Design a rectangular microstrip antenna to operate at a frequency of 2 GHz on a Montoya Corporation substrate with a relative permittivity of 2.2 and dielectric thickness of 0.064" = 64 mils, 0.5 oz. copper cladding (17 μm), and tan(δ) = 0.003. The antenna is to be matched to a 50 Ω microstrip transmission line on this substrate using an inset feed. Discuss and justify design choices. Accurately sketch a top view of the final design (all dimensions in mm). **EE 583 only**- Include a fully-labeled Smith chart showing the normalized admittances $y_1 = y_2$ and y_{2t} (i.e., y_2 translated across length $L + \Delta L$ of microstrip antenna) and discuss results.

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Design Procedure (All calculations done using to full precision of/on MathCad)

1) Specify ε_r and h of substrate, the desired resonant frequency f_r, and the impedance $Z_{c,\text{feed}}$ of the feeding transmission line.

$$\boldsymbol{\varepsilon_r = 2.2}, \quad \boldsymbol{h = 0.064\ \text{in}}(25.4\ \text{mm/in}) = \boldsymbol{1.6256\ \text{mm}}, \quad \boldsymbol{f_r = 2\ \text{GHz}}, \quad \& \quad \boldsymbol{Z_{c,\text{feed}} = 50\ \Omega}$$

2) Calculate width of patch using (14-6)

$$W = \frac{c}{2f_r}\sqrt{\frac{2}{\varepsilon_r + 1}} = \frac{2.9979\times10^8}{2(2\times10^9)}\sqrt{\frac{2}{2.2+1}} \quad \Rightarrow \quad \underline{\boldsymbol{W = 59.2517\ \text{mm}}}$$

3) Calculate effective relative permittivity using (14-1)

$$\varepsilon_{\text{r,eff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^{-0.5} = \frac{2.2+1}{2} + \frac{2.2-1}{2}\left[1 + 12\frac{1.6256}{59.2517}\right]^{-0.5} \quad \Rightarrow \quad \underline{\boldsymbol{\varepsilon_{\text{r,eff}} = 2.12042}}$$

4) Calculate fringing length using (14-2)

$$\Delta L = 0.412h\frac{\left(\varepsilon_{\text{r,eff}} + 0.3\right)\left(\frac{W}{h} + 0.264\right)}{\left(\varepsilon_{\text{r,eff}} - 0.258\right)\left(\frac{W}{h} + 0.8\right)} = 0.412(1.6256)\frac{\left(2.1204 + 0.3\right)\left(\frac{59.252}{1.6256} + 0.264\right)}{\left(2.1204 - 0.258\right)\left(\frac{59.252}{1.6256} + 0.8\right)}$$

$$\Rightarrow \quad \underline{\boldsymbol{\Delta L = 0.8579\ \text{mm}}}$$

5) Calculate the effective length (14-5) and guided wavelength λ.

$$L_{\text{eff}} = \frac{c}{2f_r\sqrt{\varepsilon_{\text{r,eff}}}} = \frac{2.9979\times10^8}{2(2\times10^9)\sqrt{2.1204}} \quad \Rightarrow \quad \underline{\boldsymbol{L_{\text{eff}} = 51.4695\ \text{mm}}}$$

and

$$\lambda = 2L_{\text{eff}} = 2(51.4695) \quad \Rightarrow \quad \underline{\boldsymbol{\lambda = 102.9390\ \text{mm}}}$$

6) Calculate the patch length L (14-7) and L/λ. Then, calculate and check that $W/L < 2$.

$L = L_{eff} - 2\Delta L = 51.4695 - 2(0.8579) \Rightarrow$ **$L = 49.7537$ mm**

$L/\lambda = 51.4695/102.939 \Rightarrow$ **$L/\lambda = 0.48333$**

$W/L = 59.2517/49.7537 \Rightarrow$ **$W/L = 1.191$ (should be OK as it's < 2)**

7) Calculate $G_1 \approx G_2$ and $B_1 \approx B_2$.

$$G_{1,\text{est}} = \frac{W}{120\lambda_0}\left[1 - \frac{1}{24}(k_0 h)^2\right] \quad \text{for } \frac{h}{\lambda_0} < \frac{1}{10} \quad (14-8a)$$

$$B_{1,\text{est}} = \frac{W}{120\lambda_0}\left[1 - 0.636\ln(k_0 h)\right] \quad \text{for } \frac{h}{\lambda_0} < \frac{1}{10} \quad (14-8b)$$

where

$\lambda_0 = c/f_r = 2.9979\times10^8/\,2\times10^9 \Rightarrow \lambda_0 = 149.8962$ mm,

$h/\lambda_0 = 1.6256/149.8962 \Rightarrow h/\lambda_0 = 0.0065$,

and $k_0 = 2\pi/\lambda_0 = 2\pi/0.14989 \Rightarrow k_0 = 41.9169$ rad/m,

$$G_{1,\text{est}} = \frac{59.2517}{120(149.896)}\left[1 - \frac{1}{24}\left(41.9169 * 1.6256 * 10^{-3}\right)^2\right] \Rightarrow \underline{G_{1,\text{est}} = 3.2934 \text{ mS}}$$

$$B_{1,\text{est}} = \frac{59.2517}{120(149.896)}\left[1 - 0.636\ln\left(41.9169 * 1.6256 * 10^{-3}\right)\right] \Rightarrow \underline{B_{1,\text{est}} = 8.9216 \text{ mS}}$$

numerically integrate (notes & 14-12a)-

$$G_1 = \frac{1}{\pi\eta_0}\int_{\theta=0}^{\pi}\left[\frac{\sin\left(\frac{k_0 W}{2}\cos\theta\right)}{\cos\theta}\right]^2 \sin^3\theta\, d\theta$$

$$= \frac{1}{\pi(376.73)}\int_{\theta=0}^{\pi}\left[\frac{\sin\left(\frac{41.917(0.05925)}{2}\cos\theta\right)}{\cos\theta}\right]^2 \sin^3\theta\, d\theta \Rightarrow \mathbf{\underline{G_1 = 1.5735 \text{ mS}}}$$

$$\text{(notes)}\ B_1 = \left(\frac{G_1}{G_{1,\text{est}}}\right)B_{1,\text{est}} = \left(\frac{1.5735}{3.2934}\right)8.9216 \Rightarrow \mathbf{\underline{B_1 = 4.2626 \text{ mS}}}.$$

Therefore, $Y_1 = Y_2 = G_1 + j\,B_1 \Rightarrow$ **$\underline{Y_1 = Y_2 = 1.5735 + j\,4.2626 \text{ mS}}$**

8) Calculate the TL characteristic impedance $Z_{c,\mathrm{ant}}$ (14-19a) and admittance $Y_{c,\mathrm{ant}}$ for the rectangular microstrip antenna.

$$Z_{c,\mathrm{ant}} = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{\mathrm{r,eff}}}} \ln\left[\dfrac{8h}{W} + \dfrac{W}{4h}\right] & \dfrac{W}{h} \le 1 \\ \dfrac{\eta_0}{\sqrt{\varepsilon_{\mathrm{r,eff}}}\left[\dfrac{W}{h} + 1.393 + 0.667 \ln\left(\dfrac{W}{h} + 1.444\right)\right]} & \dfrac{W}{h} > 1 \end{cases}$$

here W/h = 59.2517/1.6256 = 36.4491 and

$$Z_{c,\mathrm{ant}} = \frac{376.73}{\sqrt{2.1402}\left[36.449 + 1.393 + 0.667 \ln(36.449 + 1.444)\right]} \quad \Rightarrow \quad \underline{\mathbf{Z_{c,ant} = 6.4251\ \Omega}}$$

and

$$Y_{c,\mathrm{ant}} = \frac{1}{Z_{c,\mathrm{ant}}} = \frac{1}{6.4251} \quad \Rightarrow \quad \underline{\mathbf{Y_{c,ant} = 0.1556\ S}}$$

9) (EE 583) Use Smith chart or direct calculation to verify $\tilde{Y}_2 = \tilde{G}_2 + j\tilde{B}_2 \approx G_1 - jB_1 = Y_1^*$.

On Smith chart (next page), plot $y_2 = Y_2/Y_{c,\mathrm{ant}} = (1.5735 + j\,4.2626)10^{-3}(6.4251)$

$\Rightarrow$ $\underline{y_2 = 0.0101 + j0.0274 \text{ S/S}}$.

Move $(L+\Delta L)/\lambda$ =(49.7537+0.8579)/102.939 = 0.4917 WAVELENGTHS TOWARD GENERATOR on arc of constant $|\Gamma|$.

On Smith chart, read off $\Rightarrow$ $\underline{y_{2t} = 0.0101 - j0.0245 \text{ S/S}}$.

We approximately (~10.6% error for susceptance) meet the condition that-

$\underline{y_{2t} = 0.0101 - j0.0245 \text{ S/S} \approx y_1^* = 0.0101 - j0.0274 \text{ S/S}}$

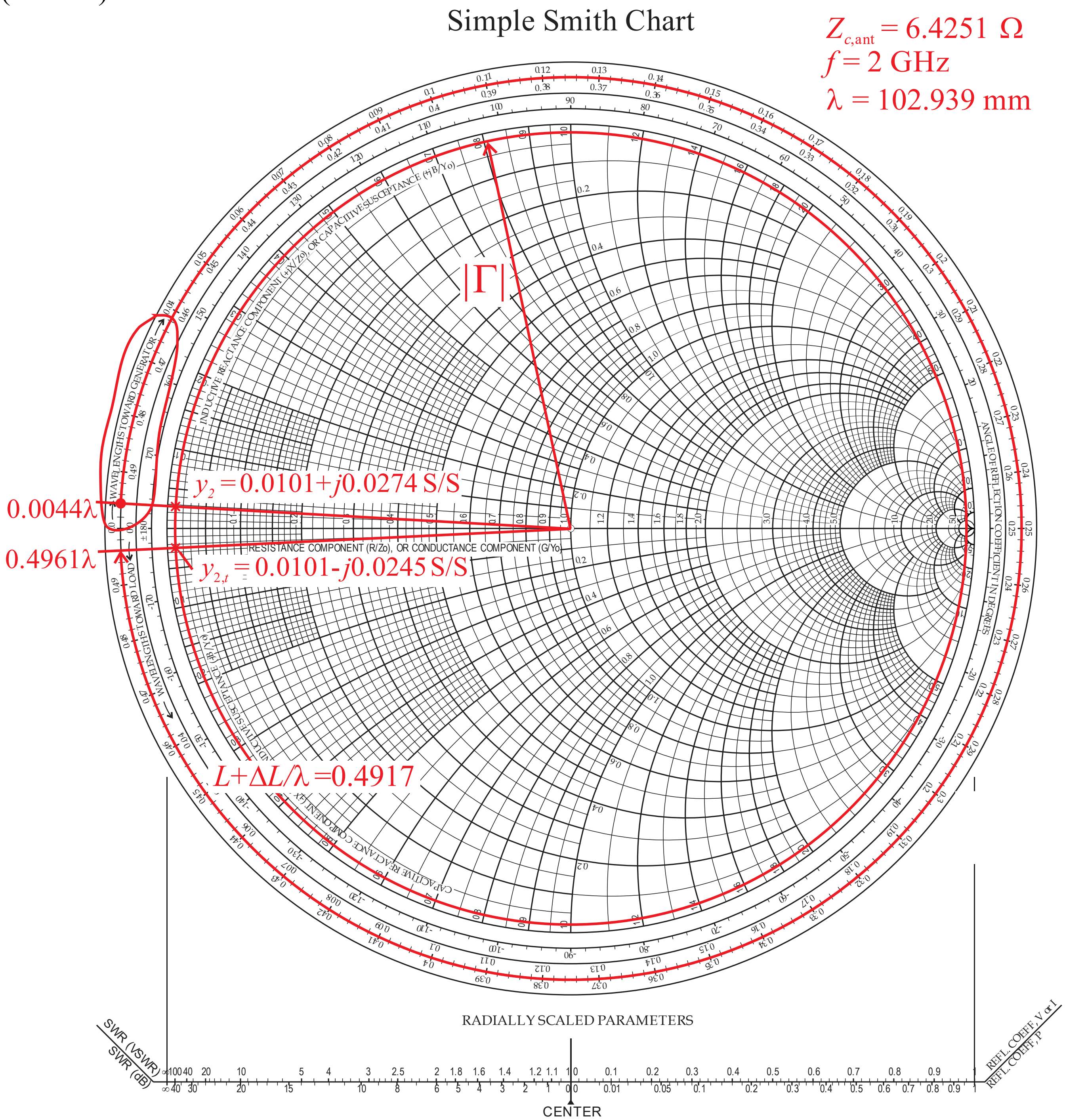
10) Numerically calculate the mutual conductance G_{12} (14-18a) between the slots

$$G_{12} = \frac{1}{\pi \eta_0} \int_{\theta=0}^{\pi} \left[\frac{\sin\left(\dfrac{k_0 W}{2}\cos\theta\right)}{\cos\theta}\right]^2 J_0(k_0 L \sin\theta) \sin^3\theta \, d\theta$$

$$= \frac{1}{\pi(376.73)} \int_{\theta=0}^{\pi} \left[\frac{\sin\left(\dfrac{41.92(0.05925)}{2}\cos\theta\right)}{\cos\theta}\right]^2 J_0(41.92(0.04975)\sin\theta) \sin^3\theta \, d\theta$$

$\Rightarrow$ $\underline{\mathbf{G_{12} = 0.46513\ mS}}$

(EE 583)



11) Calculate R_{in}, use plus (+) sign in (14-17) equation.

$$Z_{\text{in}} = R_{\text{in}} = \frac{1}{2(G_1 + G_{12})} = \frac{1}{2(1.5735 + 0.46513)10^{-3}} \quad \Rightarrow \quad \underline{\mathbf{Z_{in} = R_{in} = 245.2603\ \Omega}}$$

12) Since an inset microstrip feed is required (i.e., $R_{in} \neq Z_{c,feed}$), calculate length y_0 of the inset needed to match the rectangular patch to the feeding transmission line. When $G_1/Y_{c,feed} << 1$ and $B_1/Y_{c,feed} << 1$, a good starting point is

$$y_0 \approx \frac{L}{\pi}\cos^{-1}\left(\sqrt{\frac{Z_{c,feed}}{R_{in}}}\right) \quad \text{(14-20a).}$$

Here, $G_1/Y_{c,feed} = 0.0015735/0.02 = 0.0787 << 1$ (OK),

but $B_1/Y_{c,feed} = 0.0042626/0.02 = 0.213 << 1$ (so-so).

So, we will estimate $y_{0,est} \approx \dfrac{0.04975}{\pi}\cos^{-1}\left(\sqrt{\dfrac{50}{245.26}}\right)$ $\Rightarrow$ **$y_{0,est}$ = 17.458 mm**

Check this estimate using (14-20a)-

$$R_{in}(y_0) \simeq \frac{1}{2(G_1+G_{12})}\left[\cos^2\left(\frac{\pi y_0}{L}\right) + \frac{G_1^2+B_1^2}{Y_{c,feed}^2}\sin^2\left(\frac{\pi y_0}{L}\right) - \frac{B_1}{Y_{c,feed}}\sin\left(\frac{2\pi y_0}{L}\right)\right]$$

$$\simeq 245.26\left[\cos^2\left(\frac{\pi y_0}{0.0498}\right) + \frac{0.00157^2+0.00423^2}{0.02^2}\sin^2\left(\frac{\pi y_0}{0.0498}\right) - \frac{0.00423}{0.02}\sin\left(\frac{2\pi y_0}{0.0498}\right)\right]$$

$\Rightarrow$ **$R_{in}(y_{0,est}$ = 17.458 mm) = 17.961 Ω**

Since $R_{in}(y_{0,est})$ is too low, iteratively adjust inset length y_0 using above equation-

$\Rightarrow R_{in}$ (**y_0 = 14.3816 mm**) = 50 Ω = $Z_{c,feed}$.

13) Determine the width W_0 of the feeding microstrip transmission line.

Method 2: Iteratively guess the width W_0 of the feeding transmission line using-

$$\varepsilon_{r,eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[1+12\frac{h}{W_0}\right]^{-0.5} \quad \text{(14-1)}$$

and (14-19b)

$$Z_{c,feed} = \frac{\eta_0}{\sqrt{\varepsilon_{r,eff}}\left[\frac{W_0}{h}+1.393+0.667\ln\left(\frac{W_0}{h}+1.444\right)\right]} \qquad \frac{W_0}{h} > 1.$$

to get $Z_{c,feed}$ = 50 Ω and $\varepsilon_{r,eff}$ = 1.872, when $W_0 = 3.10368\,h$ $\Rightarrow$ **W_0 = 5.0453 mm**.

14) Select the notch width n in the range $0.2W_0 < n < 0.5W_0$ (notes).

Arbitrarily select $n \approx 0.2W_0$ $\Rightarrow$ **n = 1 mm**.

15) Draw the resulting design.