50Ω transmission line operating at 10 GHz. Assuming that the antenna is oriented so as to receive maximum power and has a physical aperture 4.5cm x 2cm, what is the maximum effective area and aperture efficiency?

$$G_0(dB) = 10\log_{10} G_0 \Rightarrow G_0 = 10^{G_0(dB)/10} = 10^{9.8/10} = \underline{9.55}$$

+ note $G_0 = e_{cd} D_0$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60-50}{60+50} = 0.\overline{09}$$

$$\lambda = \frac{u}{f} = \frac{2.998\times10^8}{10\times10^9} = 0.02998 \text{ m}$$

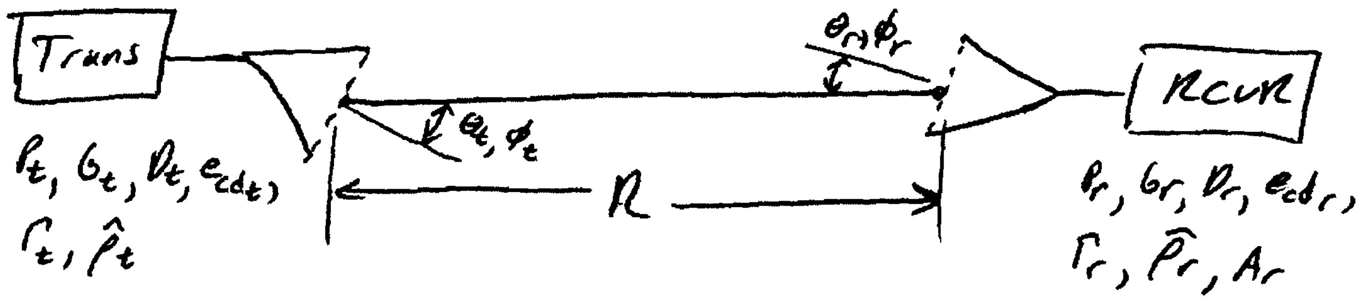
$$A_{em} = (9.55)\left[1-(0.\overline{09})^2\right]\left(\frac{0.02998^2}{4\pi}\right)|1|^2$$

$$= 6.774\times10^{-4}\, m^2 \, \frac{100^2 cm^2}{m^2} = \underline{\underline{6.774 \text{ cm}^2}}$$

$$\epsilon_{ap} = \frac{A_{em}}{A_{phys}} = \frac{6.774 \text{ cm}^2}{(4.5\text{cm})(2\text{cm})} = \underline{\underline{0.7527 \text{ or } 75.27\%}}$$

2.17 Friis Transmission Eqn & Radar Range Equation

Friis Transmission Eqn – Relates power received to power transmitted between two antennas in far-field



@ a distance R from the transmitting antenna in the direction of the Rcvr, the power density is:

$$W_t = \frac{P_t\, G_t(\theta_t, \phi_t)}{4\pi R^2} = \underset{\text{rad. effic.}}{e_t} \frac{P_t\, D_t(\theta_t, \phi_t)}{4\pi R^2}$$

How much of this power can the receiver collect?

→ Use effective area A_r of the receiver

$$A_r = e_r D_r(\theta_r, \phi)\left(\frac{\lambda^2}{4\pi}\right)$$

and

$$P_r = W_t A_r$$

2.17 cont.

This gives:

$$P_r = e_r D_r(\theta_r, \phi_r)\left(\frac{\lambda^2}{4\pi}\right) e_t \frac{D_t(\theta_t, \phi_t)}{4\pi R^2} P_t$$

↓ Now, account for T.L. mis-matches @ the receiver + transmitter, and polarization mis-matches

Friis Transmission Eqn.

$$\boxed{\frac{P_r}{P_t} = e_{cd_t} e_{cd_r}\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)\left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)\left|\hat{\rho}_t \cdot \hat{\rho}_r\right|^2}$$

Where P_r + P_t → Power received + transmitted (W)

e_{cd_t} + e_{cd_r} → conductor + dielectric efficiencies of transmitter + receiver

Γ_t + Γ_r → Refl. coeff. between transmitter and its T.L., ditto for receiver

$\left(\frac{\lambda}{4\pi R}\right)^2$ → free space loss factor (accounts for power density drop due to spherical spreading of power)

$D_t(\theta_t, \phi_t)$, $D_r(\theta_r, \phi_r)$ → Directivities of transmitter + receiver in direction(s) toward one another.

$\left|\hat{\rho}_t \cdot \hat{\rho}_r\right|^2$ → Polarization loss factor between transmitting + receiving antennas.

2.17 cont.

Often the Friis Transmission Eqn is expressed in decibel form:

$$\boxed{\begin{aligned} P_r\,(\text{dBm or dBW}) = {} & 10\log_{10} e_{cd_t} + 10\log_{10} e_{cd_r} + 10\log_{10}\left(1-|\Gamma_t|^2\right) \\ & + 10\log_{10}\left(1-|\Gamma_r|^2\right) + 20\log_{10}\left(\frac{\lambda}{4\pi R}\right) \\ & + D_t(\theta_t,\phi_t)\,(\text{dBi}) + D_r(\theta_r,\phi_r)\,(\text{dBi}) \\ & + 20\log_{10}|\hat{\rho}_t\cdot\hat{\rho}_r| + P_t\,(\text{dBm or dBW}) \end{aligned}}$$

Note: Every term except D_t, D_r, P_r, + P_t should be 0dB or less.

Note: $P\,(\text{dBm}) = 10\log_{10}\left(\frac{P}{1\text{mW}}\right)$

$P\,(\text{dBW}) = 10\log_{10}\left(\frac{P}{1\text{W}}\right)$

2.17 cont.

ex. A radio in a car is tuned to an FM radio station (95.1 MHz) located 25 Km away. If the car is climbing a 7% grade toward the radio station, how much power is received? Assume the following:

Auto/Car

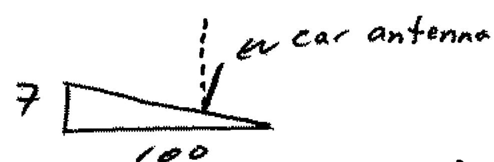
* effic. = 99%
* monopole/whip antenna (vertical when car on level ground)
* $|\Gamma_r(f = 95.1\,MHz)| = 0.2$ (VSWR = 1.5)
* $D_r(\theta_r, \phi_r) = \dfrac{4\pi U}{P_{rad}} = \dfrac{4\pi \dfrac{\cancel{\eta}\cancel{|I_0|^2}}{8\pi^2} \sin^3\theta_r}{\dfrac{\cancel{\eta}\cancel{|I_0|^2}}{8\pi} C_{in}(2\pi)}$

$$= \frac{4 \sin^3\theta_r}{C_{in}(2\pi)} = \frac{4 \sin^3\theta_r}{2.435}$$

$$= 1.643 \sin^3\theta_r$$

FM radio Station

* $P_t = 50\,KW$
* effic. = 98%
* vertical polarization
* $|\Gamma_t| \approx 0$
* $D_t(\theta_t, \theta_r) = 10$



$\tan^{-1} 7/100 = 4°$ ← angle from vertical of car antenna

2.17 cont.

ex. cont.

$$PLF = |\hat{\rho}_t \cdot \hat{\rho}_r|^2 = |\cos 4°|^2 = 0.995134$$

$$D_r(\theta_r = 94°, \phi_r) = 1.643 \sin^3(94°) = 1.631$$

wavelength → $\lambda = \frac{c}{f} = \frac{2.998 \times 10^8}{95.1 \times 10^6} = 3.152471 \text{ m}$

Method 1

$$P_r = P_t \, e_{cd_t} \, e_{cd_r} (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2)\left(\frac{\lambda}{4\pi R}\right)^2 D_t \, D_r \, |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

$$= (50 \times 10^3)(0.98)(0.99)(1)(1 - 0.2^2)\left(\frac{3.152}{4\pi(25 \times 10^3)}\right)^2 10(1.631)0.995134$$

$$\underline{P_r = 76.11 \mu W = -11.19 \text{ dBm}}$$

Method 2

$$P_t(\text{dBm}) = 10 \log_{10}\left(\frac{50 \times 10^3}{10^{-3}}\right) = 76.9897 \text{ dBm}$$

$$10 \log_{10} e_{cd_t} = -0.04365 \text{ dB}$$

$$10 \log_{10} e_{cd_r} = -0.08774 \text{ dB}$$

$$10 \log_{10}(1 - |\Gamma_t|^2) = 0 \text{ dB}$$

$$10 \log_{10}(1 - |\Gamma_r|^2) = -0.17729 \text{ dB}$$

$20 \log_{10}\left(\frac{\lambda}{4\pi R}\right) = -99.969975 \text{ dB}$ ← space loss is the killer!

$$20 \log_{10}(\cos 4°) = -0.02118 \text{ dB}$$

$D_t(\text{dBi}) = 10 \text{ dBi}$ $D_r(\text{dBi}) = 10 \log_{10}(1.631) = 2.12454 \text{ dBi}$

2.17 cont.

ex. cont.

$$P_r(dBm) = -0.04365 - 0.08774 + 0 - 0.17729 - 99.969975 + 10 + 2.12454 - 0.02118 + 76.9897 \text{ dBm}$$

$$\boxed{\begin{aligned} P_r(dBm) &= -11.1856 \text{ dBm} \\ P_r &= 76.11\ \mu W \end{aligned}} = 10^{\frac{-11.1856}{10}} \times 1 mW$$

$\Rightarrow$ Same answer!

Notes: For efficiencies $\gtrsim 90\%$, $|\Gamma| \lesssim 0.2$, + PLF > 0.9, we can usually neglect the losses due to these terms.

The most important terms are the power(s), directivities, + free space loss factor!

2.17 cont.

Radar Range Eqn

- allows us to calculate how much power will be received from a radar signal hitting a target

Background - We need a way to characterize how much power is re-radiated or scattered by the target after being illuminated by an EM signal

→ **Radar Cross Section (Scattering Cross Section, Echo area, RCS) σ (m^2)**

→ area intercepting that amount of power which, when scattered isotropically, produces at the receiver a power density which is equal to that scattered by the actual target.

$$\boxed{\begin{aligned} W_s &= \lim_{R \to \infty} \left[\frac{\sigma W_i}{4\pi R^2} \right] \\ \text{or} \quad \sigma &= \lim_{R \to \infty} \left[4\pi R^2 \frac{W_s}{W_i} \right] = \lim_{R \to \infty} \left[4\pi R^2 \frac{|E^s|^2}{|E^i|^2} \right] \end{aligned}}$$

↳ Far-field quantity

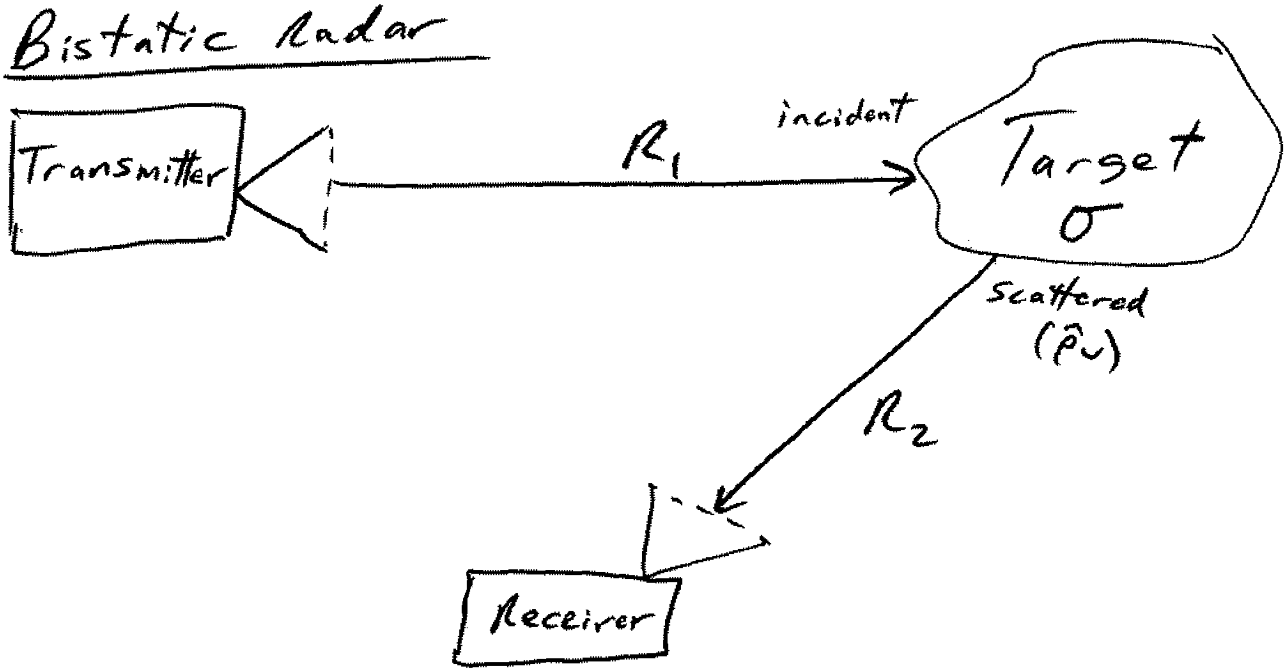
2.17 cont.

Note: * RCS is very much a function of angle (both for incident wave & scattered wave), polarization, geometry, ...

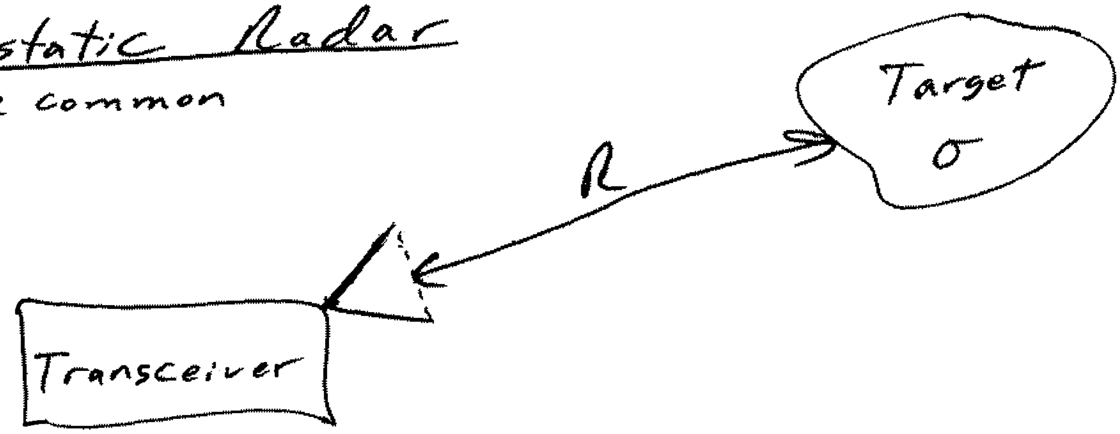
* Huge area of interest for military (e.g., aircraft, missiles, ships, ...)

* Terrifically complex to model (think about all the physical structures on a ship)

Bistatic Radar



Monostatic Radar
→ more common



2.17 cont.

Bistatic Radar Range Eqn.

$$\boxed{\frac{P_r}{P_t} = e_{cd_t}\, e_{cd_r} \left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right) \sigma \, \frac{D_t(\theta_t,\phi_t)\, D_r(\theta_r,\phi_r)}{4\pi} \times \left(\frac{\lambda}{4\pi R_1 R_2}\right)^2 |\hat{\rho}_w \cdot \hat{\rho}_r|^2}$$

↳ eq'n can be expressed in dB as before

w/ $\sigma\,(\text{dBsm}) = 10 \log_{10}\left(\frac{\sigma\,(m^2)}{1 m^2}\right)$

Monostatic Radar Range Eqn.

→ Same as above w/ $R_1 = R_2$, $D_t = D_r$, and

$e_{cd_t} = e_{cd_r}$

* Typical RCS are shown in Table 2.2

(normal incidence, scattered directly back)

Table 2.2 RCS OF SOME TYPICAL TARGETS

	Typical RCSs [22]	
Object	RCS (m^2)	RCS (dBsm)
Pickup truck	200	23
Automobile	100	20
Jumbo jet airliner	100	20
Large bomber *or* commercial jet	40	16
Cabin cruiser boat	10	10
Large fighter aircraft	6	7.78
Small fighter aircraft *or* four-passenger jet	2	3
Adult male	1	0
Conventional winged missile	0.5	−3
Bird	0.01	−20
Insect	0.00001	−50
Advanced tactical fighter	0.000001	−60

→ representative of X-Band (~10 GHz)

2.17 cont.

RCS depends on :
* target geometry/shape
* target materials
* frequency
* $\measuredangle$ of incident wave
* $\measuredangle$ of observation
* polarization of incident wave
⋮

RCS minimization

→ can control target shape + materials

→ no sharp corners, protruding wires, flat metal plates, ...

Skip antenna RCS discussion

Note: Both the Friis + Radar Range Eq'ns neglect atmospheric/environment losses due to rain, snow, fog, smoke, ...

2.17 cont.

ex. A police radar operating at 11 GHz transmits a 5W signal at targets up to 100m distant. Assuming the RCS of a typical car to be 100 m^2 and that the receiver requires 0.01 μW what gain should the antenna be? Neglect PLF + T.L. mismatching

$$\frac{P_r}{P_t} = (\sigma) \frac{G_{t/r}^2}{4\pi} \left(\frac{\lambda}{4\pi R^2} \right)^2$$

$$\frac{10^{-8}}{5} = (100) \frac{G_{t/r}^2}{4\pi} \left(\frac{\frac{2.998 \times 10^8}{11 \times 10^9}}{4\pi\, 100^2} \right)^2$$

$$G_{t/r}^2 = 5342.96$$

$$G_{t/r} = 73.096 = 18.6\ dB_i$$ ← Horn / Dish antenna

Note: $G = e_{cd} D$

2.18 Antenna Temperature

→ Account for fact that every object above absolute zero (0K = −273°C) radiates energy.

→ Use equivalent temperature T_B (AKA brightness temperature) to represent this energy

$$T_B(\theta,\phi) = \epsilon(\theta,\phi)\,T_m = \left(1-|\Gamma(\theta,\phi)|^2\right)T_m$$

T_B → brightness temp (K)

ϵ → emissivity $0 \le \epsilon \le 1$

T_m → physical or molecular temp. (K)

Γ → refl. coeff. of surf. for wave polarization

Note: $T_B \le T_m$

ex. Ground (earth) $T_B \simeq 300K$ Sky (straight up) $T_B \simeq 5K$

How do antennas receive this energy?

characterize using antenna temperature

$$T_A = \frac{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} T_B(\theta,\phi)\,G(\theta,\phi)\sin\theta\, d\theta\, d\phi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} G(\theta,\phi)\sin\theta\, d\theta\, d\phi} \quad (K)$$

where

$G(\theta,\phi)$ is the gain pattern of the antenna

2.18 cont.

This energy received by the antenna contributes to the noise power in the receiver as defined by

$$P_r = K T_A \Delta f \quad (W)$$

$P_r \rightarrow$ antenna noise power (W)

$K \rightarrow$ Boltzmann's constant 1.38×10^{-23} J/K

$T_A \rightarrow$ antenna temp. (K)

$\Delta f \rightarrow$ receiver bandwidth (Hz)

What about other conditions/factors (e.g. trans. line loss, temp. ...)? Modify T_A to account for these factors

$$T_a = T_A e^{-2\alpha \ell} + T_{AP} e^{-2\alpha \ell} + T_0 (1 - e^{-2\alpha \ell})$$

where $T_{AP} = \left(\frac{1}{e_A} - 1\right) T_P$

$T_a \rightarrow$ ant. temp. @ receiver terminals (K)

$T_A \rightarrow$ ant. temp. @ antenna terminals (K)

$T_{AP} \rightarrow$ ant. temp. @ antenna terminals due to phys. temp of ant. (K)

$T_P \rightarrow$ phys. temp of ant. (K)

$\alpha \rightarrow$ atten. coeff. of trans. line (Np/m)

$e_A \rightarrow$ thermal effic. of ant.

$\ell \rightarrow$ trans. line length (m)

$T_0 \rightarrow$ phys. temp. of trans. line (K)

2.18 cont.

Now, we can write the noise power at the receiver as

$$P_r = K T_a \Delta f$$

Now, we can also consider thermal noise in the receiver electronics/components at the receiver terminals

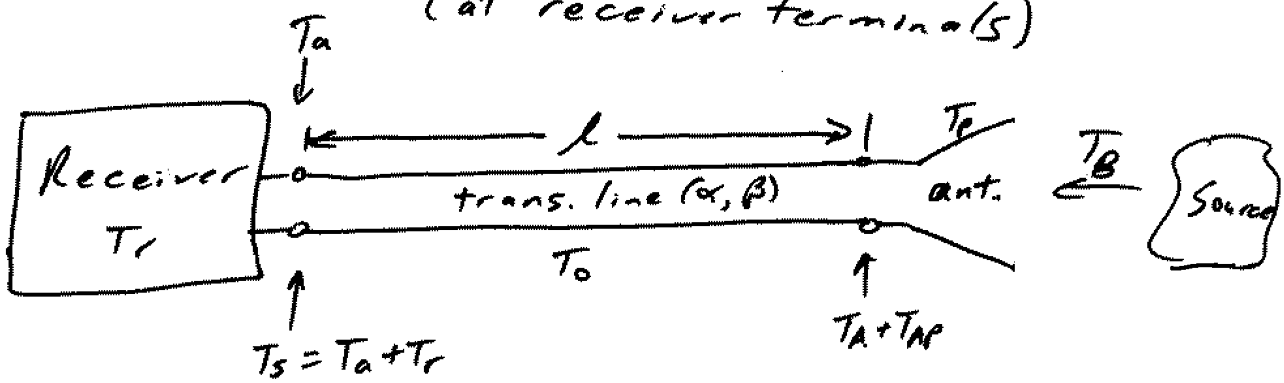
$$P_s = K(T_a + T_r)\Delta f = K T_s \Delta f$$

P_s → system noise power (at receiver terminals)

T_a → ant. noise temp. (")

T_r → receiver " " (")

$T_s = T_a + T_r$ → effective system noise temp. (at receiver terminals)



How important is all this?

Depends on application + necessary SNR.

Very important to radio astronomy. Less important to FM radio.