A PEC folded dipole used for FM radio reception has length ℓ = 1.46 m, wire spacing s = 2 cm, and wire diameter $2a$ = 1.29 mm. Find the characteristic impedance of transmission line mode and equivalent radius of the antenna mode. Then, at 88, 97.5, and 108 MHz, find: a) the length of the folded dipole in wavelengths (ℓ/λ), b) the input impedance of transmission line mode, c) the input impedance of antenna mode using NEC-2 [include input file(s) & excerpt of output file(s)], d) the exact & estimated input impedance, and e) the VSWR on a 300 Ω feeding transmission line. Show all work.

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characteristic impedance of transmission line (TL) mode-

$$Z_{0t} = \eta/\pi \cosh^{-1}(s/2a) = 376.7303/\pi \cosh^{-1}(0.02/0.00129) \Rightarrow \underline{\mathbf{Z_{0t} = 411.6984\ \Omega}}$$

equivalent radius of the antenna mode-

$$a_e = \sqrt{(a\,s)} = \sqrt{(0.000645 \cdot 0.02)} \Rightarrow \underline{\mathbf{a_e = 0.00359166\ m = 3.59166\ mm}}$$

a) length of the folded dipole in wavelengths-

$$\lambda_{low} = c/f_{low} = 2.998\times10^8/88\times10^6 = 3.40682\ m.$$

$$\text{So, } \ell/\lambda_{low} = 1.46/3.40682 \Rightarrow \underline{\mathbf{\ell/\lambda_{low} = 0.42855}}$$

$$\lambda_{ctr} = c/f_{ctr} = 2.998\times10^8/97.5\times10^6 = 3.07487\ m.$$

$$\text{So, } \ell/\lambda_{ctr} = 1.46/3.07487 \Rightarrow \underline{\mathbf{\ell/\lambda_{ctr} = 0.47482}}$$

$$\lambda_{high} = c/f_{high} = 2.998\times10^8/108\times10^6 = 2.77593\ m.$$

$$\text{So, } \ell/\lambda_{high} = 1.46/2.77593 \Rightarrow \underline{\mathbf{\ell/\lambda_{high} = 0.52595}}$$

b) input impedance of TL mode- $Z_{int} = j\,Z_{0t}\tan(k\,\ell/2) = j\,Z_{0t}\tan(\pi\,\ell/\lambda)$

$$Z_{int\_low} = j\,Z_{0t}\tan(\pi\ell/\lambda_{low}) = j\,411.6984\tan(\pi\,0.42855) \Rightarrow \underline{\mathbf{Z_{int\_low} = j\,1803.27\ \Omega}}$$

$$Z_{int\_ctr} = j\,Z_{0t}\tan(\pi\ell/\lambda_{ctr}) = j\,411.6984\tan(\pi\,0.47482) \Rightarrow \underline{\mathbf{Z_{int\_ctr} = j\,5192.86\ \Omega}}$$

$$Z_{int\_high} = j\,Z_{0t}\tan(\pi\ell/\lambda_{high}) = j\,411.6984\tan(\pi\,0.52595) \Rightarrow \underline{\mathbf{Z_{int\_high} = -j\,5038.69\ \Omega}}$$

c) input impedance of antenna mode using NEC-2 (include input file(s) & excerpt of output file(s))

NEC Input File

```
CM EE 483/583 folded_dipole_L_1_46m_FM_in.txt
CM This file is used to determine antenna-mode input impedance
CM of a folded dipole when driven at 88, 97.5, & 108 MHz.
CM spacing s = 0.02 m, 2a = 1.29 mm
CM effective radius = ae = 0.003591657 m w/ length = l = 1.46 m
CM Used 51 segments (del/a ~ 8).  DRIVEN SEGMENT IS #26.
CE
GW 1 51 0.0 0.0 -0.73 0.0 0.0 0.73 0.003591657
GE 0
EK 0 ! Use extended kernel anyway
EX 0 1 26 00 1.0 0.0
PT -1 ! suppress currents
FR 0 1 0 0  88 0.0
XQ 0
FR 0 1 0 0  97.5 0.0
XQ 0
FR 0 1 0 0 108   0.0
XQ 0
EN
```

NEC Output File excerpt

```
<snip>
 - - - - COMMENTS - - - -
 EE483 folded_dipole_L_1_46m_FM_in.txt
 This file is used to determine antenna-mode input impedance of a
folded dipole when driven at 88, 97.5, & 108 MHz.
 spacing s = 0.02 m, 2a = 1.29 mm
 effective radius = ae = 0.003591657 m w/ length = l = 1.46 m
 Used 51 segments (del/a ~ 8).  DRIVEN SEGMENT IS #26.
<snip>
 - - - - - - FREQUENCY - - - - - -
FREQUENCY= 8.8000E+01 MHZ
WAVELENGTH= 3.4068E+00 METERS
<snip>
- - - ANTENNA INPUT PARAMETERS - - -
TAG SEG.   VOLTAGE (V)    CURRENT (AMPS)         IMPEDANCE (OHMS)  <snip>
NO. NO.  REAL     IMAG.   REAL        IMAG.     REAL         IMAG.    <snip>
1   26  1.00E+00 0.0E+00 5.565E-03 8.655E-03 5.25589E+01 -8.17464E+01  <snip>
<snip>
- - - - - - FREQUENCY - - - - - -
FREQUENCY= 9.7500E+01 MHZ
WAVELENGTH= 3.0749E+00 METERS
<snip>
```

```
- - - ANTENNA INPUT PARAMETERS - - -
TAG SEG. VOLTAGE (V)     CURRENT (AMPS)        IMPEDANCE (OHMS)  <snip>
NO. NO. REAL    IMAG.    REAL        IMAG.    REAL       IMAG.    <snip>
 1  26 1.00E+00 0.00E+00 1.372E-02-5.727E-04 7.27818E+01 3.03874E+00  <snip>
<snip>
- - - - - - FREQUENCY - - - - - -
FREQUENCY= 1.0800E+02 MHZ
WAVELENGTH= 2.7759E+00 METERS
<snip>
- - - ANTENNA INPUT PARAMETERS - - -
TAG SEG. VOLTAGE (V)     CURRENT (AMPS)        IMPEDANCE (OHMS)   <snip>
NO. NO. REAL    IMAG.    REAL        IMAG.    REAL       IMAG.     <snip>
1 26 1.00E+00 0.00E+00 5.138E-03-4.772E-03 1.04500E+02 9.70494E+01  <snip>
<snip>
```

In summary, input impedance of antenna mode from NEC-2

$$@\, f = 88 \text{ MHz}, \quad \Rightarrow \quad \underline{\boldsymbol{Z_{A,\text{low}} = 52.5589 - j81.7464\ \Omega}}$$

$$@\, f = 97.5 \text{ MHz}, \quad \Rightarrow \quad \underline{\boldsymbol{Z_{A,\text{ctr}} = 72.7818 + j\,3.03874\ \Omega}}$$

$$@\, f = 108 \text{ MHz}, \quad \Rightarrow \quad \underline{\boldsymbol{Z_{A,\text{high}} = 104.5 + j\,97.0494\ \Omega}}$$

d) the exact & estimated input impedance,

Equations: $Z_{\text{in,exact}} = 4\, Z_A\, Z_{\text{int}} / (2\, Z_A + Z_{\text{int}})$ & $Z_{\text{in,est}} \approx 4\, Z_A$

@ f = 88 MHz,

$$Z_{\text{in,exact,low}} = 4\,(52.5589 - j81.7464)(j1803.27)/[2\,(52.5589 - j81.7464) + j1803.27]$$

$$\Rightarrow \quad \underline{\boldsymbol{Z_{\text{in,exact,low}} = 253.2077 - j343.3556\ \Omega}}$$

$$Z_{\text{in,est,low}} = 4\,(52.5589 - j81.7464) \quad \Rightarrow \quad \underline{\boldsymbol{Z_{\text{in,est,low}} = 210.2356 - j326.9856\ \Omega}}$$

@ f = 97.5 MHz,

$$Z_{\text{in,exact,ctr}} = 4\,(72.7818 + j3.03874)(j5192.86)/[2\,(72.7818 + j3.03874) + j5192.86]$$

$$\Rightarrow \quad \underline{\boldsymbol{Z_{\text{in,exact,ctr}} = 290.2194 + j20.2665\ \Omega}}$$

$$Z_{\text{in,est,ctr}} = 4\,(72.7818 + j3.03874) \quad \Rightarrow \quad \underline{\boldsymbol{Z_{\text{in,est,ctr}} = 291.1272 + j12.155\ \Omega}}$$

@ f = 108 MHz,

$$Z_{\text{in,exact,high}} = 4\,(104.5 + j97.0494)(-j5038.69)/[2\,(104.5 + j97.0494) - j5038.69]$$

$$\Rightarrow \quad \underline{\boldsymbol{Z_{\text{in,exact,high}} = 451.3254 + j384.2802\ \Omega}}$$

$$Z_{\text{in,est,high}} = 4\,(104.5 + j97.0494) \Rightarrow \quad \underline{\boldsymbol{Z_{\text{in,est,high}} = 418 + j388.1976\ \Omega}}$$

e) the VSWR on a 300 Ω feeding transmission line.

$$\text{Equations: } \Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \quad \& \quad \text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

@ f = 88 MHz,

$$\Gamma_{low} = \frac{(253.2077 - j343.3556) - 300}{(253.2077 - j343.3556) + 300} = 0.53222\angle -65.934°$$

$$\text{VSWR}_{low} = \frac{1+0.53222}{1-0.53222} \quad \Rightarrow \quad \underline{\textbf{VSWR}_{\textbf{low}} = \textbf{3.276}}$$

@ f = 97.5 MHz,

$$\Gamma_{ctr} = \frac{(290.2194 + j20.2665) - 300}{(290.2194 + j20.2665) + 300} = 0.0381\angle 113.795°$$

$$\text{VSWR}_{low} = \frac{1+0.0381}{1-0.0381} \quad \Rightarrow \quad \underline{\textbf{VSWR}_{\textbf{ctr}} = \textbf{1.079}}$$

@ f = 108 MHz,

$$\Gamma_{high} = \frac{(451.3254 + j384.2802) - 300}{(451.3254 + j384.2802) + 300} = 0.4894\angle 41.418°$$

$$\text{VSWR}_{high} = \frac{1+0.4894}{1-0.4894} \quad \Rightarrow \quad \underline{\textbf{VSWR}_{\textbf{high}} = \textbf{2.917}}$$