An antenna has input impedance $Z_A = 50 - j\,45\ \Omega$ at 1 GHz. Match it to a feeding transmission line ($Z_0 = 50\ \Omega$ & $u = 2.1 \times 10^8$ m/s) using a quarterwave transformer (QWT) placed as close to the antenna as possible. Assume the wavelength λ' of the QWT is 92% of λ for the feeding transmission line. Draw a fully labeled sketch of the final design.

- The wavelength is $\lambda = u/f = 2.1 \times 10^8 / 1 \times 10^9 \Rightarrow \underline{\lambda = 0.21\ \text{m} = 21\ \text{cm}}$.
- Given- QWT wavelength $\lambda' = 0.92\,\lambda = 0.92(0.21) \Rightarrow \underline{\lambda' = 0.1932\ \text{m} = 19.32\ \text{cm}}$.

Steps

1) Calculate the normalized impedance for the antenna $z_A = Z_A / Z_0 = (50 - j\,45)/50 \Rightarrow$ $\underline{z_A = 1 - j\,0.9\ \Omega/\Omega}$ and plot on **Smith chart** (see Figure 2).
2) Draw circle, centered on Smith chart, through z_A point. This circle of constant $|\Gamma|$ includes the locus of all possible z_{in} (and y_{in}) along the transmission line with this load.
3) The two match points where the circle of constant $|\Gamma|$ intersects the real axis are $r_{m,1} = r_{min} = 0.418$ [or $R_{min} = 0.418(50) = 20.9\ \Omega$] and $r_{m,2} = r_{max} = 2.39$. Select the $r_{m,1} = r_{min} = 0.418$ match point as it is closest to z_A.
4) Find the distance d_1 from z_A to the match point $r_{m,1} = r_{min} = 0.418$ using scales on Smith chart, $\underline{d_1 = 0.1587\lambda}$ or $d_1 = 0.1587(21) \Rightarrow \underline{d_1 = 3.333\ \text{cm}}$.
5) Starting at the match point, insert the QWT. The QWT will have a characteristic impedance $Z'_{0,min} = \sqrt{Z_0 R_{min}} = \sqrt{50(20.9)} \Rightarrow \underline{Z_0' = 32.33\ \Omega}$ and length $\lambda'/4 = 19.32/4 \Rightarrow \underline{\lambda'/4 = 4.83\ \text{cm}}$.
6) As shown on Figure 1, everywhere toward the source from the QWT sees an input impedance of $Z_{in} = Z_0 = 50\ \Omega$.

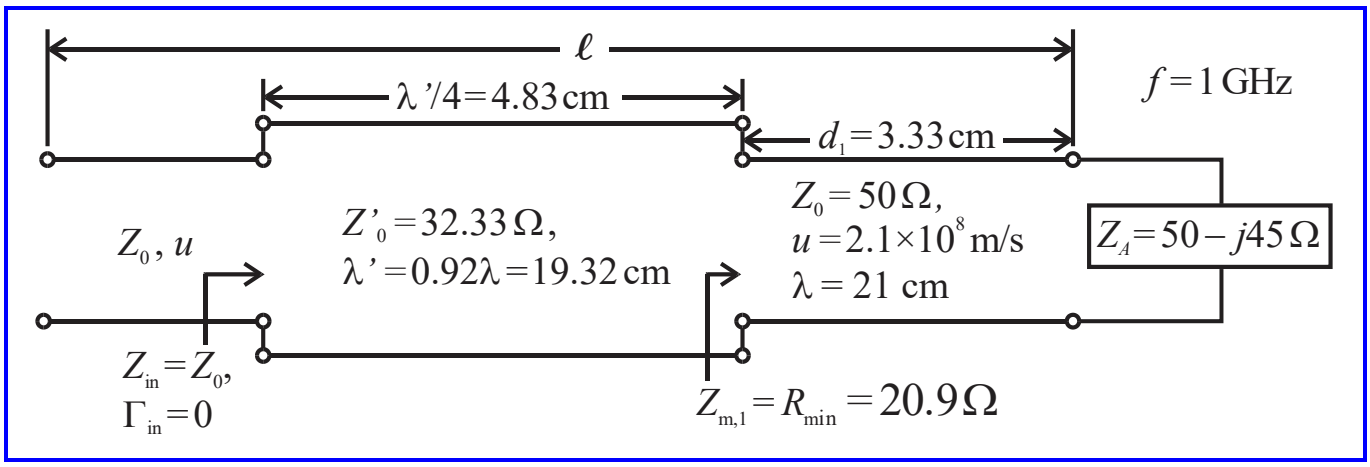

Figure 1 Matching antenna using a QWT.

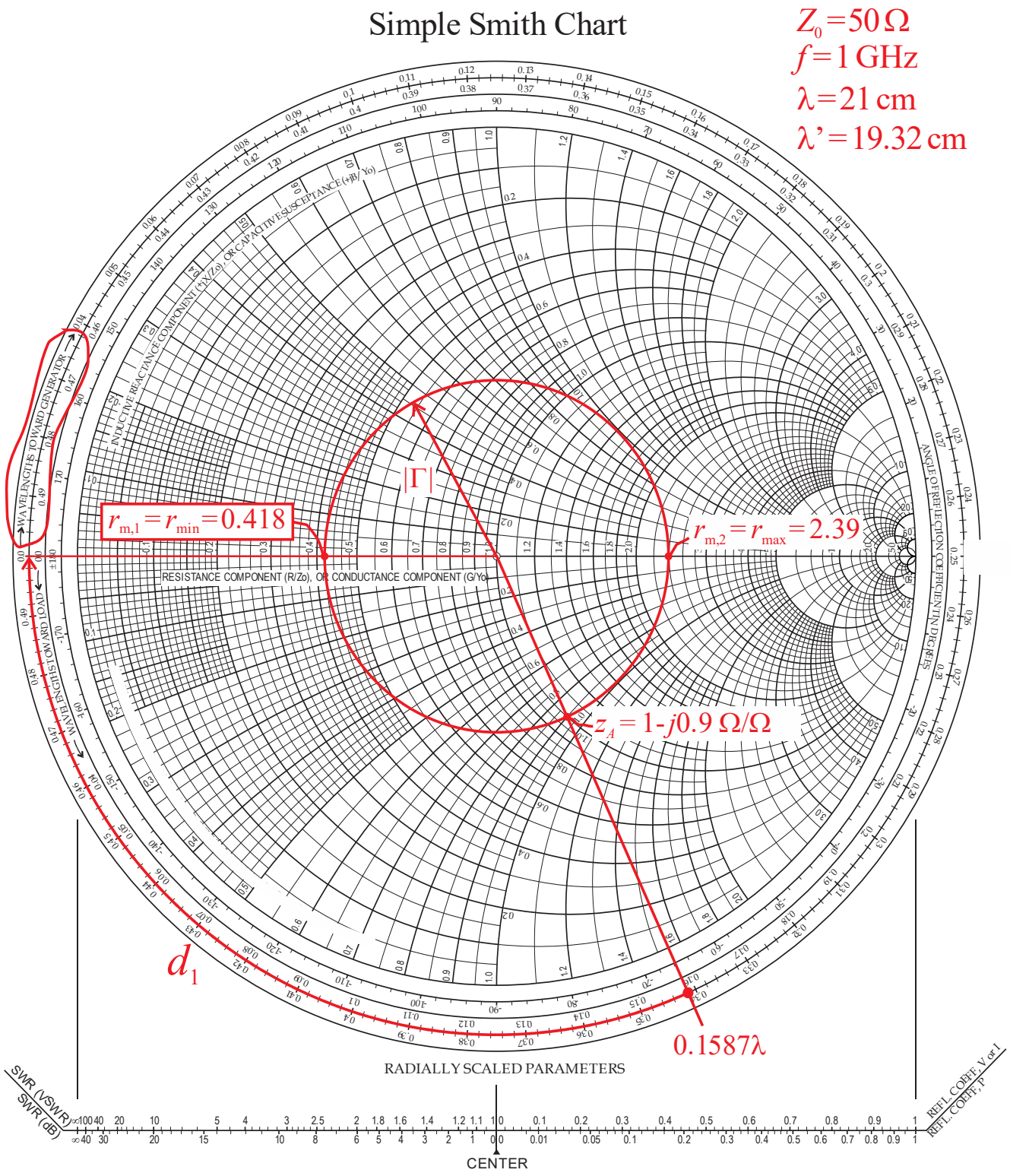

Figure 2 Smith chart for matching antenna using a QWT.