For the rectangular microstrip antenna of part 1), compute and graph the E-plane and H plane normalized directivity patterns (both unitless and in dB ) with the positive $x$-axis pointing toward top of page. For the radiation patterns in dB, use a 0 to -40 dB scale. Also, find the actual HPBW in the E- and H-planes. In a table, list the estimated HPBWs, actual HPBWs, and percent differences (\%). How do they compare?

Design a rectangular microstrip antenna to operate at a frequency of 2 GHz on a Montoya Corporation substrate with a relative permittivity of 2.2 and dielectric thickness of 0.064 " $=64$ mils, 0.5 oz . copper cladding $(17 \mu \mathrm{~m})$, and $\tan (\delta)=0.003$. The antenna is to be matched to a $50 \Omega$ microstrip transmission line on this substrate using an inset feed. Discuss and justify design choices. Accurately sketch a top view of the final design (all dimensions in mm). EE 583 only- Include a fully-labeled Smith chart showing the normalized admittances $y_{1}=y_{2}$ and $y_{2 t}$ (i.e., $y_{2}$ translated across length $L+\Delta L$ of microstrip antenna) and discuss results.

## Summary of necessary dimensions \& parameters from design-

$h=0.064 \mathrm{in}(25.4 \mathrm{~mm} / \mathrm{in})=\mathbf{1 . 6 2 5 6} \mathbf{~ m m}, f_{\mathrm{r}}=2 \mathrm{GHz}$
Free space wavelength $\underline{\boldsymbol{\lambda}_{0}}=\mathbf{1 4 9 . 8 9 6 2 ~ \mathbf { m m }}$ and wave number $\underline{\boldsymbol{k}_{0}}=\mathbf{4 1 . 9 1 6 9 \mathbf { r a d } / \mathrm { m }}$.
Patch width $\Rightarrow W=\mathbf{5 9 . 2 5 1 7} \mathrm{mm}$
effective length of patch $\Rightarrow \underline{L}_{\text {eff }}=\mathbf{5 1 . 4 6 9 5} \mathbf{~ m m}$
Patch length $\Rightarrow \underline{L}=49.7537 \mathrm{~mm}$
Slot conductance $\Rightarrow \underline{G_{1}}=\mathbf{1 . 5 7 3 5} \mathbf{~ m S}$.
mutual conductance between the slots $\Rightarrow \underline{G}_{12}=\mathbf{0 . 4 6 5 1} \mathrm{mS}$

Per (14-43), the total (far-field) electric field for the two radiating slots is-

$$
E_{\phi}^{t}=j \frac{k_{0} h W E_{0} e^{-j k_{0} r}}{\pi r}\left\{\sin \theta \frac{\sin (X)}{X} \frac{\sin (Z)}{Z}\right\} \cos \left(\frac{k_{0} L_{\text {eff }}}{2} \sin \theta \sin \phi\right)
$$

where $X(\theta, \phi)=\frac{k_{0} h}{2} \sin \theta \sin \phi(14-43 \mathrm{a})$ and $Z(\theta)=\frac{k_{0} W}{2} \cos \theta(14-43 \mathrm{~b})$.
Normalize by dividing out lead terms-

$$
E_{\phi, n}^{t}=E_{\phi}^{t} /\left(j \frac{k_{0} h W E_{0} e^{-j k_{0} r}}{\pi r}\right)=\sin \theta \frac{\sin (X)}{X} \frac{\sin (Z)}{Z} \cos \left(\frac{k_{0} L_{\mathrm{eff}}}{2} \sin \theta \sin \phi\right) .
$$

The E-plane is on the $x-y$ plane where $\theta=90^{\circ}$ and $0 \leq \phi \leq 90^{\circ} \& 270^{\circ} \leq \phi<360^{\circ}$. Note, to 'trick' MathCad, let $-90^{\circ} \leq \phi \leq 90^{\circ}$.

The H-plane is on the $x-z$ plane where $\phi=0$ and $0^{\circ} \leq \theta \leq 180^{\circ}$.
MathCad \& CorelDraw were used to produce the following plots.

E-Plane Radiation Patterns $\left(x-y\right.$ plane where $\theta=90^{\circ}$ and $\left.0 \leq \phi \leq 90^{\circ} \& 270^{\circ} \leq \phi<360^{\circ}\right)$


The exact E-Plane half-power beamwidth $\mathbf{H P B W}_{\mathbf{E}}=\mathbf{9 3 . 4 3 7 6}^{\circ}$ whereas the estimate from problem 2) was $\underline{H P B W}_{\mathbf{E}, \text { est }}=\mathbf{9 0 . 3 7 6 1 ^ { \circ }}$. Pretty good agreement.

H-Plane Radiation Patterns $\left(x-z\right.$ plane where $\phi=0$ and $\left.0^{\circ} \leq \theta \leq 180^{\circ}\right)$


The exact H-Plane half-power beamwidth is $\mathbf{H P B W}_{\mathbf{H}}=\mathbf{7 6 . 9 6 3 8}^{\circ}$ whereas the estimate from problem 2) was $\underline{\mathbf{H P B}}_{\mathbf{H}, \text { est }}=\mathbf{5 6 . 3 6 2 6}^{\circ}$. Not very good agreement.

## MathCad excerpt

Estimated HPBWs for this microstrip patch antenna using (14-58) \& (14-59)

E-plane radiation pattern ( $x-y$ plane where $\theta=\pi / 2=90 \mathrm{deg}$, symmetric around $\phi=0$ )

$$
\text { Eepln }_{\mathrm{n}}:=\mathrm{E}\left(\frac{\pi}{2}, \phi_{\mathrm{n}}\right) \quad \text { Eepln }_{-} \mathrm{dB}_{\mathrm{n}}:=20 \cdot \log \left(\text { Eepln }_{\mathrm{n}}\right)
$$

Find E-plane half power $|\mathrm{E}|=0.707107$ points:

$$
\begin{aligned}
& \mathrm{E}\left(\frac{\pi}{2}, 46.7188 \cdot \frac{\pi}{180}\right)=0.707107 \quad \mathrm{E}\left(\frac{\pi}{2},-46.7188 \cdot \frac{\pi}{180}\right)=0.707107 \\
& \text { HPBWepln }:=2 \cdot 46.7188 \quad \text { HPBWepln }=93.4376 \text { deg } \\
& \text { Fair agreement with part 2) estimate of } \Theta \mathrm{E}=90.3761 \text { degrees. }
\end{aligned}
$$

H-plane radiation pattern ( $y-z$ plane where $\phi=0$ or $\pi$, symmetric around $\theta=90 \mathrm{deg}$ )
$\operatorname{Ehpln}_{\mathrm{n}}:=\mathrm{E}\left(\theta_{\mathrm{n}}, 0\right) \quad \quad \operatorname{Ehpln} \operatorname{ldB}_{\mathrm{n}}:=\operatorname{if}\left(\operatorname{Ehpln}_{\mathrm{n}} \leq 0.01,-40,20 \cdot \log \left(\operatorname{Ehpln}_{\mathrm{n}}\right)\right)$
Find H-plane half power $|\mathbb{E}|=0.707107$ points on either side of 90 deg:

$$
\mathrm{E}\left(128.4819 \cdot \frac{\pi}{180}, 0\right)=0.707107 \quad \mathrm{E}\left(51.5181 \cdot \frac{\pi}{180}, 0\right)=0.707107
$$

$$
\text { HPBWhpln }:=128.4819-51.5181 \quad \text { HPBWhpln }=76.9638 \quad \text { deg }
$$

Not very good agreement with part 2) estimate of $\Theta H=56.3626$ degrees.

Ehalf_pwr $\mathrm{m}_{\mathrm{n}}:=0.5 \cdot \sqrt{2}$
Ehalf_dB $\mathrm{n}_{\mathrm{n}}:=20 \cdot \log (0.5 \cdot \sqrt{2})$

$$
\begin{aligned}
& \Theta \mathrm{E}:=2 \cdot \operatorname{asin}\left[\sqrt{\left.\frac{7.03 \cdot \lambda 0^{2}}{4 \cdot \pi^{2} \cdot(3 \cdot \mathrm{Leff}}{ }^{2}+\mathrm{h}^{2}\right)}\right] \\
& \Theta \mathrm{E} \cdot \frac{180}{\pi}=90.376 \quad \text { degrees } \\
& \Theta \mathrm{H}:=2 \cdot \mathrm{a} \sin \left[\sqrt{\frac{1}{(2+\mathrm{k} 0 \cdot \mathrm{~W})}}\right] \\
& \Theta H \cdot \frac{180}{\pi}=56.363 \\
& \mathrm{X}(\theta, \phi):=\frac{\mathrm{k} 0 \cdot \mathrm{~h}}{2} \cdot \sin (\theta) \cdot \cos (\phi) \quad \mathrm{Z}(\theta, \phi):=\frac{\mathrm{k} 0 \cdot \mathrm{~W}}{2} \cdot \cos (\theta) \\
& \mathrm{E}(\theta, \phi):=\sin (\theta) \cdot \frac{\sin (\mathrm{X}(\theta, \phi))}{\mathrm{X}(\theta, \phi)} \cdot \frac{\sin (\mathrm{Z}(\theta, \phi))}{\mathrm{Z}(\theta, \phi)} \cdot \cos \left[\frac{\mathrm{k} 0 \cdot \text { Leff }}{2} \cdot(\sin (\theta) \cdot \sin (\phi))\right] \\
& \mathrm{n}:=0 . .720 \quad \phi_{\mathrm{n}}:=(\mathrm{n}-360) \cdot \frac{\pi}{720} \quad \theta_{\mathrm{n}}:=\mathrm{n} \cdot \frac{\pi}{720}
\end{aligned}
$$

