For the match of problem 2, **analytically** find the input impedance Z_{in}, reflection coefficient Γ_{in}, and VSWR at 195, 200, & 205 MHz on the feeding transmission line just past the inductor toward the generator assuming Z_A remains constant. Tabulate results- col. 1 frequency, col. 2 Z_{in} (rectangular format), col. 3 Γ (polar format), and col. 4 VSWR.

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2) You have an antenna with input impedance $Z_A = 12.5 + j40\ \Omega$ at 200 MHz. Match it to a feeding transmission line ($Z_0 = 50\ \Omega$ & $u = 2.5 \times 10^8$ m/s) using a discrete inductor connected in series as close to the antenna as possible. Draw a fully labeled sketch of the final design.

   - The wavelength is $\lambda = c/f = 2.5 \times 10^8 / 200 \times 10^6 = 1.25$ m = 125 cm.

<snip>

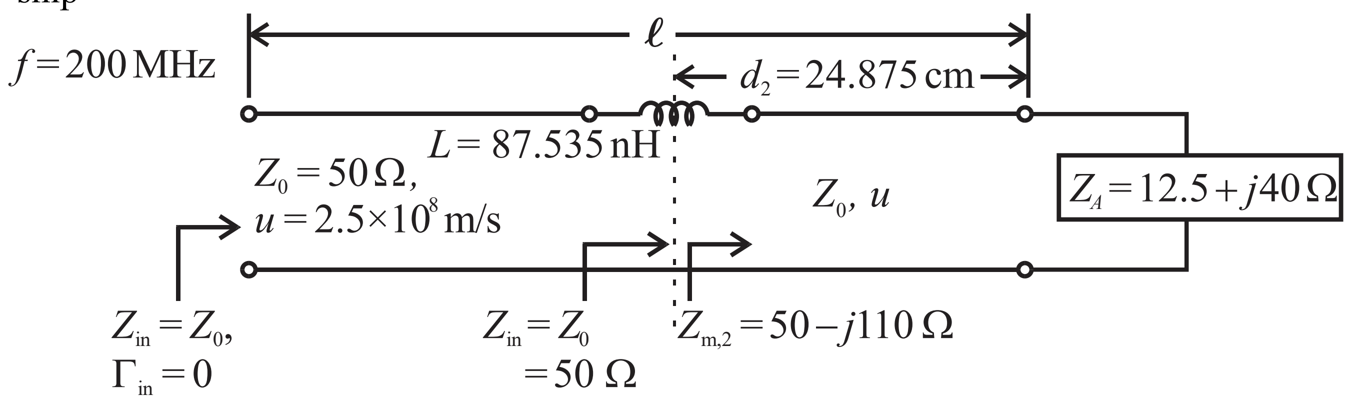

Figure 1 Matching antenna using discrete series capacitor.

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First, analytically calculate the reflection coefficient at the antenna. It is assumed to be constant with respect to frequency. [Note: All calculations checked using MathCad.]

$$\Gamma_A = \frac{Z_A - Z_0}{Z_A + Z_0} = \frac{(12.5 + j40) - 50}{(12.5 + j40) + 50} \Rightarrow \underline{\Gamma_A = 0.7389\angle 100.533^\circ}$$

Next, find the reflection coefficient at the match point ($d_2 = 24.875$ cm) for each frequency.

$$\Gamma_{\text{in},f} = \Gamma_A e^{-j2\beta d_2} = \Gamma_A e^{-j2\left(\frac{2\pi f}{u}\right)d_2} = (0.7389\angle 100.533^\circ)e^{-j\left(\frac{4\pi f}{2.5\times10^8}\right)0.24875}$$

$$\text{At 195 MHz, } \Gamma_{\text{low}} = (0.7389\angle 100.533^\circ)e^{-j\left(\frac{4\pi 195\times10^6}{2.5\times10^8}\right)0.24875} \Rightarrow \underline{\Gamma_{\text{low}} = 0.7389\angle 39.165^\circ}$$

$$\text{At 200 MHz, } \Gamma_{\text{ctr}} = (0.7389\angle 100.533^\circ)e^{-j\left(\frac{4\pi 200\times10^6}{2.5\times10^8}\right)0.24875} \Rightarrow \underline{\Gamma_{\text{ctr}} = 0.7389\angle -42.747^\circ}$$

$$\text{At 205 MHz, } \Gamma_{\text{high}} = (0.7389\angle 100.533^\circ)e^{-j\left(\frac{4\pi 205\times10^6}{2.5\times10^8}\right)0.24875} \Rightarrow \underline{\Gamma_{\text{high}} = 0.7389\angle -46.329^\circ}$$

Then, calculate the input impedance at the match point for each frequency.

$$Z_{\text{in},f} = Z_0\left(\frac{1+\Gamma_{\text{in},f}}{1-\Gamma_{\text{in},f}}\right) = 50\left(\frac{1+\Gamma_{\text{in},f}}{1-\Gamma_{\text{in},f}}\right)$$

$$Z_{\text{m,low}} = 50\left(\frac{1+\Gamma_{\text{low}}}{1-\Gamma_{\text{low}}}\right) = 50\left(\frac{1+(0.7389\angle 39.165^\circ)}{1-(0.7389\angle 39.165^\circ)}\right) \Rightarrow \underline{Z_{\text{m,low}} = 56.727 - j116.609\,\Omega}$$

$$Z_{\text{m,ctr}} = 50\left(\frac{1+\Gamma_{\text{ctr}}}{1-\Gamma_{\text{ctr}}}\right) = 50\left(\frac{1+(0.7389\angle -42.747^\circ)}{1-(0.7389\angle -42.747^\circ)}\right) \Rightarrow \underline{Z_{\text{m,ctr}} = 49.272 - j108.855\,\Omega}$$

$$Z_{\text{m,high}} = 50\left(\frac{1+\Gamma_{\text{high}}}{1-\Gamma_{\text{high}}}\right) = 50\left(\frac{1+(0.7389\angle -46.329^\circ)}{1-(0.7389\angle -46.329^\circ)}\right) \Rightarrow \underline{Z_{\text{m,high}} = 43.198 - j101.699\,\Omega}$$

Calculate the inductor impedance for each frequency.

$$Z_{\text{ind,low}} = j2\pi f_{\text{low}}L = j2\pi(195\times10^6)87.535\times10^{-9} \Rightarrow \underline{Z_{\text{ind,low}} = j107.25\,\Omega}$$

$$Z_{\text{ind,ctr}} = j2\pi f_{\text{ctr}}L = j2\pi(200\times10^6)87.535\times10^{-9} \Rightarrow \underline{Z_{\text{ind,low}} = j110\,\Omega}$$

$$Z_{\text{ind,high}} = j2\pi f_{\text{high}}L = j2\pi(205\times10^6)87.535\times10^{-9} \Rightarrow \underline{Z_{\text{ind,low}} = j112.75\,\Omega}$$

Calculate the input impedance, $Z_{\text{in},f} = Z_{\text{cap},f} + Z_{\text{m},f}$, after matching for each frequency.

$$Z_{\text{in,low}} = j107.25 + (56.727 - j116.609) \Rightarrow \underline{Z_{\text{in,low}} = 56.727 - j9.359\,\Omega}$$

$$Z_{\text{in,ctr}} = j110 + (49.272 - j108.855) \Rightarrow \underline{Z_{\text{in,ctr}} = 49.727 + j1.145\,\Omega}$$

$$Z_{\text{in,high}} = j112.175 + (43.198 - j101.699) \Rightarrow \underline{Z_{\text{in,high}} = 43.198 + j11.050\,\Omega}$$

Calculate the input reflection coefficient after matching for each frequency.

$$\Gamma_{\text{in},f} = \frac{Z_{\text{in},f} - Z_0}{Z_{\text{in},f} + Z_0} = \frac{Z_{\text{in},f} - 50}{Z_{\text{in},f} + 50}$$

$$\Gamma_{\text{in,low}} = \frac{(56.727 - j9.359) - 50}{(56.727 - j9.359) + 50} \Rightarrow \underline{\Gamma_{\text{in,low}} = 0.10758\angle -49.281.89^\circ}$$

$$\Gamma_{\text{in,ctr}} = \frac{(49.727 + j1.145) - 50}{(49.727 + j1.145) + 50} \Rightarrow \underline{\Gamma_{\text{in,ctr}} = 0.01366\angle 121.79^\circ}$$

$$\Gamma_{\text{in,high}} = \frac{(43.198 + j11.05) - 50}{(43.198 + j11.05) + 50} \Rightarrow \underline{\Gamma_{\text{in,high}} = 0.13826\angle 114.85^\circ}$$

Calculate the VSWR after matching for each frequency.

$$\mathrm{VSWR}_{\mathrm{low}} = \frac{1+|\Gamma_{\mathrm{low}}|}{1-|\Gamma_{\mathrm{low}}|} = \frac{1+0.10758}{1-0.10758} \quad \Rightarrow \quad \underline{\mathrm{VSWR}_{\mathrm{low}} = 1.241}$$

$$\mathrm{VSWR}_{\mathrm{ctr}} = \frac{1+|\Gamma_{\mathrm{ctr}}|}{1+|\Gamma_{\mathrm{ctr}}|} = \frac{1+0.01366}{1+0.01366} \quad \Rightarrow \quad \underline{\mathrm{VSWR}_{\mathrm{ctr}} = 1.028}$$

$$\mathrm{VSWR}_{\mathrm{high}} = \frac{1+|\Gamma_{\mathrm{high}}|}{1+|\Gamma_{\mathrm{high}}|} = \frac{1+0.13826}{1+0.13826} \quad \Rightarrow \quad \underline{\mathrm{VSWR}_{\mathrm{high}} = 1.321}$$

Tabulate results

f **(MHz)**	Z_{in} **(Ω)**	**Γ**	**VSWR**
195	**56.727 -** j **9.359**	**0.10758∠-49.28°**	**1.241**
200	**49.272 +** j **1.145**	**0.01366∠121.79°**	**1.028**
205	**43.198 +** j **11.05**	**0.13826∠114.85°**	**1.321**