

Match the channel 10 six-element Yagi-Uda antenna from the previous assignment with the boom omitted to a **100 Ω** twin-lead transmission line using a **T-match** so that the VSWR is less than 1.1 at the center frequency. At each step, discuss and justify design changes/choices. Accurately sketch final design. In a table, summarize the original (unmatched) and final Yagi-Uda input impedance, input reflection coefficient (polar format), VSWR, gain (dBi), and front-to-back ratio (dB). Comment on how the final design compares with the original.

- Use NEC-2 to find the antenna-mode input impedance(s). Include the input file(s) and relevant excerpts of the output file(s). Assume $c = 2.998 \times 10^8$ m/s.

Design a six-element Yagi-Uda antenna for VHF television channel 13 using a ~~copper pipe boom~~ <snip> and brass elements with an outer diameter of 3/8 inch. <snip>

Design Summary:

Directivity of a six-element Yagi-Uda antenna is $10.2 \text{ dBd} = 10.2 + 2.15 \Rightarrow$ 12.35 dBi
(Note: For a Yagi-Uda antenna, gain \approx directivity.)

Design Frequency- Channel 10 (192-198 MHz) \Rightarrow $f_c = 195 \text{ MHz}$.

Desired input impedance for T-match \Rightarrow $R_0 = 100 \Omega$

Element diameter (use brass pipe $\sigma_{\text{brass}} = 1.1 \times 10^8 \text{ S/m}$) \Rightarrow $d = 2a = 3/8'' = 0.9525 \text{ cm}$

Design wavelength λ , s_{12} (reflector-driven element spacing) & s_{ij} [driven-director & director-director spacing(s)] using Table 10.6 values.

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8}{195 \times 10^6} = 1.5374359 \text{ m} \Rightarrow \underline{\lambda = 153.7436 \text{ cm}}$$

$$\text{reflector-driven element spacing } \underline{s_{12} = 0.2\lambda = 30.7487 \text{ cm}}$$

$$\text{driven-director \& director-director spacing(s)} \underline{s_{ij} = 0.25\lambda = 38.4359 \text{ cm}}$$

Corrected (only for element diameter) lengths-

$$\text{reflector } l_1' = 0.484\lambda \Rightarrow \underline{l_1' = 74.4119 \text{ cm}}$$

$$\text{directors } l_3' = l_6' = 0.436\lambda \Rightarrow \underline{l_3' = l_6' = 67.0322 \text{ cm}}$$

$$\text{directors } l_4' = l_5' = 0.427\lambda \Rightarrow \underline{l_4' = l_5' = 65.6485 \text{ cm}}$$

$$\text{Choose driven element } l_2' \approx (l_1' + l_3')/2 = 0.46\lambda \Rightarrow \underline{l_2' = 70.722 \text{ cm}}$$

No Match**Input NEC File**

```

CM yagi_6element_ch10.txt
CM THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.
CM
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 195 MHz W/ wavelength of 153.7436 cm.
CM
CM Brass 6-element Yagi-Uda antenna dimensions:
CM element diameters d = 3/8" = 0.9525 cm = 0.375in, radius a = 0.47625 cm
CM Reflector l1' = 74.4119 cm & Driven element l2' = 70.722 cm
CM Directors l3' = l6' = 67.0322 cm & l4' = l5' = 65.6485 cm
CM Reflector-Driven spacing S12 = 30.7487 cm
CM other element spacings Sij = 38.4359 cm
CM Place antenna on the y-z plane.
CM Choose segment length approx. delta ~ 3.8 cm = 8a
CE
GW 1 21 0.0 -0.37206 0.0 0.0 0.37206 0.0 0.0047625 ! Reflector l1
GW 2 19 0.0 -0.35361 0.307487 0.0 0.35361 0.307487 0.0047625 ! Driven l2
GW 3 17 0.0 -0.335161 0.691846 0.0 0.335161 0.691846 0.0047625 ! Director l3
GW 4 17 0.0 -0.3282425 1.076205 0.0 0.3282425 1.076205 0.0047625 ! Director l4
GW 5 17 0.0 -0.3282425 1.460564 0.0 0.3282425 1.460564 0.0047625 ! Director l5
GW 6 17 0.0 -0.335161 1.844923 0.0 0.335161 1.844923 0.0047625 ! Director l6
GE 0 0
PT -1 ! suppress current outputs
EK 0 ! Use extended kernel to be safe
LD 5 0 0 0 1.1e7 ! Set conductivity of brass elements
FR 0 1 0 0 195.0 0
EX 0 2 10 0 1.0 0.0 ! Excite middle segment of driven element
RP 0 2 2 0000 0.0 0.0 180.0 90.0
EN

```

No Match NEC results

$$Z_a = 15.8053 + j17.2159 \Omega \Rightarrow \Gamma_{in} = 0.734 \angle 160^\circ \text{ \& \text{ VSWR} = 6.52 \text{ (wrt } 100 \Omega)}$$

$$G_{max} = 12.40 \text{ dBi}$$

$$G_{back} = -1.98 \text{ dBi}$$

$$\text{F/B ratio} = 14.38 \text{ dB}$$

Note: Z_a is resistive-inductive, will want to shorten l_2' to make it more resistive-capacitive.

Try 1: T-Match Design choices

Shorten driven element length from $l_2' = 70.722$ cm to $l_2' = 69.0$ cm

Select a T-Match diameter smaller than elements (3/8"): $2a' = 1/4" = 0.635$ cm

Select a T-Match length roughly a bit less than $l_2'/3$: $l' = 20$ cm

Select a T-Match spacing so that $Z_{0,t-match} < 300 \Omega$: $s = 3$ cm

From MathCad, the effective radius of T-Match section is $a_e = 1.07974$ cm.

Input NEC File- Try 1

```

CM yagi_6element_ch13_tmatach_try1.txt
CM THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.
CM
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 195 MHz W/ wavelength of 153.7436 cm.
CM
CM Brass 6-element Yagi-Uda antenna dimensions:
CM element diameters: d=0.9525 cm = 0.375in, radius a = 0.47625 cm
CM equiv. radius T-Match portion of driven element ae = 1.07974 cm
CM T-match has a length of l'= 20 cm
CM Reflector l1' = 74.4119 cm & Driven element l2' = 69 cm (new)
CM Directors l3' = l6' = 67.0322 cm & l4' = l5' = 65.6485 cm
CM Reflector-Driven spacing S12 = 30.7487 cm
CM other element spacings Sij = 38.4359 cm
CM Place antenna on the y-z plane.
CM Segment length approx. delta ~ 3.8 cm = 8a
CE
GW 1 21 0.0 -0.37206 0.0 0.0 0.37206 0.0 0.0047625 ! Reflector l1
GW 2 6 0.0 -0.345 0.307487 0.0 -0.1 0.307487 0.0047625 ! Driven end 1 l2
GW 3 7 0.0 -0.1 0.307487 0.0 0.1 0.307487 0.0107974 ! Driven middle l2
GW 4 6 0.0 0.1 0.307487 0.0 0.345 0.307487 0.0047625 ! Driven end 2 l2
GW 5 17 0.0 -0.335161 0.691846 0.0 0.335161 0.691846 0.0047625 ! Director l3
GW 6 17 0.0 -0.3282425 1.076205 0.0 0.3282425 1.076205 0.0047625 ! Director l4
GW 7 17 0.0 -0.3282425 1.460564 0.0 0.3282425 1.460564 0.0047625 ! Director l5
GW 8 17 0.0 -0.335161 1.844923 0.0 0.335161 1.844923 0.0047625 ! Director l6
GE 0 0
PT -1 ! suppress current outputs
EK 0 ! Use extended kernel to be safe
LD 5 0 0 0 1.1e7 ! Set conductivity of brass elements
FR 0 1 0 0 195.0 0
EX 0 3 4 0 1.0 0.0 ! Drive middle element of tag #3
RP 0 2 2 0000 0.0 0.0 180.0 90.0
EN

```

Try 1 NEC results- $Z_a = 12.688 - j2.00747 \Omega$, $G_{max} = 12.96$ dBi, & $G_{back} = -1.36$ dBi

From MathCad- $Z_{in} = 62.434 + j9.05 \Omega$, $|\Gamma| = 0.238$, & $VSWR = 1.623$ (too high)

Comments: On second try, make l_2' shorter (more capacitive) & l' longer (per suggestion) to make Z_{in} have less inductive reactance.

MatchCad spreadsheet:**T-Match equations- Try 1**

$$c := 2.998 \cdot 10^8 \quad \text{m/s} \quad fc := 195 \cdot 10^6 \quad \text{Hz} \quad \lambda := \frac{c}{fc} \quad \lambda = 1.53744 \quad \text{m}$$

$$k := \frac{2 \cdot \pi}{\lambda} \quad k = 4.08679 \quad \text{rad/m} \quad Z_{\text{desired}} := 100 \quad \Omega$$

$$d := 0.9525 \cdot 10^{-2} \quad a := d \cdot 0.5 \quad a = 0.0047625 \quad \text{m}$$

$$d_{\text{prime}} := 0.635 \cdot 10^{-2} \quad a_{\text{prime}} := d_{\text{prime}} \cdot 0.5 \quad \boxed{a_{\text{prime}} = 0.003175} \quad \text{m}$$

$$\boxed{s := 3 \cdot 10^{-2}} \quad \text{m} \quad \boxed{l_{\text{prime}} := 20.0 \cdot 10^{-2}} \quad \text{m}$$

$$Z_0 := \frac{376.73}{2 \cdot \pi} \cdot \text{acosh} \left[\frac{(s^2 - a^2 - a_{\text{prime}}^2)}{2 \cdot a \cdot a_{\text{prime}}} \right] \quad Z_0 = 242.767 \quad \Omega$$

$$u := \frac{a}{a_{\text{prime}}} \quad u = 1.5 \quad v := \frac{s}{a_{\text{prime}}} \quad v = 9.44882$$

$$\alpha := \frac{\text{acosh} \left[\frac{(v^2 - u^2 + 1)}{2 \cdot v} \right]}{\text{acosh} \left[\frac{(v^2 + u^2 - 1)}{2 \cdot v \cdot u} \right]} \quad ae := a_{\text{prime}} \cdot e^{\frac{1}{(1+u)^2} \cdot (u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v))}$$

$$\alpha = 1.2139 \quad ae = 0.0107974 \quad \text{m}$$

$$Z_t := j \cdot Z_0 \cdot \tan \left(\frac{k \cdot l_{\text{prime}}}{2} \right) \quad Z_t = 105.1334i \quad \Omega$$

$$Y_t := \frac{1}{Z_t} \quad Y_t = -9.512i \times 10^{-3} \quad \text{S} \quad \frac{Y_t}{2} = -4.756i \times 10^{-3} \quad \text{S}$$

Za from NEC (a MoM program)- Try 1

$$Z_a := 12.688 - j \cdot 2.00747 \quad \Omega \quad Y_a := \frac{1}{Z_a} \quad Y_a = 0.07689 + 0.01217i \quad \text{S}$$

$$Y_{\text{in}} := \frac{Y_t}{2} + \frac{Y_a}{(1 + \alpha)^2} \quad Y_{\text{in}} = 0.016 - 2.274i \times 10^{-3} \quad \text{S}$$

$$Z_{\text{in}} := \frac{1}{Y_{\text{in}}} \quad \boxed{Z_{\text{in}} = 62.434 + 9.05i} \quad \Omega \quad Z_{\text{desired}} = 100 \quad \Omega$$

$$\Gamma := \frac{(Z_{\text{in}} - Z_{\text{desired}})}{Z_{\text{in}} + Z_{\text{desired}}} \quad |\Gamma| = 0.238 \quad \text{VSWR} := \frac{(1 + |\Gamma|)}{1 - |\Gamma|} \quad \boxed{\text{VSWR} = 1.623}$$

$$l_{\text{suggested}} := \frac{2}{k} \cdot \text{atan} \left[\frac{1}{2 \cdot Z_0 \cdot \text{Im} \left[\frac{Y_a}{(1 + \alpha)^2} \right]} \right]$$

$$l_{\text{suggested}} = 0.33897 \quad \text{m}$$

$$l_{\text{suggested}} \cdot 0.5 = 0.169 \quad \text{m}$$

Try 2: T-Match Design choices

Shorten driven element length from $l_2' = 69.0$ to $l_2' = \mathbf{68.8\text{ cm}}$

Keep T-Match diameter: $2a' = 1/4'' = \mathbf{0.635\text{ cm}}$ (no change)

Lengthen T-Match length from $l' = 20\text{ cm}$ to $l' = \mathbf{24\text{ cm}}$

Keep T-Match spacing: $s = \mathbf{3\text{ cm}}$ (no change)

From MathCad, the effective radius of T-Match section remains $a_e = 1.07974\text{ cm}$.

Input NEC File- Try 2

```

CM yagi_6element_ch13_tmatach_try2.txt
CM THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.
CM
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 195 MHz W/ wavelength of 153.7436 cm.
CM
CM Brass 6-element Yagi-Uda antenna dimensions:
CM element diameters: d=0.9525 cm = 0.375in, radius a = 0.47625 cm
CM equiv. radius T-Match portion of driven element ae = 1.07974 cm
CM T-match has a length of l' = 24 cm (longer)
CM Reflector l1' = 74.4119 cm & Driven element l2' = 68.8 cm (shorter)
CM Directors l3' = l6' = 67.0322 cm & l4' = l5' = 65.6485 cm
CM Reflector-Driven spacing S12 = 30.7487 cm
CM other element spacings Sij = 38.4359 cm
CM Place antenna on the y-z plane.
CM Segment length approx. delta ~ 3.8 cm = 8a
CE
GW 1 21 0.0 -0.37206 0.0 0.0 0.37206 0.0 0.0047625 ! Reflector l1
GW 2 6 0.0 -0.3440 0.307487 0.0 -0.12 0.307487 0.0047625 ! Driven end 1 l2
GW 3 7 0.0 -0.12 0.307487 0.0 0.12 0.307487 0.0107974 ! Driven middle l2
GW 4 6 0.0 0.12 0.307487 0.0 0.3440 0.307487 0.0047625 ! Driven end 2 l2
GW 5 17 0.0 -0.335161 0.691846 0.0 0.335161 0.691846 0.0047625 ! Director l3
GW 6 17 0.0 -0.3282425 1.076205 0.0 0.3282425 1.076205 0.0047625 ! Director l4
GW 7 17 0.0 -0.3282425 1.460564 0.0 0.3282425 1.460564 0.0047625 ! Director l5
GW 8 17 0.0 -0.335161 1.844923 0.0 0.335161 1.844923 0.0047625 ! Director l6
GE 0 0
PT -1 ! suppress current outputs
EK 0 ! Use extended kernel to be safe
LD 5 0 0 0 1.1e7 ! Set conductivity of brass elements
FR 0 1 0 0 195.0 0
EX 0 3 4 0 1.0 0.0 ! Drive middle element of tag #3
RP 0 2 2 0000 0.0 0.0 180.0 90.0
EN

```

Try 2 NEC results- $Z_a = 12.8283 - j3.99679\ \Omega$, $G_{\max} = 12.85\text{ dBi}$, & $G_{\text{back}} = -1.45\text{ dBi}$

From MathCad- $Z_{\text{in}} = \mathbf{68.837 - j3.13\ \Omega}$, $|\Gamma| = \mathbf{0.185}$, & $\text{VSWR} = \mathbf{1.455}$ (too high)

Comments: Z_{in} is still too small & a bit capacitive. On third try, decrease $2a'$ and s to make α bigger.

MatchCad spreadsheet:**T-Match equations- Try 2**

$$c := 2.998 \cdot 10^8 \quad \text{m/s} \quad fc := 195 \cdot 10^6 \quad \text{Hz} \quad \lambda := \frac{c}{fc} \quad \lambda = 1.53744 \quad \text{m}$$

$$k := \frac{2 \cdot \pi}{\lambda} \quad k = 4.08679 \quad \text{rad/m} \quad Z_{\text{desired}} := 100 \quad \Omega$$

$$d := 0.9525 \cdot 10^{-2} \quad a := d \cdot 0.5 \quad a = 0.0047625 \quad \text{m}$$

$$d_{\text{prime}} := 0.635 \cdot 10^{-2} \quad a_{\text{prime}} := d_{\text{prime}} \cdot 0.5 \quad \boxed{a_{\text{prime}} = 0.003175} \quad \text{m}$$

$$\boxed{s := 3 \cdot 10^{-2}} \quad \text{m} \quad \boxed{l_{\text{prime}} := 24 \cdot 10^{-2}} \quad \text{m}$$

$$Z_0 := \frac{376.73}{2 \cdot \pi} \cdot \text{acosh} \left[\frac{(s^2 - a^2 - a_{\text{prime}}^2)}{2 \cdot a \cdot a_{\text{prime}}} \right] \quad Z_0 = 242.767 \quad \Omega$$

$$u := \frac{a}{a_{\text{prime}}} \quad u = 1.5 \quad v := \frac{s}{a_{\text{prime}}} \quad v = 9.44882$$

$$\alpha := \frac{\text{acosh} \left[\frac{(v^2 - u^2 + 1)}{2 \cdot v} \right]}{\text{acosh} \left[\frac{(v^2 + u^2 - 1)}{2 \cdot v \cdot u} \right]} \quad ae := a_{\text{prime}} \cdot e^{\frac{1}{(1+u)^2} \cdot (u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v))}$$

$$\alpha = 1.2139 \quad ae = 0.0107974 \quad \text{m}$$

$$Z_t := j \cdot Z_0 \cdot \tan \left(\frac{k \cdot l_{\text{prime}}}{2} \right) \quad Z_t = 129.61885i \quad \Omega$$

$$Y_t := \frac{1}{Z_t} \quad Y_t = -7.715i \times 10^{-3} \quad \text{S} \quad \frac{Y_t}{2} = -3.857i \times 10^{-3} \quad \text{S}$$

Za from NEC (a MoM program)- Try 2

$$Z_a := 12.8283 - j \cdot 3.99679 \quad \Omega \quad Y_a := \frac{1}{Z_a} \quad Y_a = 0.07106 + 0.02214i \quad \text{S}$$

$$Y_{\text{in}} := \frac{Y_t}{2} + \frac{Y_a}{(1 + \alpha)^2} \quad Y_{\text{in}} = 0.014 + 6.592i \times 10^{-4} \quad \text{S}$$

$$Z_{\text{in}} := \frac{1}{Y_{\text{in}}} \quad \boxed{Z_{\text{in}} = 68.837 - 3.13i} \quad \Omega \quad Z_{\text{desired}} = 100 \quad \Omega$$

$$\Gamma := \frac{(Z_{\text{in}} - Z_{\text{desired}})}{Z_{\text{in}} + Z_{\text{desired}}} \quad |\Gamma| = 0.185 \quad \text{VSWR} := \frac{(1 + |\Gamma|)}{1 - |\Gamma|} \quad \boxed{\text{VSWR} = 1.455}$$

$$l_{\text{suggested}} := \frac{2}{k} \cdot \text{atan} \left[\frac{1}{2 \cdot Z_0 \cdot \text{Im} \left[\frac{Y_a}{(1 + \alpha)^2} \right]} \right]$$

$$l_{\text{suggested}} = 0.20937 \quad \text{m}$$

$$l_{\text{suggested}} \cdot 0.5 = 0.105 \quad \text{m}$$

Try 3: T-Match Design choices

Keep driven element length $l_2' = 68.8$ cm (no change)

Shrink T-Match diameter from $2a' = 1/4'' = 0.635$ cm to $2a' = 3/16'' = 0.47625$ cm

Keep T-Match length: $l' = 24$ cm (no change)

Shrink T-Match spacing from $s = 3$ cm to $s = 2$ cm

From MathCad- the effective radius of T-Match section is now $a_e = 0.83438$ cm.

Input NEC File- Try 3

```

CM yagi_6element_ch13_tmatach_try3.txt
CM THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.
CM
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 195 MHz W/ wavelength of 153.7436 cm.
CM
CM Brass 6-element Yagi-Uda antenna dimensions:
CM element diameters: d=0.9525 cm = 0.375in, radius a = 0.47625 cm
CM equiv. radius T-Match portion of driven element ae = 0.83438 cm (new)
CM T-match has a length of l' = 24 cm
CM Reflector l1' = 74.4119 cm & Driven element l2' = 68.8 cm
CM Directors l3' = l6' = 67.0322 cm & l4' = l5' = 65.6485 cm
CM Reflector-Driven spacing S12 = 30.7487 cm
CM other element spacings Sij = 38.4359 cm
CM Place antenna on the y-z plane.
CM Segment length approx. delta ~ 3.8 cm = 8a
CE
GW 1 21 0.0 -0.37206 0.0 0.0 0.37206 0.0 0.0047625 ! Reflector l1
GW 2 6 0.0 -0.3440 0.307487 0.0 -0.12 0.307487 0.0047625 ! Driven end 1 l2
GW 3 7 0.0 -0.12 0.307487 0.0 0.12 0.307487 0.0083438 ! Driven middle l2
GW 4 6 0.0 0.12 0.307487 0.0 0.3440 0.307487 0.0047625 ! Driven end 2 l2
GW 5 17 0.0 -0.335161 0.691846 0.0 0.335161 0.691846 0.0047625 ! Director l3
GW 6 17 0.0 -0.3282425 1.076205 0.0 0.3282425 1.076205 0.0047625 ! Director l4
GW 7 17 0.0 -0.3282425 1.460564 0.0 0.3282425 1.460564 0.0047625 ! Director l5
GW 8 17 0.0 -0.335161 1.844923 0.0 0.335161 1.844923 0.0047625 ! Director l6
GE 0 0
PT -1 ! suppress current outputs
EK 0 ! Use extended kernel to be safe
LD 5 0 0 0 1.1e7 ! Set conductivity of brass elements
FR 0 1 0 0 195.0 0
EX 0 3 4 0 1.0 0.0 ! Drive middle element of tag #3
RP 0 2 2 0000 0.0 0.0 180.0 90.0
EN

```

Try 3 NEC results- $Z_a = 13.3119 - j4.41455 \Omega$, $G_{\max} = 12.7$ dBi, & $G_{\text{back}} = -1.60$ dBi

From MathCad- $Z_{\text{in}} = 88.8 + j6.017 \Omega$, $|\Gamma| = 0.067$, & $\text{VSWR} = 1.144$ (too high)

Comments: Z_{in} is better, but still a bit inductive and small. On fourth try, make l_2' a tiny bit shorter, l' a bit longer, and decrease s a tiny bit.

MatchCad spreadsheet:**T-Match equations- Try 3**

$$c := 2.998 \cdot 10^8 \quad \text{m/s} \quad fc := 195 \cdot 10^6 \quad \text{Hz} \quad \lambda := \frac{c}{fc} \quad \lambda = 1.53744 \quad \text{m}$$

$$k := \frac{2 \cdot \pi}{\lambda} \quad k = 4.08679 \quad \text{rad/m} \quad Z_{\text{desired}} := 100 \quad \Omega$$

$$d := 0.9525 \cdot 10^{-2} \quad a := d \cdot 0.5 \quad a = 0.0047625 \quad \text{m}$$

$$d_{\text{prime}} := 0.47625 \cdot 10^{-2} \quad a_{\text{prime}} := d_{\text{prime}} \cdot 0.5 \quad \boxed{a_{\text{prime}} = 0.0023813} \quad \text{m}$$

$$\boxed{s := 2 \cdot 10^{-2}} \quad \text{m} \quad \boxed{l_{\text{prime}} := 24 \cdot 10^{-2}} \quad \text{m}$$

$$Z_0 := \frac{376.73}{2 \cdot \pi} \cdot \text{acosh} \left[\frac{(s^2 - a^2 - a_{\text{prime}}^2)}{2 \cdot a \cdot a_{\text{prime}}} \right] \quad Z_0 = 209.172 \quad \Omega$$

$$u := \frac{a}{a_{\text{prime}}} \quad u = 2 \quad v := \frac{s}{a_{\text{prime}}} \quad v = 8.39895$$

$$\alpha := \frac{\text{acosh} \left[\frac{(v^2 - u^2 + 1)}{2 \cdot v} \right]}{\text{acosh} \left[\frac{(v^2 + u^2 - 1)}{2 \cdot v \cdot u} \right]} \quad ae := a_{\text{prime}} \cdot e^{\frac{1}{(1+u)^2} \cdot (u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v))}$$

$$\alpha = 1.45711 \quad ae = 0.0083438 \quad \text{m}$$

$$Z_t := j \cdot Z_0 \cdot \tan \left(\frac{k \cdot l_{\text{prime}}}{2} \right) \quad Z_t = 111.68152i \quad \Omega$$

$$Y_t := \frac{1}{Z_t} \quad Y_t = -8.954i \times 10^{-3} \quad \text{S} \quad \frac{Y_t}{2} = -4.477i \times 10^{-3} \quad \text{S}$$

Za from NEC (a MoM program)- Try 3

$$Z_a := 13.3119 - j \cdot 4.41455 \quad \Omega \quad Y_a := \frac{1}{Z_a} \quad Y_a = 0.06768 + 0.02244i \quad \text{S}$$

$$Y_{\text{in}} := \frac{Y_t}{2} + \frac{Y_a}{(1 + \alpha)^2} \quad Y_{\text{in}} = 0.011 - 7.596i \times 10^{-4} \quad \text{S}$$

$$Z_{\text{in}} := \frac{1}{Y_{\text{in}}} \quad \boxed{Z_{\text{in}} = 88.8 + 6.017i} \quad \Omega \quad Z_{\text{desired}} = 100 \quad \Omega$$

$$\Gamma := \frac{(Z_{\text{in}} - Z_{\text{desired}})}{Z_{\text{in}} + Z_{\text{desired}}} \quad |\Gamma| = 0.067 \quad \text{VSWR} := \frac{(1 + |\Gamma|)}{1 - |\Gamma|} \quad \boxed{\text{VSWR} = 1.144}$$

$$l_{\text{suggested}} := \frac{2}{k} \cdot \text{atan} \left[\frac{1}{2 \cdot Z_0 \cdot \text{Im} \left[\frac{Y_a}{(1 + \alpha)^2} \right]} \right]$$

$$l_{\text{suggested}} = 0.27966 \quad \text{m}$$

$$l_{\text{suggested}} \cdot 0.5 = 0.14 \quad \text{m}$$

Try 4: T-Match Design choices

Shorten driven element from $l_2' = 68.8$ cm to $l_2' = \underline{68.7}$ cm

Keep T-Match diameter: $2a' = 3/16'' = \underline{0.47625}$ cm (no change)

Lengthen T-Match: from $l' = 24$ cm to $l' = \underline{26}$ cm

Shrink T-Match spacing from $s = 2$ cm to $s = \underline{1.9}$ cm

From MathCad, the effective radius of T-Match section is now $a_e = \underline{0.81557}$ cm.

Input NEC File- Try 4

```

CM yagi_6element_ch13_tmatach_try4.txt
CM THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.
CM
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 195 MHz W/ wavelength of 153.7436 cm.
CM
CM Brass 6-element Yagi-Uda antenna dimensions:
CM element diameters: d=0.9525 cm = 0.375in, radius a = 0.47625 cm
CM equiv. radius T-Match portion of driven element ae = 0.81557 cm (new)
CM T-match has a length of l'= 26 cm (new)
CM Reflector l1' = 74.4119 cm & Driven element l2' = 68.7 cm (new)
CM Directors l3' = l6' = 67.0322 cm & l4' = l5' = 65.6485 cm
CM Reflector-Driven spacing S12 = 30.7487 cm
CM other element spacings Sij = 38.4359 cm
CM Place antenna on the y-z plane.
CM Segment length approx. delta ~ 3.8 cm = 8a
CE
GW 1 21 0.0 -0.37206 0.0 0.0 0.37206 0.0 0.0047625 ! Reflector l1
GW 2 6 0.0 -0.3435 0.307487 0.0 -0.13 0.307487 0.0047625 ! Driven end 1 l2
GW 3 7 0.0 -0.13 0.307487 0.0 0.13 0.307487 0.0081557 ! Driven middle l2
GW 4 6 0.0 0.13 0.307487 0.0 0.3435 0.307487 0.0047625 ! Driven end 2 l2
GW 5 17 0.0 -0.335161 0.691846 0.0 0.335161 0.691846 0.0047625 ! Director l3
GW 6 17 0.0 -0.3282425 1.076205 0.0 0.3282425 1.076205 0.0047625 ! Director l4
GW 7 17 0.0 -0.3282425 1.460564 0.0 0.3282425 1.460564 0.0047625 ! Director l5
GW 8 17 0.0 -0.335161 1.844923 0.0 0.335161 1.844923 0.0047625 ! Director l6
GE 0 0
PT -1 ! suppress current outputs
EK 0 ! Use extended kernel to be safe
LD 5 0 0 0 1.1e7 ! Set conductivity of brass elements
FR 0 1 0 0 195.0 0
EX 0 3 4 0 1.0 0.0 ! Drive middle element of tag #3
RP 0 2 2 0000 0.0 0.0 180.0 90.0
EN

```

Try 4 NEC results-

$Z_a = 13.3703 - j5.37718 \Omega$, $G_{\max} = 12.66$ dBi, $G_{\text{back}} = -1.63$ dBi, & F/B ratio = 14.29 dB

From MathCad- $Z_{\text{in}} = \underline{94.857 - j0.344 \Omega}$, $|\Gamma| = \underline{0.0265}$, & $\text{VSWR} = \underline{1.054 < 1.1}$ (DONE)

MatchCad spreadsheet:**T-Match equations- Try 4**

$$c := 2.998 \cdot 10^8 \quad \text{m/s} \quad fc := 195 \cdot 10^6 \quad \text{Hz} \quad \lambda := \frac{c}{fc} \quad \lambda = 1.53744 \quad \text{m}$$

$$k := \frac{2 \cdot \pi}{\lambda} \quad k = 4.08679 \quad \text{rad/m} \quad Z_{\text{desired}} := 100 \quad \Omega$$

$$d := 0.9525 \cdot 10^{-2} \quad a := d \cdot 0.5 \quad a = 0.0047625 \quad \text{m}$$

$$d_{\text{prime}} := 0.47625 \cdot 10^{-2} \quad a_{\text{prime}} := d_{\text{prime}} \cdot 0.5 \quad \boxed{a_{\text{prime}} = 0.0023813} \quad \text{m}$$

$$\boxed{s := 1.9 \cdot 10^{-2}} \quad \text{m} \quad \boxed{l_{\text{prime}} := 26 \cdot 10^{-2}} \quad \text{m}$$

$$Z_0 := \frac{376.73}{2 \cdot \pi} \cdot \text{acosh} \left[\frac{(s^2 - a^2 - a_{\text{prime}}^2)}{2 \cdot a \cdot a_{\text{prime}}} \right] \quad Z_0 = 202.511 \quad \Omega$$

$$u := \frac{a}{a_{\text{prime}}} \quad u = 2 \quad v := \frac{s}{a_{\text{prime}}} \quad v = 7.979$$

$$\alpha := \frac{\text{acosh} \left[\frac{(v^2 - u^2 + 1)}{2 \cdot v} \right]}{\text{acosh} \left[\frac{(v^2 + u^2 - 1)}{2 \cdot v \cdot u} \right]} \quad ae := a_{\text{prime}} \cdot e^{\frac{1}{(1+u)^2} \cdot (u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v))}$$

$$\alpha = 1.47122 \quad ae = 0.0081557 \quad \text{m}$$

$$Z_t := j \cdot Z_0 \cdot \tan \left(\frac{k \cdot l_{\text{prime}}}{2} \right) \quad Z_t = 119.00402i \quad \Omega$$

$$Y_t := \frac{1}{Z_t} \quad Y_t = -8.403i \times 10^{-3} \quad \text{S} \quad \frac{Y_t}{2} = -4.202i \times 10^{-3} \quad \text{S}$$

Za from NEC (a MoM program)- Try 4

$$Z_a := 13.3703 - j \cdot 5.37718 \quad \Omega \quad Y_a := \frac{1}{Z_a} \quad Y_a = 0.06438 + 0.02589i \quad \text{S}$$

$$Y_{\text{in}} := \frac{Y_t}{2} + \frac{Y_a}{(1 + \alpha)^2} \quad Y_{\text{in}} = 0.011 + 3.821i \times 10^{-5} \quad \text{S}$$

$$Z_{\text{in}} := \frac{1}{Y_{\text{in}}} \quad \boxed{Z_{\text{in}} = 94.857 - 0.344i} \quad \Omega \quad Z_{\text{desired}} = 100 \quad \Omega$$

$$\Gamma := \frac{(Z_{\text{in}} - Z_{\text{desired}})}{Z_{\text{in}} + Z_{\text{desired}}} \quad |\Gamma| = 0.0265 \quad \text{VSWR} := \frac{(1 + |\Gamma|)}{1 - |\Gamma|} \quad \boxed{\text{VSWR} = 1.054}$$

$$l_{\text{suggested}} := \frac{2}{k} \cdot \text{atan} \left[\frac{1}{2 \cdot Z_0 \cdot \text{Im} \left[\frac{Y_a}{(1 + \alpha)^2} \right]} \right]$$

$$l_{\text{suggested}} = 0.25807 \quad \text{m}$$

$$l_{\text{suggested}} \cdot 0.5 = 0.129 \quad \text{m}$$

Parameter	Original/unmatched	w/ T-Match
Z_{in}	$15.8053 + j17.2159 \Omega$	$94.857 - j0.344 \Omega$
Γ_{in}	$0.734 \angle 160^\circ$	$0.0265 \angle -176^\circ$
VSWR	6.52	1.054
Gain (dBi)	12.40	12.66
F/B ratio (dB)	14.38	14.29

Comments: The T-Match resulted in a dramatic improvement in matching while giving a small 0.26 dB increase in gain and incurring a slight 0.09 dB decrease in front-to-back ratio. A gain of 12.35 dBi was expected for the 6-element design.

6 element, channel 10 Yagi-Uda antenna with T-match and without boom

Dimensions: $s_{12} = 0.2\lambda = 30.7487$ cm, $s_{ij} = 0.25\lambda = 38.4359$ cm, $d = 3/8'' = 0.9525$ cm, T-Match- $2a' = 3/16'' = 0.47625$ cm, $s = 1.9$ cm, and $l' = 26$ cm

