Here, you will match the five-element Yagi-Uda antenna for channel 7 from the previous assignment **with the boom omitted** to a **100 Ω** twin-lead transmission line using a **T-match** so that the VSWR is less than 1.1 at the center frequency f_c. Steps:

a) Tabulate the element lengths and spacings (in cm) for the unmatched design (make driven element length the simple average of the reflector and first director).

b) Model **unmatched design** using NEC-2. Determine and tabulate input impedance Z_{in}, input reflection coefficient Γ_{in} (polar format), VSWR, maximum gain G_{max} (dBi), backlobe gain G_{back} (dBi), and front-to-back/FB ratio (dB) at f_c.

c) Match the antenna. At each step, discuss, list, and justify design changes/choices as well as show results/work.

d) In a **table**, summarize the original (unmatched) and final (matched)- Z_{in}, Γ_{in}, VSWR, G_{max} (dBi), G_{back} (dBi), and FB ratio (dB) at f_c. Comment on how the final design compares with the original.

e) Accurately sketch final antenna design with T-match (no boom).

- Include the input NEC-2 file(s) and relevant excerpts of the output file(s). Assume $c = 2.998 \times 10^8$ m/s.

****

a) Design Summary:

Directivity of a five-element Yagi-Uda antenna is 9.2 dBd = 9.2 + 2.15 $\Rightarrow$ **11.35 dBi**

Design Frequency- Channel 7 (**174 - 180 MHz**), so ***f* = 177 MHz**.

Desired input impedance- ***R*₀ = 100 Ω** (for T-match)

Element diameter- ***d* = 13/32" = 1.031875 cm** (use brass pipe $\sigma_{brass} = 1.1\times10^8$ S/m)

$$\lambda = \frac{c}{f} = \frac{2.998\times10^8}{177\times10^6} \Rightarrow \boldsymbol{\lambda = 1.693785\ m = 169.3785\ cm}$$

Table 1 Element lengths and spacings for the unmatched design (no boom)

Description	(λ)	(cm)
Reflector, l_1'	0.484λ	81.98
Driven, l_2'	0.460λ	77.91
1st director, l_3'	0.436λ	73.85
2nd director, l_4'	0.431λ	73.00
3rd director, l_5'	0.436λ	73.85
All spacings, $s_{12} = s_{ij}$	0.2λ	33.88

b) Model **unmatched design** (no boom) using NEC-2

NEC-2 Input file

```
CM yagi_5element_ch7.txt
CM
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 177 MHz W/ wavelength of 169.3785 cm.
CM
CM Brass 5-element Yagi-Uda antenna (no boom) dimensions:
CM element diameters d = 13/32” = 1.031875 cm, radius a = 0.5159375 cm
CM Reflector l1' = 81.98 cm & Driven element l2' = 77.91 cm
CM Directors l3' = l5' = 73.85 cm  & l4' = 73 cm
CM all element spacings Sij = 33.88 cm
CM Place antenna on the y-z plane.
CM Choose segment length approx. delta ~ 3 cm ~ 5.8a
CE
GW 1 27 0.0 -0.4099  0.0      0.0 0.4099  0.0     0.005159375 ! Reflector l1
GW 2 25 0.0 -0.38955 0.3388   0.0 0.38955 0.3388  0.005159375 ! Driven l2
GW 3 25 0.0 -0.36925 0.6776   0.0 0.36925 0.6776  0.005159375 ! Director l3
GW 4 24 0.0 -0.365   1.0164   0.0 0.365   1.0164  0.005159375 ! Director l4
GW 5 25 0.0 -0.36925 1.3552   0.0 0.36925 1.3552  0.005159375 ! Director l5
GE 0 0 ! No ground plane
PT -1  ! suppress current outputs
EK 0   ! Use extended kernel
LD 5 0 0 0 1.1e7  ! Set conductivity of brass on all segments
FR 0 1 0 0 177.0 0
EX 0 2 13 0 1.0 0.0 ! Excite middle segment of driven element
RP 0 2 2 0000 0.0 0.0 180.0 90.0 ! Get main and backlobe gains
EN
```

NEC-2 Output file excerpts

```
- - - ANTENNA INPUT PARAMETERS - - -

TAG SEG. VOLTAGE (VOLTS) CURRENT (AMPS)        IMPEDANCE (OHMS)         <snip>
NO. NO.  REAL   IMAG.     REAL        IMAG.       REAL         IMAG.     <snip>
 2 40  1.0E+00 0.0E+00 3.00499E-02-2.86371E-02 1.74397E+01 1.66197E+01 <snip>
<snip>
EFFICIENCY    =  98.67 PERCENT

- - - RADIATION PATTERNS - - -
- - ANGLES - -           - POWER GAINS -    <snip>
 THETA     PHI        MAJOR  MINOR    TOTAL  <snip>
 DEGREES  DEGREES       DB     DB        DB    <snip>
  0.00     0.00      11.21  -999.99  11.21  <snip>
180.00     0.00      -1.02  -999.99  -1.02  <snip>
<snip>
```

Table 2 Unmatched design five-element Yagi-Uda antenna for channel 7

Z_{in} (Ω)	Γ_{in}	VSWR	G_{max} (dBi)	G_{back} (dBi)	FB ratio (dB)
17.4397 + *j*16.6197	0.7100∠160.56°	5.897	11.21	-1.02	12.23

c) Match the antenna.

Try 1: T-Match Design choices

Driven element length: l_2' = **76 cm** (shorten from 77.91 cm as Z_{in} was inductive)

T-Match diameter: $2a'$ = 3/16" = **0.47625 cm** (choose smaller than $2a$ to get $\alpha > 1$)

T-Match length: l' = **19.4 cm** (choose ~$l_2'/4$)

T-Match spacing: s = **3 cm** (choose less than $s_{12}/4$ and so $200 < Z_0 < 300\ \Omega$)

From MathCad $Z_0 = 255.256\ \Omega$, eff. radius of T-Match $a_e = 1.02208$ cm, $Z_t = j96.029\ \Omega$

NEC results $Z_a = 14.0204 - j1.59243\ \Omega$, $G_{max} = 11.76$ dBi, & $G_{back} = -0.45$ dBi

From MathCad- $Z_{in} = 75.809 + j24.417\ \Omega$, $|\Gamma| = 0.194$, & VSWR = 1.48 (too high)

Comments: Z_{in} has inductive reactance. On second try, make l_2' a bit shorter to make Z_{in} more capacitive and, per suggestion from MathCad, lengthen T-match l'.

Input NEC file:

```
CM yagi_5element_ch7_try1.txt
CM
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 177 MHz W/ wavelength of 169.3785 cm.
CM
CM Brass 5-element Yagi-Uda antenna (no boom) dimensions:
CM element diameters d = 13/32" = 1.031875 cm, radius a = 0.5159375 cm
CM Reflector l1' = 81.98 cm
CM Driven element l2' = 76 cm (shorten from 77.91 cm)
CM ae = 0.0102208 m, l' = 19.4 cm
CM Directors l3' = l5' = 73.85 cm  & l4' = 73 cm
CM all element spacings Sij = 33.88 cm
CM Place antenna on the y-z plane.
CM Choose segment length approx. delta ~ 3 cm ~ 5.8a
CE
GW 1  27 0.0 -0.4099  0.0     0.0 0.4099  0.0     0.005159375 ! Reflector l1
GW 21  9 0.0 -0.38    0.3388  0.0 -0.097  0.3388  0.005159375 ! Driven l2 tip
GW 2   7 0.0 -0.097   0.3388  0.0 0.097   0.3388  0.0102208   ! Driven l2 mid
GW 22  9 0.0  0.097   0.3388  0.0 0.38    0.3388  0.005159375 ! Driven l2 tip
GW 3  25 0.0 -0.36925 0.6776  0.0 0.36925 0.6776  0.005159375 ! Director l3
GW 4  24 0.0 -0.365   1.0164  0.0 0.365   1.0164  0.005159375 ! Director l4
GW 5  25 0.0 -0.36925 1.3552  0.0 0.36925 1.3552  0.005159375 ! Director l5
GE 0 0 ! No ground plane
PT -1  ! suppress current outputs
EK 0   ! Use extended kernel
LD 5 0 0 0 1.1e7  ! Set conductivity of brass on all segments
FR 0 1 0 0 177.0 0
EX 0 2 4 0 1.0 0.0 ! Excite middle segment of driven element Tag 2
RP 0 2 2 0000 0.0 0.0 180.0 90.0 ! Get main and backlobe gains
EN
```

MatchCad spreadsheet:

T-Match equations- Try 1

$c := 2.998 \cdot 10^8$ m/s $\quad fc := 177 \cdot 10^6$ Hz $\quad \lambda := \frac{c}{fc}$ $\quad \lambda = 1.69379$ m

$k := \frac{2 \cdot \pi}{\lambda}$ $\quad k = 3.70955$ rad/m $\quad Zdesired := 100$ Ω

$d := 1.031875 \cdot 10^{-2}$ $\quad a := d \cdot 0.5$ $\quad a = 0.0051594$ m

$dprime := 0.47625 \cdot 10^{-2}$ $\quad aprime := dprime \cdot 0.5$ $\quad \boxed{aprime = 0.0023813}$ m

$\boxed{s := 3 \cdot 10^{-2}}$ m $\quad \boxed{lprime := 19.4 \cdot 10^{-2}}$ m

$$Z0 := \frac{376.73}{2 \cdot \pi} \cdot \operatorname{acosh}\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right] \qquad Z0 = 255.256 \ \Omega$$

$u := \frac{a}{aprime}$ $\quad u = 2.167$ $\quad v := \frac{s}{aprime}$ $\quad v = 12.59843$

$$\alpha := \frac{\operatorname{acosh}\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{\operatorname{acosh}\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]} \qquad ae := aprime \cdot e^{\frac{1}{(1+u)^2} \cdot \left(u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)\right)}$$

$\alpha = 1.42733$ $\quad ae = 0.0102208$ m

$$Zt := j \cdot Z0 \cdot \tan\left(\frac{k \cdot lprime}{2}\right) \qquad Zt = 96.02858i \ \Omega$$

$Yt := \frac{1}{Zt}$ $\quad Yt = -0.01i$ S $\quad \frac{Yt}{2} = -5.207i \times 10^{-3}$ S

Za from NEC (a MoM program)- Try 1

$Za := 14.0204 - j \cdot 1.59243$ Ω $\quad Ya := \frac{1}{Za}$ $\quad Ya = 0.07042 + 0.008i$ S

$$Yin := \frac{Yt}{2} + \frac{Ya}{(1+\alpha)^2} \qquad Yin = 0.012 - 3.849i \times 10^{-3} \ S$$

$Zin := \frac{1}{Yin}$ $\quad \boxed{Zin = 75.809 + 24.417i}$ Ω $\quad Zdesired = 100$ Ω

$\Gamma := \frac{(Zin - Zdesired)}{Zin + Zdesired}$ $\quad |\Gamma| = 0.194$ $\quad VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$ $\quad \boxed{VSWR = 1.48}$

$$lsuggested := \frac{2}{k} \cdot \operatorname{atan}\left[\frac{1}{2 \cdot Z0 \cdot \operatorname{Im}\left[\frac{Ya}{(1+\alpha)^2}\right]}\right] \qquad lsuggested = 0.52017 \ m$$

Try 2: T-Match Design choices:

Driven element length: $l_2' =$ **75 cm** (shorten from 76 cm as Z_{in} was inductive)

Same T-Match diameter: $2a' = 3/16" =$ 0.47625 cm

T-Match length: $l' =$ **20 cm** (lengthen from 19.4 cm per suggestion from MathCad)

Same T-Match spacing: $s =$ 3 cm

From MathCad- $Z_0 = 255.256\ \Omega$ (same), $a_e = 1.02208$ cm (same), $Z_t = j99.28504\ \Omega$

NEC results- $Z_a = 13.4159 - j9.02507\ \Omega$, $G_{max} = 11.74$ dBi, & $G_{back} = -0.45$ dBi

From MathCad- $Z_{in} = 113.801 - j10.753\ \Omega$, $|\Gamma| = 0.082$, & VSWR = 1.178

Comments: Overshot a bit. Lengthen driven element a bit.

Input NEC file:

```
CM yagi_5element_ch7_try2.txt
CM
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 177 MHz W/ wavelength of 169.3785 cm.
CM
CM Brass 5-element Yagi-Uda antenna (no boom) dimensions:
CM element diameters d = 13/32” = 1.031875 cm, radius a = 0.5159375 cm
CM Reflector l1' = 81.98 cm
CM Driven element l2' = 75 cm (shorten from 76 cm)
CM ae = 0.0102208 m, l' = 20 cm (lengthen from 19.4 cm)
CM Directors l3' = l5' = 73.85 cm  & l4' = 73 cm
CM all element spacings Sij = 33.88 cm
CM Place antenna on the y-z plane.
CM Choose segment length approx. delta ~ 3 cm ~ 5.8a
CE
GW 1  27 0.0 -0.4099  0.0     0.0 0.4099  0.0     0.005159375 ! Reflector l1
GW 21  9 0.0 -0.375   0.3388  0.0 -0.1    0.3388  0.005159375 ! Driven l2 tip
GW 2   7 0.0 -0.1     0.3388  0.0 0.1     0.3388  0.0102208  ! Driven l2 mid
GW 22  9 0.0  0.1     0.3388  0.0 0.375   0.3388  0.005159375 ! Driven l2 tip
GW 3  25 0.0 -0.36925 0.6776  0.0 0.36925 0.6776  0.005159375 ! Director l3
GW 4  24 0.0 -0.365   1.0164  0.0 0.365   1.0164  0.005159375 ! Director l4
GW 5  25 0.0 -0.36925 1.3552  0.0 0.36925 1.3552  0.005159375 ! Director l5
GE 0 0 ! No ground plane
PT -1  ! suppress current outputs
EK 0   ! Use extended kernel
LD 5 0 0 0 1.1e7  ! Set conductivity of brass on all segments
FR 0 1 0 0 177.0 0
EX 0 2 4 0 1.0 0.0 ! Excite middle segment of driven element Tag 2
RP 0 2 2 0000 0.0 0.0 180.0 90.0 ! Get main and backlobe gains
EN
```

MatchCad spreadsheet:

T-Match equations- Try 2

$c := 2.998 \cdot 10^{8}$ m/s $\quad fc := 177 \cdot 10^{6}$ Hz $\quad \lambda := \frac{c}{fc}$ $\quad \lambda = 1.69379$ m

$k := \frac{2 \cdot \pi}{\lambda}$ $\quad k = 3.70955$ rad/m $\quad Zdesired := 100$ Ω

$d := 1.031875 \cdot 10^{-2}$ $\quad a := d \cdot 0.5$ $\quad a = 0.0051594$ m

$dprime := 0.47625 \cdot 10^{-2}$ $\quad aprime := dprime \cdot 0.5$ $\quad \boxed{aprime = 0.0023813}$ m

$\boxed{s := 3 \cdot 10^{-2}}$ m $\quad \boxed{lprime := 20 \cdot 10^{-2}}$ m

$$Z0 := \frac{376.73}{2 \cdot \pi} \cdot \operatorname{acosh}\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right] \qquad Z0 = 255.256 \ \Omega$$

$u := \frac{a}{aprime}$ $\quad u = 2.167$ $\quad v := \frac{s}{aprime}$ $\quad v = 12.59843$

$$\alpha := \frac{\operatorname{acosh}\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{\operatorname{acosh}\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]} \qquad ae := aprime \cdot e^{\frac{1}{(1+u)^2} \cdot \left(u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)\right)}$$

$\alpha = 1.42733$ $\quad ae = 0.0102208$ m

$$Zt := j \cdot Z0 \cdot \tan\left(\frac{k \cdot lprime}{2}\right) \qquad Zt = 99.28504i \ \Omega$$

$Yt := \frac{1}{Zt}$ $\quad Yt = -0.01i$ S $\quad \frac{Yt}{2} = -5.036i \times 10^{-3}$ S

Za from NEC (a MoM program)- Try 1

$Za := 13.4159 - j \cdot 9.02507$ Ω $\quad Ya := \frac{1}{Za}$ $\quad Ya = 0.05132 + 0.03452i$ S

$$Yin := \frac{Yt}{2} + \frac{Ya}{(1+\alpha)^2} \qquad Yin = 8.709 \times 10^{-3} + 8.23i \times 10^{-4} \text{S}$$

$Zin := \frac{1}{Yin}$ $\quad \boxed{Zin = 113.801 - 10.753i}$ Ω $\quad Zdesired = 100$ Ω

$\Gamma := \frac{(Zin - Zdesired)}{Zin + Zdesired}$ $\quad |\Gamma| = 0.082$ $\quad VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$ $\quad \boxed{VSWR = 1.178}$

$$lsuggested := \frac{2}{k} \cdot \operatorname{atan}\left[\frac{1}{2 \cdot Z0 \cdot \operatorname{Im}\left[\frac{Ya}{(1+\alpha)^2}\right]}\right] \qquad lsuggested = 0.17395 \ \text{m}$$

Try 3: T-Match Design choices:

Driven element length: $l_2' =$ **75.4 cm** (lengthen from 75 cm as Z_{in} was capacitive)

Same T-Match diameter: $2a' = 3/16" = 0.47625$ cm

Same T-Match length: $l' = 20$ cm (leave alone as we are very close)

Same T-Match spacing: $s = 3$ cm

From MathCad- $Z_0 = 255.256\ \Omega$ (same), $a_e = 1.02208$ cm (same), $Z_t = j99.285\ \Omega$ (same)

NEC results- $Z_a = 13.6706 - j6.08355\ \Omega$, $G_{max} = 11.74$ dBi, & $G_{back} = -0.46$ dBi

From MathCad- $Z_{in} = 96.336 + j3.945\ \Omega$, $|\Gamma| = 0.027$, & VSWR = **1.056 (DONE!)**

Comments: All done, met VSWR specification.

Input NEC file:

```
CM yagi_5element_ch7_try3.txt
CM
CM Determine the antenna mode input impedance of the driven element.
CM Center frequency is 177 MHz W/ wavelength of 169.3785 cm.
CM
CM Brass 5-element Yagi-Uda antenna (no boom) dimensions:
CM element diameters d = 13/32" = 1.031875 cm, radius a = 0.5159375 cm
CM Reflector l1' = 81.98 cm
CM Driven element l2' = 75.4 cm (lengthen from 75 cm)
CM ae = 0.0102208 m, l' = 20 cm
CM Directors l3' = l5' = 73.85 cm  & l4' = 73 cm
CM all element spacings Sij = 33.88 cm
CM Place antenna on the y-z plane.
CM Choose segment length approx. delta ~ 3 cm ~ 5.8a
CE
GW 1  27 0.0 -0.4099  0.0     0.0 0.4099  0.0     0.005159375 ! Reflector l1
GW 21  9 0.0 -0.377   0.3388  0.0 -0.1    0.3388  0.005159375 ! Driven l2 tip
GW 2   7 0.0 -0.1     0.3388  0.0 0.1     0.3388  0.0102208   ! Driven l2 mid
GW 22  9 0.0  0.1     0.3388  0.0 0.377   0.3388  0.005159375 ! Driven l2 tip
GW 3  25 0.0 -0.36925 0.6776  0.0 0.36925 0.6776  0.005159375 ! Director l3
GW 4  24 0.0 -0.365   1.0164  0.0 0.365   1.0164  0.005159375 ! Director l4
GW 5  25 0.0 -0.36925 1.3552  0.0 0.36925 1.3552  0.005159375 ! Director l5
GE 0 0 ! No ground plane
PT -1  ! suppress current outputs
EK 0   ! Use extended kernel
LD 5 0 0 0 1.1e7  ! Set conductivity of brass on all segments
FR 0 1 0 0 177.0 0
EX 0 2 4 0 1.0 0.0 ! Excite middle segment of driven element Tag 2
RP 0 2 2 0000 0.0 0.0 180.0 90.0 ! Get main and backlobe gains
EN
```

MatchCad spreadsheet:

T-Match equations- Try 3

$c := 2.998 \cdot 10^8$ m/s $fc := 177 \cdot 10^6$ Hz $\lambda := \frac{c}{fc}$ $\lambda = 1.69379$ m

$k := \frac{2 \cdot \pi}{\lambda}$ $k = 3.70955$ rad/m Zdesired := 100 Ω

$d := 1.031875 \cdot 10^{-2}$ $a := d \cdot 0.5$ $a = 0.0051594$ m

$dprime := 0.47625 \cdot 10^{-2}$ $aprime := dprime \cdot 0.5$ $aprime = 0.0023813$ m

$s := 3 \cdot 10^{-2}$ m $lprime := 20 \cdot 10^{-2}$ m

$$Z0 := \frac{376.73}{2 \cdot \pi} \cdot acosh\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2 \cdot a \cdot aprime}\right] \qquad Z0 = 255.256 \ \Omega$$

$u := \frac{a}{aprime}$ $u = 2.167$ $v := \frac{s}{aprime}$ $v = 12.59843$

$$\alpha := \frac{acosh\left[\frac{\left(v^2 - u^2 + 1\right)}{2 \cdot v}\right]}{acosh\left[\frac{\left(v^2 + u^2 - 1\right)}{2 \cdot v \cdot u}\right]} \qquad ae := aprime \cdot e^{\frac{1}{(1+u)^2} \cdot \left(u^2 \cdot \ln(u) + 2 \cdot u \cdot \ln(v)\right)}$$

$\alpha = 1.42733$ $ae = 0.0102208$ m

$$Zt := j \cdot Z0 \cdot \tan\left(\frac{k \cdot lprime}{2}\right) \qquad Zt = 99.28504i \ \Omega$$

$Yt := \frac{1}{Zt}$ $Yt = -0.01i$ S $\frac{Yt}{2} = -5.036i \times 10^{-3}$ S

Za from NEC (a MoM program)- Try 1

$Za := 13.6706 - j \cdot 6.08355$ Ω $Ya := \frac{1}{Za}$ $Ya = 0.06106 + 0.02717i$ S

$$Yin := \frac{Yt}{2} + \frac{Ya}{(1+\alpha)^2} \qquad Yin = 0.01 - 4.244i \times 10^{-4} \ S$$

$Zin := \frac{1}{Yin}$ $Zin = 96.336 + 3.945i$ Ω Zdesired = 100 Ω

$\Gamma := \frac{(Zin - Zdesired)}{Zin + Zdesired}$ $|\Gamma| = 0.027$ $VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$ $VSWR = 1.056$

$$lsuggested := \frac{2}{k} \cdot atan\left[\frac{1}{2 \cdot Z0 \cdot Im\left[\frac{Ya}{(1+\alpha)^2}\right]}\right] \qquad lsuggested = 0.21656 \ m$$

d) In a **table**, summarize the original (unmatched) and final (matched)

Table 2 Unmatched vs. matched designs for six-element Yagi-Uda antenna for channel 13

Parameter	Unmatched	Matched
Z_a or Z_{in} (Ω)	$17.4397 + j\,16.6197$	$96.336 + j3.945$
Γ_{in}	$0.7100\angle 160.56°$	$0.0274\angle 131.74°$
VSWR	5.897	1.056
G_{max} (dBi)	11.21	11.74
G_{back} (dBi)	-1.02	-0.46
FB ratio (dB)	12.23	12.20

Comments- Huge improvement in impedance matching using the T-Match. Main beam gain increased slightly (0.53 dB). Back lobe increased slightly (0.56 dB). FB ratio is very slightly worse (0.03 dB).

e) Accurately sketch final antenna design with T-match (no boom).