Match the **five**-element Yagi-Uda antenna (channel 13) from the prior assignment **with the boom omitted** to a **100 Ω** twin-lead transmission line using a **T-match** so that the VSWR < 1.1. Steps:

a) Tabulate the element spacings and lengths (in cm) for the **unmatched design** (make driven element length the simple average of the reflector and first director).

b) Model **unmatched design** using NEC-2. Find and tabulate input impedance Z_{in}, input reflection coefficient Γ_{in} (polar format w/ angle in deg), VSWR, maximum gain G_{max} (dBi), backlobe gain G_{back} (dBi), and front-to-back/FB ratio (dB) at f_c.

c) Match the antenna. At each step, discuss, list, and justify design changes/choices as well as show results/work.

d) In a **table**, summarize the original (unmatched) and final (matched)- Z_{in}, Γ_{in}, VSWR, G_{max} (dBi), G_{back} (dBi), and FB ratio (dB) at f_c. Comment on how the final design compares with the original.

e) Accurately sketch final antenna design with T-match (no boom).

- Use NEC-2 to find the antenna-mode input impedance(s). Include the input file(s) and relevant excerpts of output file(s). Place antenna on y-z plane with elements spaced & centered along the positive z-axis with the reflector at $z = 0$.

****

a) Design Summary:

Directivity of a 5 element Yagi-Uda antenna is $9.2\,\text{dBd} = 9.2 + 2.15 =$ **11.35 dBi**

Design Frequency- Channel 13 210-216 MHz , so f **= 213 MHz**.

Desired input impedance- R_0 **= 100 Ω** (for T-match)

Element diameter- d **= 5/16" = 0.79375 cm** (use brass pipe)

Design wavelength $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^8}{213\times10^6} = 1.4075117\ \text{m} = \underline{140.7512\ \text{cm}}$

Table 1 Element lengths and spacings for the unmatched design (no boom)

Description	**(λ)**	**(cm)**
Reflector, l_1'	0.485λ	68.264
Driven, l_2'	0.4615λ	64.957
1st director, l_3'	0.438λ	61.649
2nd director, l_4'	0.433λ	60.945
3rd director, l_5'	0.438λ	61.649
All spacings, $s_{12} = s_{ij}$	0.2λ	28.150

b) Model **unmatched design** (no boom) using NEC-2

No Match with no boom

Input NEC file:

```
CM Yagi-Uda Antenna for channel 13(yagi_ch13_5element_no_match_in.txt)
CM THIS PROGRAM ASSUMES THAT THERE IS NO BOOM.
CM
CM THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE &  OF THE
CM DRIVEN ELEMENT OF A 5 ELEMENT ANTENNA.  CENTER FREQUENCY IS
CM 213 MHz W/ WAVELENGTH OF 140.7512 cm.
CM
CM THE DIMENSIONS ARE:
CM element diameters: d = 0.79375 cm = 0.3125in, radius a = 0.396875 cm
CM Reflector l1 = 68.264 cm,  Driven element l2 = 64.957 cm
CM Directors l3 = l5 = 61.649 cm, and l4 = 60.945 cm
CM Element spacings S12 = Sij = 28.150 cm
CM Segment length Delta ~ 3.2 cm ~ 8a
CE
GW 1 21 0.0 -0.34132  0.0     0.0 0.34132  0.0    0.00396875 !Reflector l1
GW 2 21 0.0 -0.32478  0.2815  0.0 0.32478  0.2815 0.00396875 !Driven l2
GW 3 19 0.0 -0.308245 0.5630  0.0 0.308245 0.5630 0.00396875 !Director l3
GW 4 19 0.0 -0.304725 0.8445  0.0 0.304725 0.8445 0.00396875 !Director l4
GW 5 19 0.0 -0.308245 1.1260  0.0 0.308245 1.1260 0.00396875 !Director l5
GE 0  ! No ground plane
EK 0  ! Use extended kernel
PT -1 ! suppress current outputs
LD 5 0 0 0 1.1e7   ! Set conductivity of brass on all segments
FR 0 1 0 0 213.0 0 ! Center frequency of CH13
EX 0 2 11 0 1.0 0.0
RP 0 2 2 0000 0.0 0.0 180.0 90.0 ! Front & back gains
EN
```

NEC-2 Output file excerpts

```
- - - ANTENNA INPUT PARAMETERS - - -

TAG SEG. VOLTAGE (VOLTS) CURRENT (AMPS)        IMPEDANCE (OHMS)        <snip>
NO. NO.  REAL   IMAG.     REAL        IMAG.       REAL        IMAG.    <snip>
2 32  1.0E+00 0.0E+00 2.81723E-02 -2.87610E-02 1.73809E+01 1.77441E+01 <snip>
<snip>
EFFICIENCY    =  98.41 PERCENT

- - - RADIATION PATTERNS - - -
- - ANGLES - -           - POWER GAINS -    <snip>
 THETA     PHI       MAJOR  MINOR    TOTAL  <snip>
 DEGREES  DEGREES      DB     DB       DB    <snip>
  0.00     0.00      11.20  -999.99  11.21  <snip>
180.00     0.00      -0.77  -999.99  -1.02  <snip>
<snip>
```

Reflection coefficient $\Gamma_{in} = \dfrac{Z_{in} - Z_0}{Z_{in} + Z_0} = \dfrac{(17.3809 + j17.7441) - 100}{(17.3809 + j17.7441) + 100} = \underline{0.71182\angle 159.3°}.$

$$\text{VSWR} = \frac{1+\Gamma_{in}}{1-\Gamma_{in}} = \frac{1+0.71182}{1-0.71182} = \underline{5.940}$$

Table 2 Unmatched design five-element Yagi-Uda antenna for channel 13

Z_{in} (Ω)	Γ_{in}	VSWR	G_{max} (dBi)	G_{back} (dBi)	FB ratio (dB)
17.3809 + *j*17.7441	0.7118∠159.3°	5.940	11.20	-0.77	11.97

c) Match the antenna.

Try 1 T-Match Design choices:

Driven element length: $l_2' =$ **64 cm** (shorten from 64.957 cm as Z_{in} was inductive)

All other element lengths & spacings are unchanged from original design.

T-Match diameter: $2a' = 1/8" =$ 0.3175 cm (smaller than $2a =$ 0.79375 cm to get $\alpha > 1$)

T-Match length: $l' =$ 16 cm (choose ~$l_2'/4$)

T-Match spacing: $s =$ 2.5 cm (choose less than $s_{12}/4 = 7$ cm and $200 < Z_0 < 300$ Ω)

From MathCad $Z_0 = 273.852$ Ω, $\alpha = 1.48729$, $a_e = 0.78056$ cm, & $Z_t = j102.18$ Ω

NEC results $Z_a = 14.5762 + j3.48568$ Ω, $G_{max} = 11.71$ dBi, & $G_{back} = -0.24$ dBi

From MathCad- $Z_{in} = 63.644 + j44.91$ Ω, $|\Gamma| = 0.34$, & VSWR = 2.033 (too high)

Comments: Z_{in} is inductive. On second try, make l_2' shorter to make Z_{in} more capacitive.

Input NEC file:

```
CM Yagi-Uda Antenna for channel 13(yagi_ch13_5_element_tmatch_in1.txt)
CM
CM THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN ELEMENT
CM OF A 5 ELEMENT YAGI-UDA ANTENNA (NO BOOM).  CENTER FREQUENCY IS 213 MHz
CM W/ WAVELENGTH OF 140.75 cm.
CM
CM THE DIMENSIONS ARE:
CM element diameters: d = 0.79375 cm = 0.3125in, radius a = 0.396875 cm
CM equiv. radius T-Match portion of driven element ae = 0.78056 cm
CM which has a length of l'= 16 cm
CM Reflector l1 = 68.264 cm
CM Driven element l2 = 64 cm
CM Directors l3 = l5 = 61.649 cm, and l4 = 60.945 cm
CM Element spacings S12 = Sij = 28.150 cm
CM Segment length Delta ~ 3.2 cm ~ 8a
CE
GW 1 21 0.0 -0.34132  0.0     0.0  0.34132  0.0    0.00396875 !Reflector l1
GW 2 8  0.0 -0.32     0.2815  0.0 -0.08     0.2815 0.00396875 !Driven tip L
GW 3 5  0.0 -0.08     0.2815  0.0  0.08     0.2815 0.0078056  !Driven middle
GW 4 8  0.0  0.08     0.2815  0.0  0.32     0.2815 0.00396875 !Driven tip R
GW 5 19 0.0 -0.308245 0.5630  0.0  0.308245 0.5630 0.00396875 !Director l3
GW 6 19 0.0 -0.304725 0.8445  0.0  0.304725 0.8445 0.00396875 !Director l4
GW 7 19 0.0 -0.308245 1.1260  0.0  0.308245 1.1260 0.00396875 !Director l5
```

```
GE 0  ! No ground plane
EK 0  ! Use extended kernel
PT -1 ! suppress current outputs
LD 5 0 0 0 1.1e7   ! Set conductivity of brass on all segments
FR 0 1 0 0 213.0 0 ! Center frequency of CH13
EX 0 3 3 0 1.0 0.0
RP 0 2 2 0000 0.0 0.0 180.0 90.0 ! Front & back gains
EN
```

Mathcad T-Match equations- 5-element, Ch 13, Yagi-Uda: Try 1

$c := 2.998\cdot10^{8}$ m/s $fc := 213\cdot10^{6}$ Hz $\lambda := \frac{c}{fc}$ $\lambda = 1.40751$ m

$k := \frac{2\cdot\pi}{\lambda}$ $k = 4.46404$ rad/m $Zdesired := 100$ Ω

$d := 0.79375\cdot10^{-2}$ $a := d\cdot0.5$ $a = 0.0039688$ m

$dprime := 0.3175\cdot10^{-2}$ $aprime := dprime\cdot0.5$ $aprime = 0.0015875$ m

$s := 2.5\cdot10^{-2}$ m $lprime := 16.0\cdot10^{-2}$ m

$$Z0 := \frac{376.73}{2\cdot\pi}\cdot\operatorname{acosh}\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2\cdot a\cdot aprime}\right] \qquad \boxed{Z0 = 273.852}\ \Omega$$

$u := \frac{a}{aprime}$ $u = 2.5$ $v := \frac{s}{aprime}$ $v = 15.74803$

$$\alpha := \frac{\operatorname{acosh}\left[\frac{\left(v^2 - u^2 + 1\right)}{2\cdot v}\right]}{\operatorname{acosh}\left[\frac{\left(v^2 + u^2 - 1\right)}{2\cdot v\cdot u}\right]} \qquad ae := aprime\cdot e^{\frac{1}{(1+u)^2}\cdot\left(u^2\cdot\ln(u)+2\cdot u\cdot\ln(v)\right)}$$

$\boxed{\alpha = 1.48729}$ $ae = 0.0078056$ m $\boxed{ae\cdot100 = 0.78056}$ cm

$Zt := j\cdot Z0\cdot\tan(0.5k\cdot lprime)$ $\boxed{Zt = 102.18013i}$ Ω

$Yt := \frac{1}{Zt}$ $Yt = -9.787i\times10S^{3}$ $0.5\cdot Yt = -4.893i\times10^{-3}$ S

$\boxed{Za := 14.5762 + j\cdot3.48568}$ Ω $Ya := \frac{1}{Za}$ $Ya = 0.06489 - 0.01552i$ S

$$Yin := \frac{Yt}{2} + \frac{Ya}{(1+\alpha)^2} \qquad Yin = 0.01049 - 0.0074i \text{ S}$$

$Zin := \frac{1}{Yin}$ $\boxed{Zin = 63.644 + 44.91i}$ Ω $Zdesired = 100$ Ω

$\Gamma := \frac{(Zin - Zdesired)}{Zin + Zdesired}$ $VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$

$\Gamma = -0.137 + 0.312i$ $\boxed{|\Gamma| = 0.34}$ $\boxed{VSWR = 2.033}$

$$lsuggested := \frac{2}{k}\cdot\operatorname{atan}\left[\frac{1}{2\cdot Z0\cdot\operatorname{Im}\left[\frac{Ya}{(1+\alpha)^2}\right]}\right] \qquad lsuggested = -0.28189 \text{ m}$$

Try 2 T-Match Design choices:

Driven element length: $l_2' =$ **63 cm** (shorten from 64 cm as Z_{in} was inductive)

All other element lengths & spacings are unchanged from original design.

T-Match diameter: $2a' = 1/8" =$ 0.3175 cm (same)

T-Match length: $l' =$ 16 cm (same)

T-Match spacing: $s =$ 2.5 cm (same)

From MathCad $Z_0 = 273.852\ \Omega$, $\alpha = 1.48729$, $a_e = 0.78056$ cm, & $Z_t = j102.18\ \Omega$

NEC results $Z_a = 13.8042 - j5.77254\ \Omega$, $G_{max} = 11.70$ dBi, & $G_{back} = -0.23$ dBi

From MathCad- $Z_{in} = 99.806 + j7.266\ \Omega$, $|\Gamma| = 0.036$, & VSWR = 1.075 (< 1.1!)

Comments: Z_{in} is slightly inductive, but meets spec!

Input NEC file:

```
CM Yagi-Uda Antenna for channel 13(yagi_ch13_5_element_tmatch_in2.txt)
CM
CM THIS FILE IS USED TO DETERMINE THE INPUT IMPEDANCE OF THE DRIVEN ELEMENT
CM OF A 5 ELEMENT YAGI-UDA ANTENNA (NO BOOM).  CENTER FREQUENCY IS 213 MHz
CM W/ WAVELENGTH OF 140.75 cm.
CM
CM THE DIMENSIONS ARE:
CM element diameters: d = 0.79375 cm = 0.3125in, radius a = 0.396875 cm
CM equiv. radius T-Match portion of driven element ae = 0.78056 cm
CM which has a length of l'= 16 cm
CM Reflector l1 = 68.264 cm
CM Driven element l2 = 63 cm
CM Directors l3 = l5 = 61.649 cm, and l4 = 60.945 cm
CM Element spacings S12 = Sij = 28.150 cm
CM Segment length Delta ~ 3.2 cm ~ 8a
CE
GW 1 21 0.0 -0.34132  0.0     0.0  0.34132  0.0    0.00396875 !Reflector l1
GW 2 8  0.0 -0.315    0.2815  0.0 -0.08     0.2815 0.00396875 !Driven tip L
GW 3 5  0.0 -0.08     0.2815  0.0  0.08     0.2815 0.0078056  !Driven middle
GW 4 8  0.0  0.08     0.2815  0.0  0.315    0.2815 0.00396875 !Driven tip R
GW 5 19 0.0 -0.308245 0.5630  0.0  0.308245 0.5630 0.00396875 !Director l3
GW 6 19 0.0 -0.304725 0.8445  0.0  0.304725 0.8445 0.00396875 !Director l4
GW 7 19 0.0 -0.308245 1.1260  0.0  0.308245 1.1260 0.00396875 !Director l5
GE 0  ! No ground plane
EK 0  ! Use extended kernel
PT -1 ! suppress current outputs
LD 5 0 0 0 1.1e7   ! Set conductivity of brass on all segments
FR 0 1 0 0 213.0 0 ! Center frequency of CH13
EX 0 3 3 0 1.0 0.0
RP 0 2 2 0000 0.0 0.0 180.0 90.0 ! Front & back gains
EN
```

Mathcad T-Match equations- 5-element, Ch 13, Yagi-Uda: Try 2

$c := 2.998\cdot10^{8}$ m/s $\quad fc := 213\cdot10^{6}$ Hz $\quad \lambda := \frac{c}{fc}$ $\quad \lambda = 1.40751$ m

$k := \frac{2\cdot\pi}{\lambda}$ $\quad k = 4.46404$ rad/m $\quad Zdesired := 100$ Ω

$d := 0.79375\cdot10^{-2}$ $\quad a := d\cdot0.5$ $\quad a = 0.0039688$ m

$dprime := 0.3175\cdot10^{-2}$ $\quad aprime := dprime\cdot0.5$ $\quad aprime = 0.0015875$ m

$s := 2.5\cdot10^{-2}$ m $\quad lprime := 16.0\cdot10^{-2}$ m

$$Z0 := \frac{376.73}{2\cdot\pi}\cdot\operatorname{acosh}\left[\frac{\left(s^2 - a^2 - aprime^2\right)}{2\cdot a\cdot aprime}\right] \qquad \boxed{Z0 = 273.852}\ \Omega$$

$u := \frac{a}{aprime}$ $\quad u = 2.5$ $\quad v := \frac{s}{aprime}$ $\quad v = 15.74803$

$$\alpha := \frac{\operatorname{acosh}\left[\frac{\left(v^2 - u^2 + 1\right)}{2\cdot v}\right]}{\operatorname{acosh}\left[\frac{\left(v^2 + u^2 - 1\right)}{2\cdot v\cdot u}\right]} \qquad ae := aprime\cdot e^{\frac{1}{(1+u)^2}\cdot\left(u^2\cdot\ln(u)+2\cdot u\cdot\ln(v)\right)}$$

$\boxed{\alpha = 1.48729}$ $\quad ae = 0.0078056$ m $\quad \boxed{ae\cdot100 = 0.78056}$ cm

$Zt := j\cdot Z0\cdot\tan(0.5k\cdot lprime)$ $\quad \boxed{Zt = 102.18013i}\ \Omega$

$Yt := \frac{1}{Zt}$ $\quad Yt = -9.787i\times10\,S^{3}$ $\quad 0.5\cdot Yt = -4.893i\times10^{-3}$ S

$\boxed{Za := 13.8042 - j\cdot5.77254}\ \Omega$ $\quad Ya := \frac{1}{Za}$ $\quad Ya = 0.06166 + 0.02578i$ S

$$Yin := \frac{Yt}{2} + \frac{Ya}{(1+\alpha)^2} \qquad Yin = 0.00997 - 7.25569i\times10^{-4}\ S$$

$Zin := \frac{1}{Yin}$ $\quad \boxed{Zin = 99.806 + 7.266i}\ \Omega$ $\quad Zdesired = 100\ \Omega$

$\Gamma := \frac{(Zin - Zdesired)}{Zin + Zdesired}$ $\quad VSWR := \frac{(1 + |\Gamma|)}{1 - |\Gamma|}$

$\Gamma = 3.53\times10^{-4} + 0.036i$ $\quad \boxed{|\Gamma| = 0.036}$ $\quad \boxed{VSWR = 1.075}$

$$lsuggested := \frac{2}{k}\cdot\operatorname{atan}\left[\frac{1}{2\cdot Z0\cdot\operatorname{Im}\left[\frac{Ya}{(1+\alpha)^2}\right]}\right] \qquad lsuggested = 0.18499\ m$$

d) In a **table**, summarize the original (unmatched) and final (matched) …

Table 3 Unmatched vs. matched designs for six-element Yagi-Uda antenna for channel 13

Parameter	Unmatched	Matched
Z_a or Z_{in} (Ω)	$17.3809 + j\,17.7441$	$99.806 + j7.266$
Γ_{in}	$0.7118\angle 159.3°$	$0.036\angle 89.44°$
VSWR	5.940	1.075
G_{max} (dBi)	11.20	11.70
G_{back} (dBi)	-0.77	-0.23
FB ratio (dB)	11.97	11.93

Comments-

- Huge improvement in impedance matching using the T-Match, went from VSWR =5.94 to VSWR = 1.075.
- Main beam gain increased slightly (0.50 dB). Back lobe gain increased slightly (0.54 dB). FB ratio is very slightly worse (0.04 dB).

e) Accurately sketch final antenna design with T-match (no boom).