For the match of problem 2, <u>analytically</u> find the input impedance Z_{in}, reflection coefficient Γ, and VSWR at 95, 100, & 105 MHz on the feeding transmission just past the inductor toward the generator assuming Z_A remains constant. Tabulate results- col. 1 frequency, col. 2 Z_{in} (rectangular format), col. 3 Γ (polar format), and col. 4 VSWR.

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An antenna has input impedance $Z_A = 37.5 - j75\ \Omega$ at 100 MHz. Match it to a feeding transmission line ($Z_0 = 75\ \Omega$ & $u = 2.25 \times 10^8$ m/s) using a discrete <u>inductor</u> connected in parallel as close to the antenna as possible. Draw a fully labeled sketch of the final design.

- The wavelength is $\lambda = c/f = 2.25 \times 10^8 / 100 \times 10^6 = 2.25$ m.

<snip>

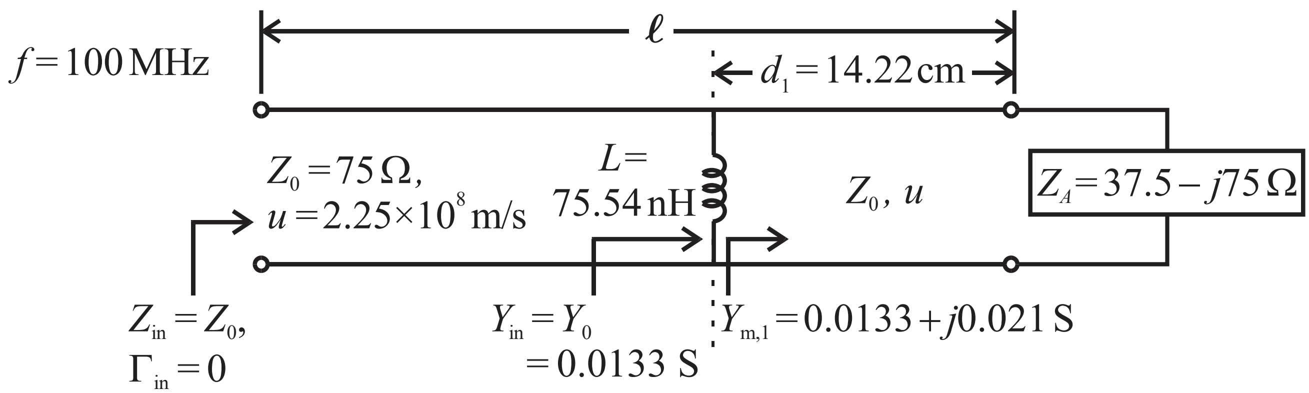

Figure 1 Matching antenna using discrete <u>parallel</u> inductor.

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First, analytically calculate the reflection coefficient at the antenna. It is assumed to be constant with respect to frequency. [Note: All calculations checked using MathCad.]

$$\Gamma_A = \frac{Z_A - Z_0}{Z_A + Z_0} = \frac{(37.5 - j75) - 75}{(37.5 - j75) + 75} \quad \Rightarrow \quad \underline{\Gamma_A = 0.6202\angle -82.875^\circ}$$

Next, calculate the reflection coefficient at the match point (d_1 = 14.22 cm) for each frequency.

$$\Gamma_{in,f} = \Gamma_A e^{-j2\beta d_1} = \Gamma_A e^{-j2\left(\frac{2\pi f}{u}\right)d_1} = (0.6202\angle -82.875^\circ)e^{-j\left(\frac{4\pi f}{2.25\times 10^8}\right)0.1422}$$

$$\text{At 95 MHz, } \Gamma_{low} = (0.62\angle -82.875^\circ)e^{-j\left(\frac{4\pi 95\times 10^6}{2.25\times 10^8}\right)0.1422} \quad \Rightarrow \quad \underline{\Gamma_{low} = 0.6202\angle -126.104^\circ}$$

$$\text{At 100 MHz, } \Gamma_{ctr} = (0.62\angle -82.875^\circ)e^{-j\left(\frac{4\pi 100\times 10^6}{2.25\times 10^8}\right)0.1422} \quad \Rightarrow \quad \underline{\Gamma_{ctr} = 0.6202\angle -128.379^\circ}$$

$$\text{At 105 MHz, } \Gamma_{high} = (0.62\angle -82.875^\circ)e^{-j\left(\frac{4\pi 105\times 10^6}{2.25\times 10^8}\right)0.1422} \quad \Rightarrow \quad \underline{\Gamma_{high} = 0.6202\angle -130.654^\circ}$$

Then, calculate the input impedance at the match point for each frequency.

$$Z_{\text{in},f} = Z_0\left(\frac{1+\Gamma_{\text{in},f}}{1-\Gamma_{\text{in},f}}\right) = 75\left(\frac{1+\Gamma_{\text{in},f}}{1-\Gamma_{\text{in},f}}\right)$$

$$Z_{\text{m,low}} = 75\left(\frac{1+\Gamma_{\text{low}}}{1-\Gamma_{\text{low}}}\right) = 75\left(\frac{1+(0.62\angle-126.1^\circ)}{1-(0.62\angle-126.1^\circ)}\right) \Rightarrow \underline{Z_{\text{m,low}} = 21.817 - j35.529\,\Omega}$$

$$Z_{\text{m,ctr}} = 75\left(\frac{1+\Gamma_{\text{ctr}}}{1-\Gamma_{\text{ctr}}}\right) = 75\left(\frac{1+(0.62\angle-128.38^\circ)}{1-(0.62\angle-128.38^\circ)}\right) \Rightarrow \underline{Z_{\text{m,ctr}} = 21.42 - j33.845\,\Omega}$$

$$Z_{\text{m,high}} = 75\left(\frac{1+\Gamma_{\text{high}}}{1-\Gamma_{\text{high}}}\right) = 75\left(\frac{1+(0.62\angle-130.65^\circ)}{1-(0.62\angle-130.65^\circ)}\right) \Rightarrow \underline{Z_{\text{m,high}} = 21.049 - j32.186\,\Omega}$$

Calculate the input admittance at the match point for each frequency.

$$Y_{\text{m,low}} = \frac{1}{Z_{\text{m,low}}} = \frac{1}{21.817 - j35.529} \Rightarrow \underline{Y_{\text{m,low}} = 0.01255 + j0.02044\,\text{S}}$$

$$Y_{\text{m,ctr}} = \frac{1}{Z_{\text{m,ctr}}} = \frac{1}{121.42 - j33.845} \Rightarrow \underline{Y_{\text{m,ctr}} = 0.01335 + j0.0211\,\text{S}}$$

$$Y_{\text{m,high}} = \frac{1}{Z_{\text{m,high}}} = \frac{1}{21.049 - j32.186} \Rightarrow \underline{Y_{\text{m,high}} = 0.01423 + j0.02176\,\text{S}}$$

Calculate the inductor admittance for each frequency.

$$Y_{\text{ind,low}} = \frac{1}{j2\pi f_{\text{low}}L} = \frac{1}{j2\pi(95\times10^6)75.54\times10^{-9}} \Rightarrow \underline{Y_{\text{ind,low}} = -j0.02218\,\text{S}}$$

$$Y_{\text{ind,ctr}} = \frac{1}{j2\pi f_{\text{ctr}}L} = \frac{1}{j2\pi(100\times10^6)75.54\times10^{-9}} \Rightarrow \underline{Y_{\text{ind,ctr}} = -j0.02107\,\text{S}}$$

$$Y_{\text{ind,high}} = \frac{1}{j2\pi f_{\text{high}}L} = \frac{1}{j2\pi(105\times10^6)75.54\times10^{-9}} \Rightarrow \underline{Y_{\text{ind,high}} = -j0.02007\,\text{S}}$$

Calculate the input admittance after matching, $Y_{\text{in},f} = Y_{\text{ind},f} + Y_{\text{m},f}$, for each frequency.

$$Y_{\text{in,low}} = -j0.02218 + (0.01255 + j0.02044) \Rightarrow \underline{Y_{\text{in,low}} = 0.01255 - j0.00174\,\text{S}}$$

$$Y_{\text{in,ctr}} = -j0.02107 + (0.01335 + j0.0211) \Rightarrow \underline{Y_{\text{in,ctr}} = 0.01335 + j0.0000275\,\text{S}}$$

$$Y_{\text{in,high}} = -j0.02007 + (0.01423 + j0.02176) \Rightarrow \underline{Y_{\text{in,high}} = 0.01423 + j0.0017\,\text{S}}$$

Calculate the input impedance after matching for each frequency.

$$Z_{\text{in,low}} = \frac{1}{Y_{\text{in,low}}} = \frac{1}{0.01255 - j0.00174} \Rightarrow \underline{Z_{\text{in,low}} = 78.174 + j10.83\,\Omega}$$

$$Z_{\text{in,ctr}} = \frac{1}{Y_{\text{in,ctr}}} = \frac{1}{0.01335 + j0.0000275} \Rightarrow \underline{Z_{\text{in,ctr}} = 74.896 - j0.1542\,\Omega}$$

$$Z_{\text{in,high}} = \frac{1}{Y_{\text{in,high}}} = \frac{1}{0.01423 + j0.0017} \Rightarrow \underline{Z_{\text{in,high}} = 69.282 - j8.258\,\Omega}$$

Calculate the input reflection coefficient after matching for each frequency.

$$\Gamma_{\text{in},f} = \frac{Z_{\text{in},f} - Z_0}{Z_{\text{in},f} + Z_0} = \frac{Z_{\text{in},f} - 75}{Z_{\text{in},f} + 75}$$

$$\Gamma_{\text{in,low}} = \frac{(78.174 + j10.83) - 75}{(78.174 + j10.83) + 75} \Rightarrow \underline{\Gamma_{\text{in,low}} = 0.0735\angle 69.62^\circ}$$

$$\Gamma_{\text{in,ctr}} = \frac{(74.896 - j0.1542) - 75}{(74.896 - j0.1542) + 75} \Rightarrow \underline{\Gamma_{\text{in,ctr}} = 0.00124\angle -123.97^\circ}$$

$$\Gamma_{\text{in,high}} = \frac{(69.282 - j8.258) - 75}{(69.282 - j8.258) + 75} \Rightarrow \underline{\Gamma_{\text{in,high}} = 0.0695\angle -121.43^\circ}$$

Calculate the VSWR after matching for each frequency.

$$\text{VSWR}_{\text{low}} = \frac{1 + |\Gamma_{\text{low}}|}{1 - |\Gamma_{\text{low}}|} = \frac{1 + 0.0735}{1 - 0.0735} \Rightarrow \underline{\text{VSWR}_{\text{low}} = 1.15865}$$

$$\text{VSWR}_{\text{ctr}} = \frac{1 + |\Gamma_{\text{ctr}}|}{1 - |\Gamma_{\text{ctr}}|} = \frac{1 + 0.00124}{1 - 0.00124} \Rightarrow \underline{\text{VSWR}_{\text{ctr}} = 1.00249}$$

$$\text{VSWR}_{\text{high}} = \frac{1 + |\Gamma_{\text{high}}|}{1 - |\Gamma_{\text{high}}|} = \frac{1 + 0.0695}{1 - 0.0695} \Rightarrow \underline{\text{VSWR}_{\text{high}} = 1.14939}$$

Table 1 Summary of specified results

f **(MHz)**	***Z*in (Ω)**	**Γ**	**VSWR**
95	**78.174 + *j*10.830**	**0.0735∠69.62°**	**1.1586**
100	**74.896 - *j* 0.154**	**0.0012∠-123.97°**	**1.0025**
105	**69.282 – *j*8.258**	**0.0695∠-121.43°**	**1.1494**