Here, you will match the six-element Yagi-Uda antenna for channel 13 from the previous assignment **with the boom omitted** to a **100 Ω** twin-lead transmission line using a **T-match** so that the VSWR is less than 1.1 at the center frequency f_c. Steps:

a) Tabulate the element lengths and spacings (in cm) for the unmatched design (make driven element length the simple average of the reflector and first director).

b) Model **unmatched design** using NEC-2 and determine and tabulate input impedance Z_a, input reflection coefficient Γ_{in} (polar format), VSWR, maximum gain G_{max} (dBi), backlobe gain G_{back} (dBi), and front-to-back/FB ratio (dB) at f_c.

c) Match the antenna. At each step, discuss, list, and justify design changes/choices as well as show results/work.

d) In a **table**, summarize the original (unmatched) and final (matched)- Z_a or Z_{in}, Γ_{in}, VSWR, G_{max} (dBi), G_{back} (dBi), and FB ratio (dB) at f_c. Comment on how the final design compares with the original.

e) Accurately sketch final antenna design with T-match (no boom).

- Include the input NEC-2 file(s) and relevant excerpts of the output file(s). Assume $c = 2.998 \times 10^8$ m/s.

****

a) Design Summary:

Directivity of a six-element Yagi-Uda antenna is 10.2 dBd = 10.2 + 2.15 = **12.35 dBi**

Design Frequency- Channel 13 (210-216 MHz), so ***f* = 213 MHz**.

Desired input impedance- ***R*₀ = 100 Ω** (for T-match)

Element diameter- ***d* = 3/8" = 0.9525 cm** (use brass pipe $\sigma_{brass} = 1.1\times10^8$ S/m)

$$\lambda = \frac{c}{f} = \frac{2.998\times10^8}{213\times10^6} = 1.4075117 \text{ m} = \underline{140.7512 \text{ cm}}$$

Table 1 Element lengths and spacings for the unmatched design

Description	(cm)
Reflector, $l_1' = 0.483\lambda$	67.983
Driven, $l_2' = 0.4585\lambda$	64.534
1st director, $l_3' = 0.434\lambda$	61.086
2nd director, $l_4' = 0.426\lambda$	59.960
3rd director, $l_5' = 0.426\lambda$	59.960
4th director, $l_6' = 0.434\lambda$	61.086
Reflector-driven spacing, $s_{12} = 0.2\lambda$	28.1502
Reflector-driven spacing, $s_{ij} = 0.25\lambda$	35.1878

b) Model **unmatched design** using NEC-2

NEC-2 Input file

```
CM yagi_6element_ch13.txt
CM THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 213 MHz W/ wavelength of 140.75 cm.
CM 6-element Yagi-Uda antenna dimensions:
CM element diameters: d = 0.9525 cm = 0.375in, radius a = 0.47625 cm
CM Reflector l1 = 67.983 cm
CM Driven element l2 = 64.534 cm
CM Directors l3 = l6 = 61.086 cm, and l4 = l5 = 59.960 cm
CM Reflector-Driven spacing S12 = 28.1502 cm
CM other element spacings Sij = 35.1878 cm
CM
CM Segment length approx. delta = 3.8 cm = 8a
CE
GW 1 17 -0.339915 0.0 0.0        0.339915 0.0 0.0       0.0047625 ! Reflector
l1
GW 2 17 -0.32267  0.0 0.281502  0.32267  0.0 0.281502 0.0047625 ! Driven l2
GW 3 16 -0.30543  0.0 0.63338   0.30543  0.0 0.63338  0.0047625 ! Director l3
GW 4 15 -0.2998   0.0 0.985258  0.2998   0.0 0.985258 0.0047625 ! Director l4
GW 5 15 -0.2998   0.0 1.337136  0.2998   0.0 1.337136 0.0047625 ! Director l5
GW 6 16 -0.30543  0.0 1.689014  0.30543  0.0 1.689014 0.0047625 ! Director l6
GE 0    ! free space
EK 0    ! use extended kernel for better accuracy
PT -1   ! No currents
FR 0 1 0 0 213.0 0 ! center freq of CH 13
EX 0 2 9 0 1.0 0.0 ! center segment of driven element
RP 0 2 2 0000 0.0 0.0 180.0 90.0 ! Main beam and backlobe directivities
EN
```

NEC-2 Output file excerpts

```
TAG SEG. VOLTAGE (V)    CURRENT (A)          IMPEDANCE (OHMS)      <snip>
NO. NO. REAL IMAG.  REAL       IMAG.        REAL      IMAG.       <snip>
2 26    1.0   0.0  3.12734E-02-3.15129E-02 1.58661E+01 1.59875E+01 <snip>
<snip>
       - - - RADIATION PATTERNS - - -
- - ANGLES - -    - POWER GAINS -       - POLARIZATION - <snip>
THETA    PHI      MAJOR   MINOR  TOTAL   AXIAL     TILT  SENSE  <snip>
DEGREES  DEGREES   DB      DB      DB     RATIO     DEG.         <snip>
0.00     0.00     12.46 -999.99   12.46    0.0       0.0  LINEAR  <snip>
180.00   0.00     -2.39 -999.99   -2.39    0.0       0.0  LINEAR  <snip>
```

Table 2 Unmatched design six-element Yagi-Uda antenna for channel 13

Z_a (Ω)	Γ_{in}	VSWR	G_{max} (dBi)	G_{back} (dBi)	FB ratio (dB)
15.8661 + j15.9875	0.7322∠161.4°	6.468	12.46	-2.38	14.85

c) Match the antenna.

Try 1: T-Match Design choices

Driven element length: l_2' = 63 cm (shorten from 64.534 cm as Z_a was inductive)

T-Match diameter: $2a'$ = 1/8" = 0.3175 cm (choose smaller than $2a$ to get $\alpha > 1$)

T-Match length: l' = 12 cm (choose less than $l_2'/4$)

T-Match spacing: s = 3 cm (choose less than $s_{12}/4$ and so $Z_0 \sim 300\ \Omega$)

From MathCad $Z_0 = 284.86\ \Omega$, eff. radius of T-Match $a_e = 0.88664$ cm, $Z_t = j78.176\ \Omega$

NEC results $Z_a = 12.8849 - j2.53918\ \Omega$, $G_{max} = 13$ dBi, & $G_{back} = -1.79$ dBi

From MathCad- $Z_{in} = 78.436 + j29.43\ \Omega$, $|\Gamma| = 0.202$, & VSWR = 1.505 (too high)

Comments: Z_{in} has inductive reactance. On second try, make l_2' a bit shorter to make Z_{in} more capacitive and, per suggestion from MathCad, lengthen T-match l'.

Input NEC file:

```
CM yagi_6element_ch13_tmatch_try1.txt
CM THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 213 MHz W/ wavelength of 140.75 cm.
CM 6-element Yagi-Uda antenna dimensions:
CM element diameters: d=0.9525 cm = 0.375in, radius a = 0.47625 cm
CM equiv. radius T-Match portion of driven element ae = 0.88664 cm
CM which has a length of l'= 12 cm
CM Reflector l1 = 67.983 cm
CM Driven element l2 = 63 cm
CM Directors l3 = l6 = 61.086 cm, and l4 = l5 = 59.960 cm
CM Reflector-Driven spacing s12 = 28.1502 cm
CM other element spacings sij = 35.1878 cm
CM Segment length approx. delta = 3.8 cm = 8a
CE
GW 1 17 -0.339915 0.0 0.0       0.339915 0.0 0.0      0.0047625 ! Refl l1
GW 2  7 -0.315    0.0 0.281502 -0.06    0.0 0.281502 0.0047625 ! Drive end l2
GW 3  3 -0.06     0.0 0.281502  0.06    0.0 0.281502 0.0088664 ! Drive mid l2
GW 4  7  0.06     0.0 0.281502  0.315   0.0 0.281502 0.0047625 ! Drive end l2
GW 5 16 -0.30543  0.0 0.63338   0.30543  0.0 0.63338  0.0047625 ! Director l3
GW 6 15 -0.2998   0.0 0.985258  0.2998   0.0 0.985258 0.0047625 ! Director l4
GW 7 15 -0.2998   0.0 1.337136  0.2998   0.0 1.337136 0.0047625 ! Director l5
GW 8 16 -0.30543  0.0 1.689014  0.30543  0.0 1.689014 0.0047625 ! Director l6
GE 0    ! free space
EK 0    ! use extended kernel for better accuracy
PT -1   ! No currents
FR 0 1 0 0 213.0 0 ! center freq of CH 13
EX 0 3 2 0 1.0 0.0 ! center of l2
RP 0 2 2 0000 0.0 0.0 180.0 90.0 ! Main beam and backlobe directivities
EN
```

MatchCad spreadsheet:

T-Match equations- Try 1

$c := 2.998 \cdot 10^{8}$ m/s $\quad fc := 213 \cdot 10^{6}$ Hz $\quad \lambda := \frac{c}{fc}$ $\lambda = 1.40751$ m

$k := \frac{2 \cdot \pi}{\lambda}$ $\quad k = 4.46404$ rad/m $\quad Zdesired := 100$ Ω

$d := 0.9525 \cdot 10^{-2}$ $\quad a := d \cdot 0.5$ $\quad a = 0.0047625$ m

$dprime := 0.3175 \cdot 10^{-2}$ $\quad aprime := dprime \cdot 0.5$ $\quad aprime = 0.0015875$ m

$s := 3 \cdot 10^{-2}$ m $\quad lprime := 12.0 \cdot 10^{-2}$ m

$$Z0 := \frac{376.73}{2 \cdot \pi} \cdot \operatorname{acosh}\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right] \qquad Z0 = 284.862 \ \Omega$$

$u := \frac{a}{aprime}$ $\quad u = 3$ $\quad v := \frac{s}{aprime}$ $\quad v = 18.89764$

$$\alpha := \frac{\operatorname{acosh}\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{\operatorname{acosh}\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]} \qquad ae := aprime \cdot e^{\frac{1}{(1+u)^2} \cdot \left(u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)\right)}$$

$\alpha = 1.5855$ $\quad ae = 0.0088664$ m

$$Zt := j \cdot Z0 \cdot \tan\left(\frac{k \cdot lprime}{2}\right) \qquad Zt = 78.17645i \ \Omega$$

$Yt := \frac{1}{Zt}$ $\quad Yt = -0.013i$ S $\quad \frac{Yt}{2} = -6.396i \times 10^{-3}$ S

Za from NEC (a MoM program)- Try 1

$Za := 12.8849 - j \cdot 2.53918$ Ω $\quad Ya := \frac{1}{Za}$ $\quad Ya = 0.07471 + 0.01472i$ S

$$Yin := \frac{Yt}{2} + \frac{Ya}{(1+\alpha)^2} \qquad Yin = 0.011 - 4.193i \times 10^{-3} \ S$$

$Zin := \frac{1}{Yin}$ $\quad Zin = 78.436 + 29.43i$ Ω $\quad Zdesired = 100$ Ω

$\Gamma := \frac{(Zin - Zdesired)}{Zin + Zdesired}$ $\quad |\Gamma| = 0.202$ $\quad VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$ $\quad VSWR = 1.505$

$$lsuggested := \frac{2}{k} \cdot \operatorname{atan}\left[\frac{1}{2 \cdot Z0 \cdot \operatorname{Im}\left[\frac{Ya}{(1+\alpha)^2}\right]}\right]$$

$lsuggested = 0.30147$ m

$lsuggested \cdot 0.5 = 0.151$ m

Try 2: T-Match Design choices:

Driven element length: l_2' = 62.5 cm (shorten from 63 cm as Z_{in} was inductive)

Same T-Match diameter: $2a'$ = 1/8" = 0.3175 cm

T-Match length: l' = 15 cm (lengthen from 12 cm per suggestion from MathCad)

Same T-Match spacing: s = 3 cm

From MathCad- Z_0 = 284.86 Ω (same), a_e = 0.88664 cm (same), $Z_t = j99.1034$ Ω

NEC results- $Z_a = 12.6259 - j6.91473$ Ω, G_{max} = 12.96 dBi, & G_{back} = -1.80 dBi

From MathCad- $Z_{in} = 109.713 + j0.646$ Ω, $|\Gamma| = 0.0464$, & VSWR = 1.097 (DONE!)

Comments: Real part of Z_{in} is a bit high, but we have met the specification.

Input NEC file:

```
CM yagi_6element_ch13_tmatch_try2.txt
CM THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 213 MHz W/ wavelength of 140.75 cm.
CM 6-element Yagi-Uda antenna dimensions:
CM element diameters: d=0.9525 cm = 0.375in, radius a = 0.47625 cm
CM equiv. radius T-Match portion of driven element ae = 0.88664 cm
CM which has a length of l'= 15 cm (new)
CM Reflector l1 = 67.983 cm
CM Driven element l2 = 62.5 cm (new)
CM Directors l3 = l6 = 61.086 cm, and l4 = l5 = 59.960 cm
CM Reflector-Driven spacing s12 = 28.1502 cm
CM other element spacings sij = 35.1878 cm
CM Segment length approx. delta = 3.8 cm = 8a
CE
GW 1 17 -0.339915 0.0 0.0       0.339915 0.0 0.0     0.0047625 ! Refl l1
GW 2  6 -0.3125   0.0 0.281502 -0.075   0.0 0.281502 0.0047625 ! Drive end l2
GW 3  5 -0.075    0.0 0.281502  0.075   0.0 0.281502 0.0088664 ! Drive mid l2
GW 4  6  0.075    0.0 0.281502  0.3125  0.0 0.281502 0.0047625 ! Drive end l2
GW 5 16 -0.30543  0.0 0.63338   0.30543  0.0 0.63338  0.0047625 ! Director l3
GW 6 15 -0.2998   0.0 0.985258  0.2998   0.0 0.985258 0.0047625 ! Director l4
GW 7 15 -0.2998   0.0 1.337136  0.2998   0.0 1.337136 0.0047625 ! Director l5
GW 8 16 -0.30543  0.0 1.689014  0.30543  0.0 1.689014 0.0047625 ! Director l6
GE 0    ! free space
EK 0    ! use extended kernel for better accuracy
PT -1   ! No currents
FR 0 1 0 0 213.0 0 ! center freq of CH 13
EX 0 3 3 0 1.0 0.0 ! center segment of l2
RP 0 2 2 0000 0.0 0.0 180.0 90.0 ! Main beam and backlobe directivities
EN
```

MatchCad spreadsheet:

T-Match equations- Try 2

$c := 2.998 \cdot 10^{8}$ m/s $\quad fc := 213 \cdot 10^{6}$ Hz $\quad \lambda := \frac{c}{fc}$ $\quad \lambda = 1.40751$ m

$k := \frac{2 \cdot \pi}{\lambda}$ $\quad k = 4.46404$ rad/m $\quad Zdesired := 100$ Ω

$d := 0.9525 \cdot 10^{-2}$ $\quad a := d \cdot 0.5$ $\quad a = 0.0047625$ m

$dprime := 0.3175 \cdot 10^{-2}$ $\quad aprime := dprime \cdot 0.5$ $\quad aprime = 0.0015875$ m

$s := 3 \cdot 10^{-2}$ m $\quad lprime := 15.0 \cdot 10^{-2}$ m

$$Z0 := \frac{376.73}{2 \cdot \pi} \cdot \operatorname{acosh}\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right] \qquad Z0 = 284.862 \ \Omega$$

$u := \frac{a}{aprime}$ $\quad u = 3$ $\quad v := \frac{s}{aprime}$ $\quad v = 18.89764$

$$\alpha := \frac{\operatorname{acosh}\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{\operatorname{acosh}\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]} \qquad ae := aprime \cdot e^{\frac{1}{(1+u)^2} \cdot \left(u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)\right)}$$

$\alpha = 1.5855$ $\quad ae = 0.0088664$ m

$$Zt := j \cdot Z0 \cdot \tan\left(\frac{k \cdot lprime}{2}\right) \qquad Zt = 99.1034i \ \Omega$$

$Yt := \frac{1}{Zt}$ $\quad Yt = -0.01i$ S $\quad \frac{Yt}{2} = -5.045i \times 10^{-3}$ S

Za from NEC (a MoM program)- Try 1

$Za := 12.6259 - j \cdot 6.91473$ Ω $\quad Ya := \frac{1}{Za}$ $\quad Ya = 0.06093 + 0.03337i$ S

$$Yin := \frac{Yt}{2} + \frac{Ya}{(1+\alpha)^2} \qquad Yin = 9.114 \times 10^{-3} - 5.364i \times 10^{-3} \ S$$

$Zin := \frac{1}{Yin}$ $\quad Zin = 109.713 + 0.646i$ Ω $\quad Zdesired = 100$ Ω

$\Gamma := \frac{(Zin - Zdesired)}{Zin + Zdesired}$ $\quad |\Gamma| = 0.0464$ $\quad VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$ $\quad VSWR = 1.097$

$$lsuggested := \frac{2}{k} \cdot \operatorname{atan}\left[\frac{1}{2 \cdot Z0 \cdot \operatorname{Im}\left[\frac{Ya}{(1+\alpha)^2}\right]}\right]$$

$lsuggested = 0.15149$ m

$lsuggested \cdot 0.5 = 0.076$ m

d) In a **table**, summarize the original (unmatched) and final (matched)

Table 2 Unmatched vs. matched designs for six-element Yagi-Uda antenna for channel 13

Parameter	Unmatched	Matched
Z_a or Z_{in} (Ω)	$15.8661 + j\,15.9875$	$109.713 + j\,0.646$
Γ_{in}	$0.7322\angle 161.4°$	$0.0464\angle 3.627°$
VSWR	6.468	1.097
G_{max} (dBi)	12.46	12.96
G_{back} (dBi)	-2.38	-1.80
FB ratio (dB)	14.85	14.76

Comments- Obviously, there was a huge improvement in impedance matching using the T-Match. Main beam gain increased slightly (0.5 dB). Back lobe increased slightly (0.58 dB). FB ratio is very slightly worse (0.09 dB).

e) Accurately sketch final antenna design with T-match (no boom).

6 element, channel 13 Yagi-Uda antenna with T-match and without boom

Dimensions: $s_{12} = 0.2\lambda = 28.1502\,\text{cm}$, $s_{ij} = 0.25\lambda = 35.1878\,\text{cm}$, $d = 3/8" = 0.9525\,\text{cm}$,
T-Match- $d' = 1/8" = 0.3175\,\text{cm}$, $s = 3\,\text{cm}$, and $l' = 15\,\text{cm}$